Ambegaokar-Baratoff–Ginzburg-Landau crossover effects on the critical current density of granular superconductors

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The critical current density of a granular superconductor, modeled as an array of Josephson-coupled grains, is calculated using a Ginzburg-Landau approach that accounts for suppression of the superconducting gap parameter in the grains by supercurrent. For a wide range of experimental parameters, the critical current density versus temperature is found to have an Ambegaokar-Baratoff dependence at low temperatures but to exhibit a crossover to a Ginzburg-Landau $(1-T/T_c)^{3/2}$ dependence near T_c, the crossover occurring at the temperature for which the Josephson coupling energy of a junction is approximately equal to the superconducting condensation energy of a grain. Experimental results displaying this behavior are reported for a NbN film.

I. INTRODUCTION

Certain granular superconductors, such as NbN, have high values of both the critical current density J_c and the normal-state resistivity ρ_n, in ranges desirable for device applications.[1–8] To identify the most important specimen parameters and to guide the preparation of materials with optimized J_c and ρ_n, Kampwirth and Gray[4] introduced a model by which such materials are represented as arrays of Josephson-coupled superconducting grains. This model, however, neglects the current-induced suppression of the order parameter and thus leads to an incorrect temperature dependence of the critical current density near T_c; the model yields a proportionality to $(1-T/T_c)$, rather than to $(1-T/T_c)^{3/2}$, a behavior which is expected from the Ginzburg-Landau theory and is closer to that seen experimentally.[4] The purpose of this paper, therefore, is to incorporate the original Josephson-coupling model[4] into a Ginzburg-Landau[9–13] theory of granular superconductors and to use this theory to derive a critical-current-density expression that is valid over a wide range of temperatures.

In Sec. II it is shown that our Ginzburg-Landau approach yields an expression for J_c that for a wide range of experimental parameters has the Ambegaokar-Baratoff[14] temperature dependence, characteristic of Josephson tunneling, at low temperatures and the usual Ginzburg-Landau[15] $(1-T/T_c)^{3/2}$ temperature dependence near T_c. It also will be shown that the crossover between the two forms occurs at that temperature where the Josephson-coupling energy of a junction is approximately equal to the superconducting condensation energy of a grain. In Sec. III we use the theory to analyze the temperature dependence of the critical current of a 225-Å thick granular NbN film, and in Sec. IV we summarize our findings.

II. GINZBURG-LANDAU THEORY

Our starting point is the Josephson-coupled grain model of Ref. 4, which assumes that the grains are arranged on a cubic lattice with lattice parameter a_0 and that the junctions between adjacent grains are identical. For the case of a two-dimensional (2D) array of grains of average thickness $d\,(d<a_0)$, the grains may be assumed to lie on a square lattice with lattice parameter a_0. In such a model, the critical current density due to tunneling between grains is $J_c=J_0=I_0/A$, where I_0 is the Ambegaokar-Baratoff[14] expression for the maximum dc Josephson current,

$$I_0(T)=[\pi\Delta(T)/2eR_n]\tanh[\Delta(T)/2k_BT]\ , \tag{1}$$

and $A=a_0^2$ [three-dimensional (3D)] or $A=a_0d$ (2D) is the cross section of a grain. Here $\Delta(T)$ is the temperature-dependent gap parameter and R_n is the normal-state tunneling resistance of a junction. The effective normal-state resistivity, measured using a sample volume much larger than the grain volume $V=a_0^3$ (3D) or $V=a_0^2d$ (2D), is then $\rho_n=R_na_0$ (3D) or $\rho_n=R_nd$ (2D), such that at low temperatures $\rho_nJ_c=\pi\Delta(0)/2ea_0$. Close to the superconducting transition temperature T_c, where I_0 is proportional to Δ^2, this model predicts that J_c is proportional to $(1-T/T_c)$.

The above model does *not* account for the ability of the supercurrent to suppress the gap parameter. To correct this deficiency, we use a Ginzburg-Landau[9–13] approach to account for current-induced gap suppression. When the array carries supercurrent at density $J=I/A$ along one of the symmetry directions, the Gibbs free energy per grain near T_c is

$$\Delta G=(H_c^2/4\pi)V(-f^2+f^4/2)+(\hbar/2e)I_0f^2(1-\cos\phi)-(\hbar/2e)I\phi\ , \tag{2}$$

where H_c is the bulk thermodynamic critical field, ϕ is the gauge-invariant phase difference across a current-carrying junction, and I_0 is the Ambegaokar-Baratoff[14] critical current near T_c,

$$I_0(T)=\frac{\pi\Delta^2(T)}{4eR_nk_BT_c}\ . \tag{3}$$

The factor f in Eq. (2) is the fraction by which both the gap parameter and the order parameter are reduced in the presence of the supercurrent,

$$I = I_0 f^2 \sin\phi \ . \tag{4}$$

Note that in Eq. (3) Δ is the value of the gap parameter *before* it is suppressed by the current.

Minimization of ΔG with respect to f or ϕ at constant I yields the condition

$$f^2 = 1 - \epsilon(1-\cos\phi) \ , \tag{5}$$

where $\epsilon = E_J/2E_s$ is the ratio of the Josephson-coupling energy of a junction, $E_j = (\hbar/2e)I_0$, to twice the superconducting condensation energy of a grain, $E_s = (H_c^2/8\pi)V$. Substituting Eq. (5) into Eq. (4), we find that the value of ϕ that maximizes I is given by

$$\cos\phi_m = \frac{(1-2\epsilon+9\epsilon^2)^{1/2} - 1 + \epsilon}{4\epsilon} \ . \tag{6}$$

The corresponding critical current density is thus given by $J_c(T) = J_0(T) g(\epsilon)$, where $g = f_m^2 \sin\phi_m$ (the subscript m indicating evaluation at $\phi = \phi_m$) is the factor by which gap suppression reduces the net critical current J_c below the Ambegaokar-Baratoff value J_0. Shown in Fig. 1 is a plot of g versus ϵ.

The normalized Josephson-coupling energy ϵ can be written with the help of Eq. (1) and the Bardeen, Cooper, Schrieffer (BCS) relations[16,17] $H_c^2(0)/4\pi = N(0)\Delta^2(0)$, $\gamma = (2\pi^2/3)N(0)k_B^2$, and $2\Delta(0) = 3.53 k_B T_c$ as

$$\epsilon(T) = \epsilon_0 \frac{[\Delta(T)/\Delta(0)]\tanh[\Delta(T)/2k_B T_c]}{[H_c(T)/H_c(0)]^2} \ , \tag{7}$$

where

$$\epsilon_0 = \epsilon(0) = 2.93 \frac{\hbar k_B}{e^2 \gamma T_c \rho_n a_0^2} \ . \tag{8}$$

To see the underlying physics, however, it is helpful to recognize that, aside from numerical factors of order unity, $\epsilon(T)$ is simply the ratio $\xi^2(T)/a_0^2$ where $\xi(T)$ is the temperature-dependent coherence length and a_0 is the effective grain diameter. We show later that $\epsilon(T) = 2\xi^2(T)/a_0^2$ near T_c. Evaluating Eq. (8) for two specific examples, we obtain $\epsilon_0 = 0.16$ for granular NbN with (see Sec. III) $\gamma = 1.3\times10^3$ erg cm^{-3} K^{-2}, $T_c = 11.9$ K, $\rho_n = 145$ $\mu\Omega$cm, and $a_0 = 219$ Å; and $\epsilon_0 = 160$ for granular Al with[18,19] $\gamma = 1.35\times10^3$ erg cm^{-3} K^{-2}, $T_c = 1.49$ K, $\rho_n = 8.3$ $\mu\Omega$cm, and $a_0 = 79$ Å.

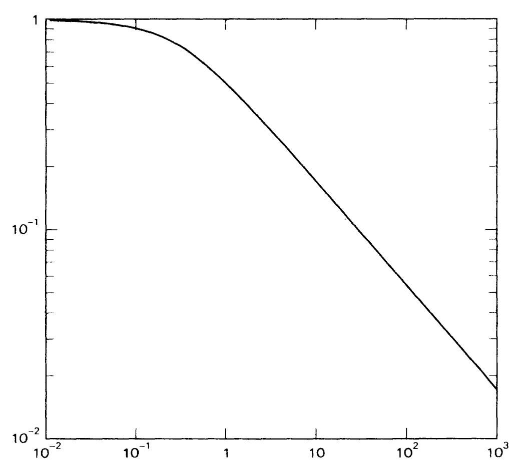

FIG. 1. Critical-current suppression factor $g = f_m^2 \sin\phi_m$ [Eqs. (5) and (6)] vs the normalized Josephson-coupling energy ϵ.

Equation (7) can be evaluated at arbitrary reduced temperature T/T_c with the help of the BCS functions tabulated in Ref. 20; $\epsilon(T)$ is a monotonically increasing function of T, diverging as $\epsilon(T) = 0.882\epsilon_0(1-T/T_c)^{-1}$ near T_c. Thus, when $\epsilon_0 \ll 1$, the crossover temperature at which $\epsilon(T) = 1$ is $T_x = T_c(1-0.882\epsilon_0)$.

When $\epsilon \gg 1$, the critical current is reached at sufficiently small values of ϕ that $\sin\phi \simeq \phi$ and $(1-\cos\phi) \simeq \phi^2/2$, and we obtain $\phi_m \simeq (2/3\epsilon)^{1/2} \ll 1$, $f_m^2 \simeq 2/3$, and $g \simeq (2/3)^{3/2}\epsilon^{-1/2} \ll 1$; i.e., when the condensation energy of a grain is much smaller than the Josephson-coupling energy, gap suppression is severe, and the temperature dependence of the critical current density reduces to that of the Ginzburg-Landau theory in the dirty limit. To show this, we recall that the Ginzburg-Landau critical current density can be expressed as[15]

$$J_c(T) = \frac{cH_c(T)}{3\sqrt{6}\pi\lambda(T)} \ . \tag{9}$$

In the dirty limit the penetration depth λ can be expressed as[21]

$$\lambda(T) = \left[\frac{c^2 \hbar k_B T_c \rho_n}{2\pi^2 \Delta^2(T)} \right]^{1/2} \ . \tag{10}$$

The present theory [Eqs. (4)–(6)] also yields J_c in the form of Eq. (9), except that playing the role of the penetration depth is the quantity[22]

$$\lambda(T) = (\hbar c^2 / 8\pi J_0 e a_0)^{1/2} \ . \tag{11}$$

This has the same form as the familiar Josephson penetration depth,[23] except that the usual effective barrier thickness $d_{\text{eff}} = d_i + 2\lambda_s$ is here replaced by a_0. Eliminating J_0 in favor of Δ with the help of Eq. (3), we recover Eq. (10), but with the normal-state resistivity defined as $\rho_n = R_n a_0$ (3D) or $\rho_n = R_n d$ (2D). Thus the present theory in the large-ϵ limit yields exactly the same critical current density as the Ginzburg-Landau theory in the dirty limit. Moreover, near T_c we obtain

$$[J_c(T)]^{2/3} = 1.840(k_B\gamma/\hbar\rho_n)^{1/3}(T_c - T) \ . \tag{12}$$

This equation permits the determination of γ from measurements of J_c versus T near T_c.

If the Josephson-coupling term in Eq. (2) is generalized as in Refs. 13 and 24 to permit f to vary from grain to grain, minimization of the resulting expression for ΔG yields equations that reduce to the usual Ginzburg-Landau theory in the large-ϵ limit. Playing the role of the Ginzburg-Landau coherence distance is the quantity

$$\xi(T) = \left[\frac{\pi^2 \hbar \Delta^2(T)}{4e^2 k_B T_c \rho_n H_c^2(T)} \right]^{1/2} \ . \tag{13}$$

At T_c, the ratio of λ [Eq. (10)] to ξ is

$$\kappa = 0.0646 c e \rho_n \gamma^{1/2}/k_B \ , \tag{14}$$

which is identical to the expression first derived by Gor'kov[25] for the dirty limit. Similar results have been obtained in previous theories of granular superconductors using a different approach.[26,27] Comparison of Eq. (13) and the defining equation for ϵ reveals that $\epsilon(T)=2\xi^2(T)/a_0^2$ near T_c, such that the limit $\epsilon \gg 1$ corresponds to $\xi \gg a_0$.

When $\epsilon \ll 1$ ($\xi \ll a_0$), we obtain $\phi_m \simeq \pi/2 - 3\epsilon/8$, $f_m^2 \simeq 1-\epsilon$, and $g \simeq 1-\epsilon$; in other words, when the condensation energy of a grain is much larger than the Josephson-coupling energy, current-induced gap suppression plays an insignificant role, and the critical current density is well represented by the Ambegaokar-Baratoff result Eq. (1).

When $\epsilon = 1$ ($\xi = a_0/\sqrt{2}$), we obtain $\phi_m = \pi/4$, $f_m^2 = 1/\sqrt{2}$, and $g = \frac{1}{2}$; in other words, the transition between the Ambegaokar-Baratoff regime and the Ginzburg-Landau regime occurs when the condensation energy of a grain is of the order of the Josephson-coupling energy, or, equivalently, when the Ginzburg-Landau coherence distance is of the order of the grain size.

Shown in Fig. 2 are calculated curves of $[J_c(T)/J_0(0)]^{2/3}$ versus T/T_c for several different values of ϵ_0. Near T_c, we find $[J_c(T)/J_0(0)]^{2/3} = 1.335\epsilon_0^{-1/3}(1-T/T_c)$. The top curve in Fig. 2 ($\epsilon_0 = 0$) is the corresponding Ambegaokar-Baratoff result [Eq. (1)], which has infinite slope at T_c.

III. COMPARISON WITH EXPERIMENT

Of the quantities entering the theory, probably the most difficult to obtain experimentally is a_0, which can be interpreted as the effective grain diameter. In the model of Ref. 4, a_0 is calculated from $a_0 = \pi\Delta/2e\rho_n J_c$, all quantities being evaluated in the limit of zero temperature. In the present theory, however, a_0 must be obtained self-consistently as follows. Combining $J_c(0) = J_0(0)g(\epsilon_0)$ with Eqs. (1) and (8) and making use of the BCS relations that led to Eq. (7), we find

$$\epsilon_0 g^2(\epsilon_0) = 0.382 \frac{\hbar \rho_n J_c^2(0)}{k_B \gamma T_c^3} \ , \tag{15}$$

which is solved for ϵ_0. We then obtain a_0 from Eq. (8). This procedure, which reduces to that of Ref. 4 when $\epsilon_0 \ll 1$, works best when $\epsilon_0 < 1$ or $a_0 > a_x$, where

$$a_x = 1.71(\hbar k_B/\gamma T_c e^2 \rho_n)^{1/2} \tag{16}$$

is the value of a_0 at which $\epsilon_0 = 1$. This method of determining a_0 has poor accuracy, on the other hand, when $\epsilon_0 \gg 1$ or $a_0 \ll a_x$, because in this limit $J_c(0)$ reduces to

$$J_c(0) = 0.881(k_B \gamma T_c^3/\hbar\rho_n)^{1/2} \ , \tag{17}$$

which is independent of a_0.

Implicit in the theory of Sec. II are the assumptions that (a) the current density flows nearly uniformly throughout the specimen's cross section and (b) self-field and vortex-pinning effects are negligible. The first assumption is well satisfied for samples of average thickness d and width w satisfying $d \ll \lambda(T)$ and $w \ll \lambda_\perp(T)$, where $\lambda_\perp = 2\lambda^2/d$ is the effective screening length for thin films. The second assumption requires that the critical current density as calculated in Sec. II obeys $J_c(T) \ll J_{c1}(T)$, where J_{c1} is the current density at which the self-field at the edge of the film is equal to H_{c1}, sufficient to nucleate a vortex. Self-field and pinning effects become important when $J_c \simeq J_{c1}$. If $J_c \gg J_{c1}$, the critical current density is no longer limited by the mechanisms of Sec. II but by the larger of J_{c1} or J_{cd}, the critical depinning current density.[28] To estimate J_{c1}, we first assume uniform current density and apply the Biot-Savart law to obtain $H_{c1} = (2J_{c1}d/c)\ln(w/d)$. We next use the Ginzburg-Landau theory for high-κ superconductors[9–12] to approximate $H_{c1} \simeq (\ln\kappa/\kappa\sqrt{2})H_c$, where $H_c(T) \simeq H_c(0)(1-T^2/T_c^2)$. Finally, using Eq. (14) and the BCS result[16,17] $H_c(0) = 2.438\gamma^{1/2}T_c$, we obtain

$$J_{c1}(T) \simeq 13.3 \left[\frac{k_B T_c \ln\kappa}{e\rho_n d \ln(w/d)} \right] (1 - T^2/T_c^2) \ . \tag{18}$$

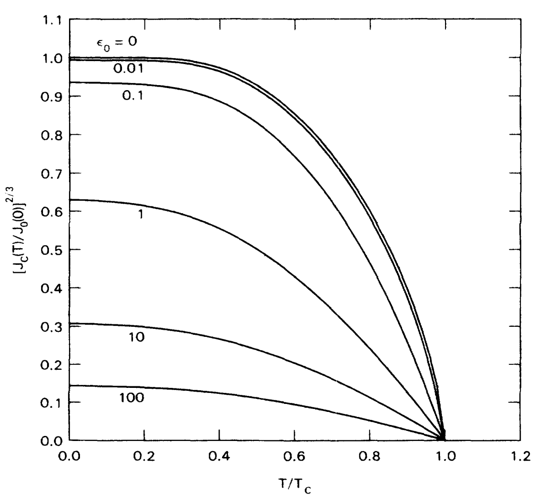

FIG. 2. $[J_c(T)/J_0(0)]^{2/3}$ versus T/T_c for various values of ϵ_0, where $J_c(T) = J_0(T)g(\epsilon)$, J_0 is the Ambegaokar-Baratoff critical-current density [Ref. 14 and Eq. (1)], $g(\epsilon)$ is shown in Fig. 1, and $\epsilon(T)$ is given by Eq. (7).

Shown in Fig. 3 is a comparison between theory and experiment for the temperature dependence of J_c in a NbN film of thickness 225 Å, width 6 μm, and length 6μm. The film was prepared by reactive dc magnetron sputtering onto an oxidized Si substrate. The film thickness was monitored during deposition with a quartz-crystal deposition rate monitor, and the final thickness cross-checked with a surface height profiler. The plotted points are experimental values, plotted as $[J_c(T)/J_c(0)]^{2/3}$, versus T/T_c, where we assume $J_c(0) = 7.7 \times 10^6$ A/cm^2. The values of $T_c = 11.9$ K and $\gamma = 1.3 \times 10^3$ erg/cm^3 K^2 are obtained from the slope of $J_c^{2/3}$ versus T near T_c [Eq. (12)], and the value of $\rho_n = 145$ $\mu\Omega$ cm is taken to be the measured resistivity at $T = 20$ K. The above value of γ is to be compared with previous experimental values for bulk specimens in the range[29–31]

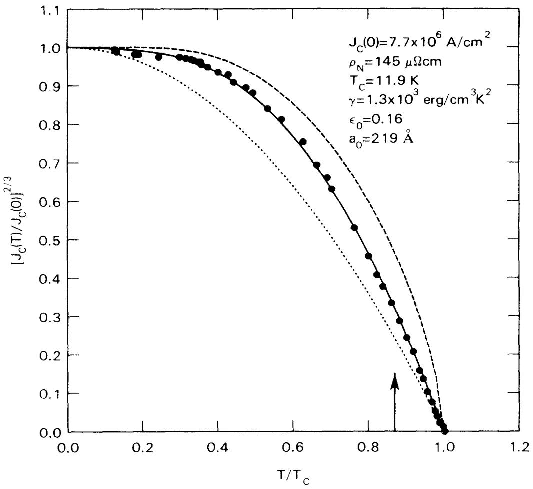

FIG. 3. $[J_c(T)/J_c(0)]^{2/3}$ vs T/T_c. Solid points: experimental data for a 225-Å NbN film; solid curve: theoretical curve calculated as described in the text; dashed curve: Ambegaokar-Baratoff critical current density [Ref. 14 and Eq. (1)]; dotted curve: Bardeen's (Ref. 36) expression, $[J_c(T)/J_c(0)]^{2/3} = 1-(T/T_c)^2$, which approximates numerical calculations (Refs. 37–39) extending the Ginzburg-Landau theory to lower temperatures. The vertical arrow indicates the crossover temperature T_x.

$\gamma=(1.7-3.6)\times10^3$ erg/cm^3 K^2. The solid curve shows $[J_c(T)/J_c(0)]^{2/3}$ as calculated from the above theory, where the values of $\epsilon_0=0.16$ and $a_0=219$ Å are obtained from Eqs. (15) and (8), respectively. The calculated junction resistance is $R_n=66$ Ω.

For comparison with the effective grain size obtained in the above manner, the actual grain size and phase structure of the NbN film were determined by using transmission electron microscopy (TEM). The results obtained by selective area diffraction TEM indicate that the NbN film is of face-centered-cubic polycrystalline structure with a lattice parameter of 4.46 Å. Lattice images were taken and the number of lattice fringes of (111) planes with interplanar spacing of 2.58 Å were then counted to measure grain sizes of the polycrystalline NbN film. The grain sizes of the film were distributed from 30–90 Å with an average size around 60 Å. Grains showed no preferential orientation, but an examination of the (111) deflection dark field image revealed that grains of similar orientation were frequently clustered together. The cluster sizes approached 200 Å.

Our finding that the effective grain size inferred from critical-current measurements is somewhat larger than the average grain size measured from electron micrographs can be understood in terms of the expected spread of intergrain Josephson-coupling strengths, using percolation arguments similar to those used in Refs. 32–35. For example, two adjacent grains with a larger-than-average coupling strength will behave essentially as a single larger grain, while two adjacent grains with a smaller-than-average coupling strength will behave essentially as if insulated from each other. It is possible that the similarly oriented grains are among those that are more strongly coupled together.

The crossover temperature T_x between Ambegaokar-Baratoff behavior and Ginzburg-Landau behavior (at which $\epsilon=1$ and $\xi=a_0/\sqrt{2}$) is calculated to be about $0.87T_c$, and Eq. (16) yields $a_x=86$ Å. Using estimates of $\lambda(T)$ via Eq. (10), we find that the assumption of nearly uniform current density is justified at all temperatures. Equation (18) yields $J_{c1}(0)=2.7\times10^7$ A/cm^2, from which we conclude that self-field and pinning effects are unimportant at all temperatures.

The dashed curve in Fig. 3 shows the Ambegaokar-Baratoff temperature dependence expected in the absence of current-induced gap suppression, i.e., $[J_0(T)/J_0(0)]^{2/3}$ [Eq. (1)] versus T/T_c. The deviation of this temperature dependence from that of the measured critical current density is most pronounced near T_c. The dotted curve in Fig. 3 shows the temperature dependence of the phenomenological expression $[J_c(T)/J_c(0)]^{2/3}=1-(T/T_c)^2$ proposed by Bardeen.[36] This simple expression is a good approximation to $[J_c(T)/J_c(0)]^{2/3}$ calculated numerically from the dirty-limit microscopic theory[37–39] that extends the Ginzburg-landau critical-current theory to low temperatures. The Bardeen expression, however, overestimates $[J_c(T)/J_c(0)]^{2/3}$ by about 6% near T_c, where it has slope -2, rather than the value -1.89 obtained from the dirty-limit microscopic theory.[37–39]

IV. SUMMARY AND CONCLUSIONS

In this paper we incorporated the Josephson-coupled-grain model of Ref. 4 into a Ginzburg-Landau theory of a grangular superconductor. We then used this theory to derive an expression for the critical current density, which provides a good description of experimental results from a 225-Å NbN film. The relevant dimensionless parameter in the theory is ϵ_0 [Eq. (8)], which, aside from numerical factors of order unity, is the ratio of the Josephson-coupling energy of a junction to the superconducting condensation energy of a grain, both energies being evaluated at zero temperature. Only when $\epsilon_0=0$ is the original Josephson-coupled-grain model valid over the entire temperature range from zero to T_c. In this case the temperature dependence of J_c is that of Ambegaokar and Baratoff.[14] For nonzero values of ϵ_0, however, the Ambegaokar-Baratoff temperature dependence holds only at low temperatures, the behavior giving way to the Ginzburg-Landau[15] $(1-T/T_c)^{3/2}$ temperature dependence above the crossover temperature $T_x = T_c(1-0.882\epsilon_0)$. For large values of ϵ_0, current-induced gap suppression is dominant at all temperatures, and J_c does not obey the Ambegaokar-Baratoff behavior at any temperature. Instead, the temperature dependence is governed by Ginzburg-Landau-like behavior at all temperatures. That is, J_c is proportional to $(1-T/T_c)^{3/2}$ near T_c, where the present theory should be valid, but further extensions of the theory[27,37–45] would be needed to calculate the low-temperature behavior of J_c more precisely.

In the strongly Josephson-coupled regime ($\epsilon\gg1$) the above model gives results in agreement with the Ginzburg-Landau theory. That is, as in Refs. 26 and 27, the resulting penetration depth $\lambda(T)$, coherence distance $\xi(T)$, and Ginzburg-Landau parameter κ are given by the same expressions [Eqs. (10), (13), and (14)] as in the microscopic dirty-limit theory,[25] except that playing the role of the normal-state resistivity ρ_n of the microscopic theory is $\rho_n = R_n a_0$(3D) or $\rho_n = R_n d$ (2D), the effective normal-state resistivity measured using a sample volume much larger than the grain volume. Our theory remains valid even when the intragrain electronic mean-free path is of the order of a_0 but the tunneling resistance R_n is so high that the mean-free path l calculated from $l = mv_F/ne^2\rho_n$ is smaller than the interatomic spacing.

The above approach also can be used to investigate the properties of isolated Josephson junctions. It is easy to see that current-induced gap suppression is important in reducing the critical current of small, high-current junctions whenever the ratio of the Josephson-coupling energy to the condensation energy of the superconducting counterelectrodes is of order unity or larger. This effect therefore can reduce the low-temperature $I_c R_n$ product below that expected from Eq. (1) and always causes I_c to be proportional to $(1-T/T_c)^{3/2}$ sufficiently close to T_c.

The theory of Sec. II also can be extended directly to square 2D arrays of Josephson junctions with lattice parameter a_0. For this case, $\rho_n = R_n d$, where R_n is equal to $R_\square$, the normal-state (tunneling) resistance per square, and $d = V/a_0^2$ is the average thickness of superconductor. The physics of Sec. II has an important consequence for the vortex structure in 2D arrays in a transverse magnetic field: When the Ginzburg-Landau coherence distance $\xi(T)$ [Eq. (13)] exceeds a_0, which always occurs sufficiently close to T_c, current-induced gap suppression produces a core of radius approximately equal to $\xi(T)$. With decreasing temperature, however, the core shrinks. For sufficiently weak Josephson coupling that $\epsilon_0 < 1$, the low-temperature vortex structure is determined by the array's geometry, and the core size is set by a_0, the lattice parameter of the array.[46]

Since the above theory is a mean-field theory that ignores charging effects, it is expected to fail at those combinations of the junction resistance R_n and capacitance C for which thermal and quantum-mechanical fluctuations destroy phase coherence among grains.[47] Such fluctuations, however, become significant only at high values[47,48] of R_n, generally in excess of the quantum of resistance $R_Q = h/4e^2 = 6.45$ kΩ. Since ϵ_0 can be written as

$$\epsilon_0 = \frac{R_Q/R_n}{2N(0)\Delta(0)V}, \tag{19}$$

we see that if the grains are sufficiently large that $N(0)\Delta(0)V \gg 1$, then when $R_n > R_Q$, $\epsilon_0 \ll 1$, and the mean-field behavior is expected to be Ambegaokar-Baratoff-like except very close to T_c. On the other hand, the simultaneous appearance of both charging-induced fluctuation effects (with $R_n \simeq R_Q$) and Ambegaokar-Baratoff–Ginzburg-Landau crossover effects (with $\epsilon_0 \simeq 1$) might be observable in specimens containing very tiny grains for which $N(0)\Delta(0)V \simeq 1$, i.e., for which size effects become important[49] and the normal-state single-particle level splittings near the Fermi energy E_F are of order $\Delta(0)$. The number of metal atoms per grain required to meet this condition, however, is of order $E_F/\Delta(0) \simeq 10^4$.

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