Surface mobility fluctuations in metal-oxide-semiconductor field-effect transistors

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We have calculated, from first principles, two-dimensional mobility fluctuations in metal-oxide-semiconductor field-effect transistors (MOSFET's) due to scattering between the induced charge carriers and the interfacial traps. The results are used to evaluate the behavior of the $1/f^{\gamma}$ noise in MOSFET's, particularly the gate-bias dependence of both the functional form and magnitude of the noise power spectrum. Similar to previous calculations that assumed the noise to be due to number fluctuations, the new model accurately accounts for the change of the spectral distribution of noise at different gate biases. However, when the surface mobility fluctuations are included in the noise computation, a much better fit to the experimentally measured noise power magnitude is obtained.

I. INTRODUCTION

The flicker noise in metal-oxide-semiconductor field-effect transistors (MOSFET's) has been a subject of research interest for many years.[1–8] Recently, we have reported[7] on a series of detailed measurements on the noise in p-channel MOSFET's. In addition to spectral analyses, we have measured both spatial and time-correlation functions of the noise. To account for the wide range of data, a modified McWhorter model was proposed whereby the noise origin was attributed to the interaction between charge carriers and interfacial trap states at the oxide-semiconductor boundary.

This model was lent credence by experiments that measured noise in submicrometer devices at low temperatures.[9–11] In these experiments, the (device area) $\times$(energy range) product of the specimen was made small enough that the effects of a single trap state on the FET channel conductance were observed.

Our model[7] was specifically intended to explain how the spectral distribution of noise power changed as the gate bias on the device was increased above the threshold voltage, in addition to accounting for the statistical properties of the noise. Experimentally, the noise was found to have a power spectrum of the form of $1/f^{\gamma}$ with γ ranging from 0.8 to 1.3 and dependent on the gate voltage V_g. The model showed that, with a general energy-dependent form of the trap distribution, the spectral dependence of the noise on gate bias would occur as a result of the change of the band bending near the oxide-silicon interface by the applied gate voltage. When compared with experimental data, our model produced an excellent fit to the observed form of $\gamma(V_g)$. The important consequence of this good fit was that, with the functional form of the noise spectra now determined, the model could then be compared with experimentally measured noise power at *any* frequency. This avoids the difficulty of noise models that are successful at a single frequency but would fail when compared with experimental data at frequencies a few decades away,[7] as a result of the changing value of γ.

One example of this latter category is the heuristic model of Hooge[2,12] which stated that the current noise spectrum in a specimen of total carriers N is given by

$$\frac{S_I(f)}{I^2}=\frac{\alpha}{Nf}\ , \tag{1}$$

where I is the average dc current in the sample and α is a universal constant of magnitude 2×10^{-3}. This equation has been widely used to model low-frequency excess noise in MOSFET's,[2,13] but clearly would not be suitable in situations where the *form* of the noise spectral density differs from Eq. (1) and changes even for fixed current I. Another serious difficulty arose when it was discovered[14–16] that the measured α in MOSFET's can deviate significantly from the universal value for different specimens. This deviation, however, can be easily accounted for in the trap model, wherein the noise spectrum and magnitude are determined by trap distribution that varies from sample to sample.

In the trap model previously used by us,[7] it was assumed that the effect of the traps was to modify the electric field at the FET inversion layer as the carriers enter and exit from the traps. This, in turn, causes the charge in the inversion layer to fluctuate in proportion to the trap-density fluctuation. This scheme is often called the "number-fluctuation model." Despite its general success, the model nevertheless underestimates noise magnitude when the device is at strong inversion. We suggested[7] that a possible reason for the discrepancy is that charged traps may also act as interfacial scattering centers to reduce the carrier mobility.

In this paper we derive from first principles calculations of the scattering effect of charged traps, following the above suggestion. We show that, in the case of strong inversion, the mobility fluctuation induced by trapping and detrapping of charged carriers indeed becomes the dominating noise mechanism. We also show that a better fit to the experimental noise magnitude can be obtained when the scattering mechanism is properly accounted for, without sacrificing the integrity of the good fit to the experimental value of γ.

II. THEORY

In this section we will consider the effect on carrier mobility due to the scattering between the carriers and the charged traps. The starting point is similar to those done elsewhere[17–21] and the carriers are represented by two-dimensional (2D) plane-wave states. When the effect of a single carrier is determined, we can then relate the mobility fluctuation to the trap-density fluctuation. The total amount of the drain-voltage noise, including both number and mobility fluctuations, is calculated to provide the basis for comparison with experimental data.

A. 2D mobility fluctuation in the presence of charged traps

We first consider a single trap, shown in Fig. 1, located near the oxide-silicon interface. For a charge carrier at $\mathbf{r}$, the scattering potential it experiences from the trap is given by[17,18]

$$V(\mathbf{r})=\frac{Q_i}{4\pi\epsilon_{\mathrm{av}}\,|\,\mathbf{r}-\mathbf{R}_i\,|}\ , \tag{2}$$

where $\mathbf{R}_i$ is the location of the trap, and ϵ_{av}, the average permittivity, is given by

$$\epsilon_{\mathrm{av}}=\tfrac{1}{2}(\epsilon_{\mathrm{si}}+\epsilon_{\mathrm{ox}})\ . \tag{3}$$

Equation (2) can be generalized to form the scattering Hamiltonian, $H(\mathbf{r})$, by including all traps:

$$H(\mathbf{r})=-\frac{e}{4\pi\epsilon_{\mathrm{av}}}\sum_i \frac{\Delta Q_i}{|\,\mathbf{r}-\mathbf{R}_i\,|}\ , \tag{4}$$

where ΔQ_i is the effective amount of scattering charge in an elemental volume $\Delta x\ \Delta y\ \Delta z$ around $\mathbf{R}_i$.

The transition rate between the two-dimensional plane-wave states $\mathbf{k}$ and $\mathbf{k}'$ is then

$$\Gamma_{\mathbf{k}\to\mathbf{k}'}=\frac{2\pi}{\hbar}\,|\,\langle \mathbf{k}'\,|\,H\,|\,\mathbf{k}\rangle\,|^2\,\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})\ . \tag{5}$$

Substituting Eq. (4) into Eq. (5) and carrying out the planar integration, one obtains

$$\Gamma_{\mathbf{k}\to\mathbf{k}'}=\frac{2\pi}{\hbar}\left[\frac{e^2}{2\epsilon_{\mathrm{av}}q}\right]^2\frac{1}{A}\int dE\int dz\,N_t(E,z)\times\exp(-2qz)\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})\ , \tag{6}$$

where $q=|\,\mathbf{k}-\mathbf{k}'\,|$ and A, the gate area, was used to normalize the plane-wave states. The total scattering rate, defined by the following equation,

$$R_s=\frac{A}{(2\pi)^2}\int d^2\mathbf{k}\,\Gamma_{\mathbf{k}\to\mathbf{k}'}[1-\cos(2\phi)]\ , \tag{7}$$

where 2ϕ is the angle between $\mathbf{k}$ and $\mathbf{k}'$, can then be evaluated from Eq. (6). The momentum relaxation time Γ_s, which is the reciprocal value of the scattering rate, is related to the mobility[18,22]

$$\mu_t=\frac{e\langle\Gamma_s\rangle}{m_h}\ , \tag{8}$$

where $\langle\Gamma_s\rangle$, the mean value of Γ_s, is defined as

$$\langle\Gamma_s\rangle=\frac{\int \Gamma_s Eg(E)f(E)dE}{\int Eg(E)f(E)dE}\ , \tag{9}$$

in which $g(E)$ is the density of states and $f(E)$ is the Fermi-Dirac distribution function. If Γ_s varies more slowly in E than the product $Eg(E)f(E)$, then $\langle\Gamma_s\rangle$ can be approximated by $\Gamma_s(E_p)$, and E_p is where $Eg(E)f(E)$ peaks. This is found to be the case as shown in Fig. 2. Equation (8) is then integrated to be

$$\mu_t\cong\frac{e\Gamma_s(E_p)}{m_h}\ . \tag{10}$$

Substituting Eq. (6) into Eq. (10), one obtains

$$\mu_t^{-1}=\frac{m_h e^3}{8\pi\hbar\epsilon_{\mathrm{av}}^2E_pA}\int d\Omega\int dz\int dE\,2\exp(-4kz\sin\phi)\times N_t(E,z)\ , \tag{11}$$

where the area variable Ω spans the gate of the device. Equation (11) now describes the total contribution to the carrier mobility by all the traps at the interface.

In the presence of trap-density fluctuation ΔN_t, the mobility fluctuation $\Delta\mu_t$ is given by

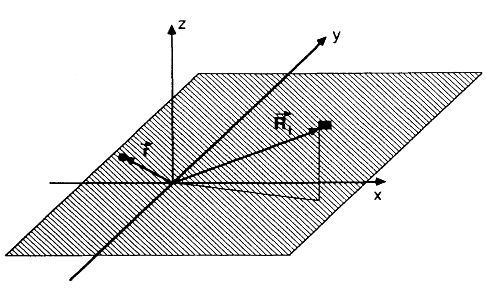

FIG. 1. Coulomb interaction between the charged-oxide trap (solid rectangle) at $\mathbf{R}_i$ and a mobile carrier at $\mathbf{r}$ (solid circle) on the 2D conduction plane (shaded area).

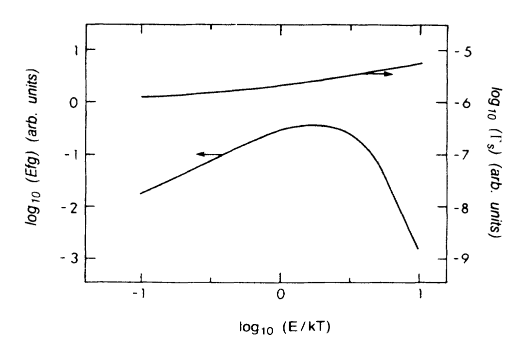

FIG. 2. A comparison between the energy dependence of the relaxation time $\Gamma_s(E)$ and the product $Eg(E)f(E)$. (See text.)

$$-\frac{\Delta\mu_t}{\mu_t^2}=\frac{m_h e^3}{8\pi\hbar\epsilon_{\rm av}^2 E_p A}\int_A d\Omega \int_z dz \int_0^{\pi/2} d\phi \int_E dE\, 2\exp(-4kz\sin\phi)\Delta N_t \ . \tag{12}$$

The carrier mobility is the combined effect of both interface trap and lattice scattering,

$$\frac{1}{\mu}=\frac{1}{\mu_t}+\frac{1}{\mu_l} \ , \tag{13}$$

where μ_l is the mobility component due to lattice scattering. A fluctuation in μ_t will thus produce a corresponding fluctuation in the carrier mobility μ,

$$\Delta\mu=\left[\frac{\mu}{\mu_t}\right]^2\Delta\mu_t \ . \tag{14}$$

In Eq. (14) we have assumed that the lattice scattering does not have any appreciable contribution to the mobility fluctuation in the frequency range where our experiments were conducted.

The spectral density of the fluctuation in the number of scattering centers can be found following McWhorter's method,[23] as detailed in our previous paper [Eqs. (15)–(20) in Ref. 7]. Using Eqs. (12) and (14) above and Eq. (2) in Ref. 7, one finds the power spectral density of the fluctuations in the carrier mobility to be given by

$$S_\mu(\omega)=\left[\frac{\mu^2 m_h e^3}{8\pi\hbar\epsilon_{\rm av}^2 E_p}\right]^2\frac{1}{A}\int dz\int dE\, G^2(k,z) \times f_t(1-f_t)\frac{4N_t\tau}{1+\omega^2\tau^2} \ , \tag{15}$$

where the scattering kernel G is given by

$$G(k,z)=\int_0^{\pi/2} 2\exp(-4kz\sin\phi)d\phi \ . \tag{16}$$

When a constant current is applied to the device, the mobility fluctuation manifests itself as a noise in the drain voltage V_d. If V_d is small (linear bias), the channel conductance is approximately constant over the entire channel length. The voltage noise is then linear in the conductance fluctuation. The drain-voltage noise is now given by

$$S_{V\mu}(\omega)=\frac{V_d^2}{\mu^2}S_\mu(\omega) \ . \tag{17}$$

The symbol $S_{V\mu}(\omega)$ is used here to indicate that Eq. (17) includes only the mobility contribution to the total noise.

B. The combined effect of number fluctuation and mobility fluctuation

The noise power spectrum computed for mobility fluctuation is quite different from previous models that evaluated the effect of interfacial traps: Prior to this work, it was generally thought that the interfacial traps primarily changed the carrier concentration through the relationship that equates carrier-density fluctuation with trap-density fluctuation,

$$S_n(\omega)=S_{N_t}(\omega) \ . \tag{18}$$

In the linear region, the noise-voltage spectrum would then be

$$S_{Vn}(\omega)=\frac{V_d^2}{N^2}S_n(\omega) \ , \tag{19}$$

where N, the total number of charge carriers in the conduction channel, is given by

$$N=AC_{\rm ox}(V_g-V_t)/e \ , \tag{20}$$

and $C_{\rm ox}$ is the gate capacitance per unit area, V_g is the gate voltage, and V_t is the threshold voltage.

A detailed calculation of S_{Vn} was previously presented by us.[7] There is, of course, no *a priori* reason why Eq. (19) should dominate over the contribution of Eq. (17). In fact, a quick order-of-magnitude estimation shows that the two terms S_{Vn} and $S_{V\mu}$ *have comparable magnitudes for most bias conditions.*

It is tempting to add the mobility- and number-fluctuation contributions to obtain the total noise spectrum, i.e.,

$$S_V(\omega)=S_{V\mu}(\omega)+S_{Vn}(\omega) \ . \tag{21}$$

However, we note that the *same* trap-density fluctuation results in the mobility and carrier-density fluctuations: The fluctuating voltages are therefore correlated. This situation becomes obvious when one examines the autocorrelation function $A(t)$ of the voltage fluctuations due to mobility, $V_\mu(t)$, and due to carrier density, $V_n(t)$:

$$\begin{aligned}A(t)&=\langle[V_n(0)+V_\mu(0)][V_n(t)+V_\mu(t)]\rangle\\&=\langle V_\mu(0)V_\mu(t)\rangle+\langle V_n(0)V_n(t)\rangle\\&\quad+\langle V_n(0)V_\mu(t)\rangle+\langle V_\mu(0)V_n(t)\rangle \ .\end{aligned} \tag{22}$$

By use of the Wiener-Khinchine theorem, the Fourier transform of the first two terms on the right-hand side of Eq. (22) corresponds to $S_{V\mu}(\omega)$ and $S_{Vn}(\omega)$, respectively, while the left-hand side corresponds to the total fluctuation $S_V(\omega)$. Equation (21) is then valid, provided that the cross-correlation terms (third and fourth terms) in Eq. (22) are negligible, compared to the first two terms. This is true provided that (i) the variance of V_μ is significantly larger than V_n, or vice versa, or (ii) trapping action occurs equally on charged and uncharged traps, such that when a carrier is captured by a trap, the mobility is equally likely to increase or to decrease. There is some experimental evidence for the latter situation.[9,10] For the moment, we shall *assume* that Eq. (21) to be true at least approximately in order to get a handle on evaluating $S_V(\omega)$.

C. Screening effects

In Sec. II A we have assumed that the bare Coulomb potential could be used for scattering calculations. Stern and Howard,[17] however, showed that the charge carriers in the inversion layer has a screening effect. In the presence of screening, the transition rate $\Gamma_{\mathbf{k}\to\mathbf{k}'}$ in Eq. (6) must be modified to be[19]

$$\Gamma'_{\mathbf{k}\to\mathbf{k}'}=\frac{2\pi}{\hbar}\left[\frac{e^2}{2\epsilon_{\rm av}}\right]^2\frac{1}{A(q+b)^2}$$

$$\times\int_E dE\int_z dz\,N_t\exp(-2qz)\delta(E_{\mathbf{k}}-E_{\mathbf{k}'})\ , \quad (23)$$

where b is the screening constant given by

$$b=\frac{2e^2d_v m_h}{4\pi\epsilon_{si}\hbar^2}\left\{1-\exp\left[-\left[\frac{\pi\hbar^2 n}{k_BTd_v m_h}\right]\right]\right\}, \quad (24)$$

in which d_v is the degeneracy of holes, m_h the effective mass of holes per ellipse, ϵ_{si} the permittivity of silicon, k_B the Boltzmann constant, and n the number of free holes per unit area.

The net effect of screening on the noise spectrum is that the scattering kernel in Eq. (16) must now be changed to

$$G'(k,z)=\int_0^{\pi/2}4\frac{[1-\cos(2\phi)]\exp(-4kz\sin\phi)}{(2k\sin\phi+b)^2}d\phi\ . \quad (25)$$

Equation (24) shows that, since n depends on the gate voltage V_g, there is now an additional V_g dependence of $S_V(\omega)$, and that the effect of screening is to decrease the contribution of mobility fluctuation by the traps.

III. EXPERIMENTAL RESULTS

In order to investigate the validity of this model, we performed detailed studies on the gate-bias dependence of the voltage-noise spectra with the drain biased at a low value. Using the same experimental procedures as in Ref. 7, the voltage-noise-power spectral density from a commercial p-channel MOSFET (SK9159) (Ref. 24) was measured at various gate biases. At least 1000 averages were made for each bias to reduce the experimental scatter. The values of γ at different gate biases were measured.

The experimental results were then compared with models using only number fluctuations, using both number and mobility fluctuations without screening, and finally using number and mobility fluctuations with screening effects included. The fitting procedure is the same as previously used by us.[7] To find a trap-density distribution that accurately fits the experimental value of $\gamma(V_g)$, the first step is to use the experimentally measured *change* in γ with V_g to find $[1/N_t(E)][\partial N_t(E)/\partial E]$. The next step is to find the $[1/N_t(z)][\partial N_t(z)/\partial z]$ that best fits the absolute magnitude of γ. As before,[7] in each step we used only one fitting parameter that corresponds to one particular feature of the experimental data. It turns out that the best-fitted values are strongly dependent only on the functional form of $N_t(E)$ that one chooses rather than on the mechanism assumed. This indicates that, in this model, the primary factor that determines the distribution of the fluctuation time constant is the trap distribution, not the mechanism by which the trap changes the drain voltage. A typical set of $\gamma(V_g)$'s is shown in Fig. 3. The particular model used included both number fluctuation and mobility fluctuation with screening. The resultant value of $[1/N_t(E)][\partial N_t(E)/\partial E]$ was 0.04 ± 0.005 meV^{-1} and

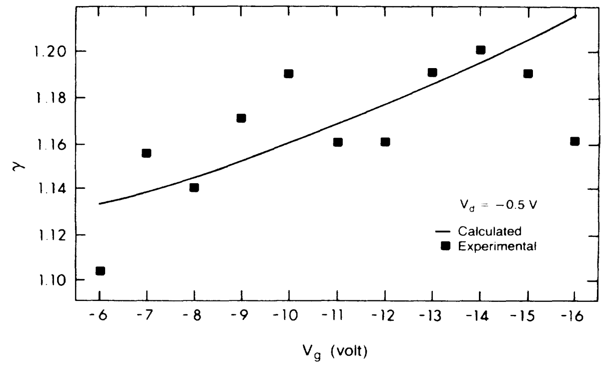

FIG. 3. Gate-bias dependence of the exponent γ in the $1/f^\gamma$ noise-power spectrum. The solid line is the theoretical best fit calculated from a trap distributed similar to that of Eq. (27) in Ref. 7.

$[1/N_t(z)][\partial N_t(z)/\partial z]$ was 3.0 ± 0.5 nm^{-1}. The former value is within the range of values found in other experiments.[25–27]

When including the mobility fluctuation, an important parameter is the total mobility, μ. This was determined by experimentally measuring the channel conductance G_c at various gate biases and using

$$\mu=\frac{LG_c}{WC_{\rm ox}(V_g-V_t)}\ , \quad (26)$$

where L and W are the channel length and width, respectively.

With the integrity of the $\gamma(V_g)$ fit intact, the final fitting step is to compare the *magnitude* of $S_V(\omega)$ with the calculated results, using a single data point of $S_V(\omega)$ to obtain a fitted value of the magnitude of the trap density.[7] An example of the fitted comparison of $S_V(\omega)$ as a function of V_g is shown in Fig. 4. The threshold voltage for

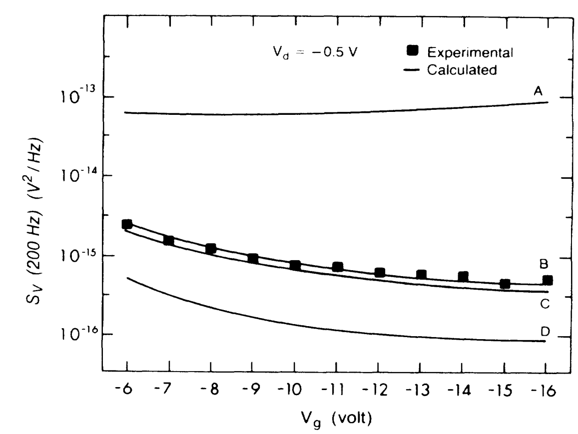

FIG. 4. Gate-bias dependence of the noise power at 200 Hz across the conduction channel. The solid lines are theoretical best fits and the squares are experimental data. Curve A is the mobility-fluctuation spectrum without screening. Curve C is the mobility-fluctuation spectrum with screening taken into account. Curve D is number-fluctuation spectrum and curve B is the sum of curves C and D.

this particular sample was $V_t = -2.5$ V. The theoretical curve (curve D) corresponds to number fluctuation only. As before,[7] we find the theoretical value to underestimate $S_V(\omega)$, especially at high V_g. Curve A is the calculated result for mobility fluctuation without screening effects. It predicts a nearly-gate-bias-independent form of $S_V(\omega)$ and clearly does not fit the experimental results. A much better fit is obtained when screening is included (curve C). Curve B sums up C and D and basically does not significantly alter the theoretical values of $S_V(\omega)$ from curve C. Experimentally, for the samples that we tested, it was not possible to discern the difference between curves B and C and to exclude the contribution of number fluctuation.

IV. CONCLUSION

We have shown in this paper how one can calculate from first principles the change in carrier mobility as a result of interfacial trapping in MOSFET's. The total noise spectrum expected for trapping model has been calculated and compared with experimental results. It is shown that when the two-dimensional screening effect is properly accounted for, the model produced an excellent fit to the experimentally measured gate-bias dependence of the noise power spectrum. *Both* the functional form and the magnitude of $S_V(\omega)$ are now in agreement with the model calculations.

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