Nonlinear balance equations for hot-electron transport with finite phonon-relaxation time

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The balance-equation approach developed by Lei and Ting for steady-state hot-electron transport is extended here to include nonequilibrium phonon occupation under the assumption that the effect of phonon-phonon interaction can be represented by an effective phonon relaxation time τ_p, which may be mode dependent. Both single heterojunctions and multilayer quantum-well superlattices, as well as three-dimensional (3D) bulk systems, are discussed. A 3D phonon model and a quasi-2D phonon model are employed in describing the various interacting electron-phonon systems. The expressions for the frictional forces and energy transfer rates obtained in steady state are structurally similar to those without hot-phonon effects and the balance equations with finite phonon relaxation can be solved with the same computational effort as in the case of $\tau_p=0$. We have examined hot-phonon effects on the energy-transfer rate, the electron temperature, and the linear and nonlinear mobilities. It is shown that finite phonon relaxation generally decreases the phonon-induced Ohmic resistivity at a given lattice temperature. However, it significantly increases the electron temperature so that the nonlinear resistivity of the system is enhanced at large drift velocities. For an n-type GaAs heterosystem the normalized electron-energy-loss rate at $\tau_p=3.5$ ps is seen to be a factor of 5—10 smaller than that at $\tau_p=0$. The functional dependence of the energy-loss rate on carrier temperature shows considerable difference from the prediction of a carrier temperature-type theory, and is in reasonably good agreement with experiments.

I. INTRODUCTION

Considerable current interest is focused on the development of a full understanding of the hot-electron energy-loss rate in quasi-two-dimensional heterosystems, under conditions of steady-state high-field electronic transport and also for time-resolved photoluminescence.[1-5] The energy-loss rate in multilayer GaAs quantum wells, measured by Shah *et al.*[5] in experiments on steady-state transport, is about a factor of 8 smaller than the theoretically predicted value. They proposed an explanation for this anomalously low electron-energy-loss rate based on the presence of a nonequilibrium population of optical phonons in excess of what is to be expected at the lattice temperature. Although the question is still open as to whether the reduced energy loss rate is due to the presence of hot phonons or due to reduced dimensionality and screening,[6,7] substantial study has been stimulated on the interplay between hot electrons and nonequilibrium phonons.[8-11] Hot-phonon dynamics have been extensively explored for bulk semiconductors by means of a phenomenological Boltzmann equation and by Monte Carlo techniques.[9,10] The theoretical analysis of the interaction of quasi-2D electrons (localized in a thin layer) with three-dimensional (3D) phonons is still in an early stage of development. It is believed that the lattice modes in a GaAs heterosystem are essentially those of the bulk, and most earlier work is based on a 3D plane-wave description of the phonon modes. For heterostructures this brings up the matter of how one integrates into the theory an appropriate description of the spatial extension over which the phonon modes participate in interaction with the quasi-2D electrons.[8] Very recently, Cai, Marchetti, and Lax[11] advanced an alternative description of the optical phonon coupling with quasi-2D electrons in a single-heterojunction system based on a localized "physical phonon wave packet" matching with the wave function of the 2D electrons.

The main concern of this paper lies with closely packed multilayer quantum-well systems, which are experimentally widely used, and are theoretically something in between a 3D bulk system and a single quasi-2D layer. Furthermore, we shall explore the ramifications of hot-phonon dynamical interactions on other transport properties in the presence of an electric field. The investigations reported here are based on the recently developed balance equation description of high-field electronic transport of Lei and Ting,[12] without invoking the phenomenological Boltzmann equation. The balance equation approach has proven useful in the analysis of steady-state transport in bulk,[12] quasi-2D heterojunctions,[13] and semiconductor superlattices[14] in the presence of a uniform electric field. This theoretical approach has also been successfully extended to studies of high-frequency conduction,[15] transient[16] and fluctuation[17] phenomena, as well as magneto transport.[18] In this paper we shall extend the hot-electron-transport balance-equation formulation to encompass finite-phonon relaxation, setting the stage for the examination of all the transport properties cited above in the presence of a nonequilibrium phonon population. The organization of this paper is as follows: In Sec. II we discuss 3D bulk systems, in Sec. III quasi-2D systems are examined, and in Sec. IV the problem of multilayer quantum wells is treated, all for hot electron transport with a

nonequilibrium phonon population. Numerical results are presented and discussed in Sec. V for linear and nonlinear mobilities, electron temperatures, and electron-energy transfer rates in single and multilayer heterostructures.

II. 3D SYSTEMS

We consider first a bulk 3D electron-phonon system in the presence of a uniform electric field $\mathbf{E}$. There are N electrons, interacting with each other through the Coulomb potential and scattered by randomly distributed impurities (H_{ei}). The phonon normal modes (Hamiltonian H_p) are the usual lattice waves specified by wave vector $\mathbf{q}$ and branch index λ. They are coupled among themselves (H_{pp}), and their coupling with electrons ($H_{\rm ep}$) provides the only mechanism for energy exchange between these two subsystems. If the electronic degrees of freedom are separated into those of center-of-mass motion (H_c) and relative motion (H_e), the total Hamiltonian of the system can be written as

$$H=H_c+H_e+H_p+H_{ei}+H_{ep}+H_{pp} ,$$

where

$$H_c=\frac{\mathbf{p}^2}{2Nm}-Ne\mathbf{E}\cdot\mathbf{R} , \tag{1}$$

$$H_e=\sum_{\mathbf{k},\sigma}\varepsilon_{\mathbf{k}}c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+\frac{1}{2V}\sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q}\\ \sigma,\sigma'}}v_c(q)c^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma}c^{\dagger}_{\mathbf{k}'-\mathbf{q},\sigma'}c_{\mathbf{k}'\sigma'}c_{\mathbf{k}\sigma} , \tag{2}$$

$$H_p=\sum_{\mathbf{q},\lambda}\hbar\Omega_{\mathbf{q}\lambda}b^{\dagger}_{\mathbf{q}\lambda}b_{\mathbf{q}\lambda} , \tag{3}$$

$$H_{ep}=\frac{1}{V^{1/2}}\sum_{\mathbf{q},\lambda}M(\mathbf{q},\lambda)(b_{\mathbf{q}\lambda}+b^{\dagger}_{-\mathbf{q}\lambda})e^{i\mathbf{q}\cdot\mathbf{R}}\rho_{\mathbf{q}} . \tag{4}$$

The form of H_{ei} and all the results related to impurity scattering are exactly the same as in Ref. 12 and will not be given in the present paper.

In these equations, $\mathbf{P}$ and $\mathbf{R}$ are the center-of-mass momentum and position, $c^{\dagger}_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are creation and annihilation operators for relative electrons with wave vector $\mathbf{k}$, spin σ and energy $\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m$. $\rho_{\mathbf{q}}=\sum_{\mathbf{k},\sigma}c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}$ is the electron-density operator, $v_c(q)=e^2/\varepsilon_0\kappa q^2$ is the Coulomb potential (κ is the dielectric constant of the material); $b^{\dagger}_{\mathbf{q}\lambda}$ and $b_{\mathbf{q}\lambda}$ are creation and annihilation operators for phonons of wave vector $\mathbf{q}$, branch λ, and energy $\hbar\Omega_{\mathbf{q}\lambda}$. Finally $M(\mathbf{q},\lambda)$ is the electron-phonon matrix element, whose dependence on volume V is explicitly written in Eq. (4). The number operator of phonons in mode $\mathbf{q},\lambda$ is

$$\hat{N}_{\mathbf{q}\lambda}=b^{\dagger}_{\mathbf{q}\lambda}b_{\mathbf{q}\lambda} , \tag{5}$$

and the rate of change of $\hat{N}_{\mathbf{q}\lambda}$ due to electron-phonon interaction is given by

$$\frac{\partial}{\partial t}\hat{N}_{\mathbf{q}\lambda}\Big|_{ep}=-\frac{i}{\hbar}[\hat{N}_{\mathbf{q}\lambda},H_{ep}]$$
$$=\frac{i}{\hbar V^{1/2}}M(\mathbf{q},\lambda)e^{i\mathbf{q}\cdot\mathbf{R}}b_{\mathbf{q}\lambda}\rho_{\mathbf{q}}$$
$$-\frac{i}{\hbar V^{1/2}}M(-\mathbf{q},\lambda)e^{-i\mathbf{q}\cdot\mathbf{R}}b^{\dagger}_{\mathbf{q}\lambda}\rho_{-\mathbf{q}} . \tag{6}$$

The density matrix ρ for the relative electron plus phonon system satisfies the Liouville equation

$$i\hbar\frac{\partial\rho}{\partial t}=[H_e+H_p+H_I,\rho] , \tag{7}$$

where $H_I=H_{ep}+H_{ei}+H_{pp}$. We consider the electron-electron Coulomb interaction to be strong and we always include it in H_e. However, the electron-impurity interaction H_{ei}, the electron-phonon interaction H_{ep}, and the phonon-phonon interaction H_{pp} are all assumed to be relatively weak and we treat them on an equal footing. The initial condition for the Liouville equation can be obtained as follows. Imagine we turn off the interaction H_I after the system has reached the steady state. Then, not only are the center-of-mass and the relative electrons decoupled from one another, but so are the different phonon modes. Therefore, they will independently approach their own equilibrium states with separate temperatures. To be more specific, the relative electron system will quickly reach a thermal equilibrium state with a single temperature T_e due to the strong Coulomb interaction between electrons. The different phonon normal modes, however, may have different temperatures since they are decoupled from each other. The density matrix for these decoupled systems takes the form

$$\rho_0=\frac{1}{Z_e}\exp(-H_e/k_BT_e)\frac{1}{Z_p}\exp\left[-\sum_{\mathbf{q},\lambda}H(\mathbf{q}\lambda)/T_{\mathbf{q}\lambda}\right]$$
$$=\frac{1}{Z}e^{-H_0/k_BT_e} . \tag{8}$$

(Z_e, Z_p, and Z are defined by the requirement that the trace of the density matrices be unity.) Here $T_{\mathbf{q}\lambda}$ is the phonon temperature of mode $\mathbf{q}\lambda$, $H(\mathbf{q}\lambda)=\hbar\Omega_{\mathbf{q}\lambda}b^{\dagger}_{\mathbf{q}\lambda}b_{\mathbf{q}\lambda}$, and we have defined H_0 by

$$H_0\equiv H_e+\sum_{\mathbf{q},\lambda}\alpha_{\mathbf{q}\lambda}H(\mathbf{q}\lambda) , \tag{9}$$

$$\alpha_{\mathbf{q}\lambda}\equiv T_e/T_{\mathbf{q}\lambda} . \tag{10}$$

For an adiabatic introduction of the interaction H_I from $t=-\infty$, this ρ_0 will serve as our initial condition for the differential equation (7). To the first order in H_I, the Liouville equation (7) with the initial condition $\rho(t=-\infty)=\rho_0$ has the solution

$$\rho(t)=\rho_0+\frac{i}{\hbar}\int_{-\infty}^{t}e^{\varepsilon t'}\exp\left[\frac{i}{\hbar}\sum_{\mathbf{q},\lambda}(1-\alpha_{\mathbf{q}\lambda})H(\mathbf{q}\lambda)(t'-t)\right][\rho_0,H_I(t'-t)]\exp\left[-\frac{i}{\hbar}\sum_{\mathbf{q},\lambda}(1-\alpha_{\mathbf{q}\lambda})H(\mathbf{q}\lambda)(t'-t)\right]dt' , \tag{11}$$

where

$$H_I(t)\equiv e^{(i/\hbar)H_0 t}H_I e^{-(i/\hbar)H_0 t} . \tag{12}$$

The statistical average of any dynamical variable $\hat{O}$ can be calculated from

$$\langle \hat{O} \rangle = \mathrm{Tr}[\rho(t)\hat{O}] .$$

The results for (a) the frictional forces experienced by the center-of-mass, $\mathbf{F}=\mathbf{F}_i+\mathbf{F}_p$ ($\mathbf{F}_i$ is due to impurities and $\mathbf{F}_p$ is due to phonons), and (b) the energy transfer rate from the electron system to phonon system W, are readily determined as in Ref. 12:

$$\mathbf{F}_p = -\frac{2}{V}\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \mathbf{q}\Pi_2(\mathbf{q},\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v}) \left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] - n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right]\right] , \tag{13}$$

$$W = \frac{2}{V}\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda}\Pi_2(\mathbf{q},\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v}) \left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] - n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right]\right] . \tag{14}$$

The expression for $\mathbf{F}_i$ is the same as that in Ref. 12. The rate of change of the phonon number in mode $\mathbf{q}\lambda$ due to electron-phonon interaction can similarly be obtained as

$$\begin{aligned}\left[\frac{\partial N_{\mathbf{q}\lambda}}{\partial t}\right]_{ep} &= -\frac{i}{\hbar}\langle [\hat{N}_{\mathbf{q}\lambda}, H_{ep}]\rangle \\ &= \frac{2}{\hbar V}|M(\mathbf{q},\lambda)|^2 \Pi_2(\mathbf{q},\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v}) \left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] - n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right]\right] .\end{aligned} \tag{15}$$

In these equations $n(x)=(e^x-1)^{-1}$ is the Bose function, $\Pi(\mathbf{q},\omega)=\Pi_1(\mathbf{q},\omega)+i\Pi_2(\mathbf{q},\omega)$ is the electron density-density correlation function, and $\mathbf{v}$ is the velocity of the center of mass, i.e., the drift velocity of the whole electron system. It is interesting to note that Eqs. (13) and (14) can be written as

$$\begin{aligned}\mathbf{F}_p &= -\sum_{\mathbf{q},\lambda}\hbar\mathbf{q}\left[\frac{\partial N_{\mathbf{q}\lambda}}{\partial t}\right]_{ep} , \\ W &= \sum_{\mathbf{q},\lambda}\hbar\Omega_{\mathbf{q}\lambda}\left[\frac{\partial N_{\mathbf{q}\lambda}}{\partial t}\right]_{ep} .\end{aligned} \tag{16}$$

The determination of the rate of change of phonon occupation number due to all effects other than the electron-phonon interaction, $(\partial N_{\mathbf{q}\lambda}/\partial t)_{pp}$, cannot be carried out without a detailed analysis of anharmonic coupling, as well as structure and boundary effects, which are sample dependent. Therefore we will represent them by an effective phonon relaxation time parameter τ_p, writing

$$\left[\frac{\partial N_{\mathbf{q}\lambda}}{\partial t}\right]_{pp} = -\left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] - n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T}\right]\right] \Big/ \tau_p , \tag{17}$$

where T is the lattice-bath temperature. The phonon relaxation time τ_p may be mode dependent. In steady state, the occupation number for each phonon mode should be constant, whence

$$0 = \frac{d}{dt}N_{\mathbf{q}\lambda} = \left[\frac{\partial N_{\mathbf{q}\lambda}}{\partial t}\right]_{ep} - \frac{1}{\tau_p}\left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] - n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T}\right]\right] ,$$

leading to

$$n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T_{\mathbf{q}\lambda}}\right] = n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right] + [1+\tau_p\Gamma(\mathbf{q}\lambda,\Omega_{\mathbf{q}\lambda}-\mathbf{q}\cdot\mathbf{v})]^{-1}\left[n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T}\right] - n\left[\frac{\hbar(\Omega\mathbf{q}\lambda-\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right]\right] ,$$

in which

$$\Gamma(\mathbf{q}\lambda,\omega) = -\frac{2}{\hbar V}|M(\mathbf{q},\lambda)|^2\Pi_2(\mathbf{q},\omega) . \tag{18}$$

Therefore the phonon-induced frictional force and the energy transfer rate in steady state may be written as

$$\mathbf{F}_p = \sum_{\mathbf{q},\lambda}\hbar\mathbf{q}\frac{\Gamma(\mathbf{q}\lambda,\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(\mathbf{q}\lambda,\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}\left[n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}{k_B T_e}\right] - n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_B T}\right]\right] , \tag{19}$$

$$W=\sum_{\mathbf{q},\lambda}\hbar\Omega_{\mathbf{q}\lambda}\frac{\Gamma(\mathbf{q}\lambda,\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(\mathbf{q}\lambda,\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}\left[n\left[\frac{\hbar(\Omega_{\mathbf{q}\lambda}+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega_{\mathbf{q}\lambda}}{k_BT}\right]\right]. \tag{20}$$

Equations (18)—(20) are the main results of this section. If the phonon relaxation is fast enough that $\tau_p\Gamma\ll 1$, Eqs. (18)—(20) reduce to the results of Ref. 12. Otherwise the energy transfer rate for given T, T_e, and v, is evidently reduced. For a finite phonon relaxation time, the force and energy balance equations in the steady state

$$N_e\mathbf{E}+\mathbf{F}=0\ ,$$

$$\mathbf{v}\cdot\mathbf{F}+W=0\ ,$$

with the $\mathbf{F}_p$ and W given in Eqs. (19) and (20), constitute a complete set of equations to determine electron temperature and drift velocity for a given electric field $\mathbf{E}$ and phonon relaxation time τ_p.

III. QUASI-2D SYSTEMS

In a quasi-two-dimensional system, e.g., a single GaAs-$Al_xGa_{1-x}As$ heterojunction, electrons move freely parallel to the interface (x-y plane) but their z motion is confined to the vicinity of the surface. The single-electron states can be represented by a 2D wave vector $\mathbf{k}=(k_x,k_y)$ and a subband index n:

$$\psi_{n\mathbf{k}}(\mathbf{r},z)=\frac{1}{S^{1/2}}e^{i\mathbf{k}\cdot\mathbf{r}}\zeta_n(z)\quad(n=0,1,\ldots)\ , \tag{21}$$

with energy

$$E_{n\mathbf{k}}=E_n+\frac{\hbar^2k^2}{2m}\ .$$

Here $\zeta_n(z)$ is the subband envelope function,[19] S is the layer area and m is the electron-band effective mass.

The balance equations for electron transport parallel to the interface in the presence of a uniform electric field $\mathbf{E}$ (within the x-y plane) will be discussed on the basis of a separation of 2D center-of-mass variables,[13] $\mathbf{P}=(P_x,P_y)$ and $\mathbf{R}=(X,Y)$, from the relative variables of the electrons. The Hamiltonian for the relative electron system can be written as

$$H_e=\sum_{n,\mathbf{k},\sigma}E_{n\mathbf{k}}c^\dagger_{n\mathbf{k}\sigma}c_{n\mathbf{k}_\sigma}+\tfrac{1}{2}\sum_{\substack{m'm,n'n\\ \mathbf{k},\mathbf{k}_1,\mathbf{q}\\ \sigma,\sigma_1}}\frac{e^2}{2\varepsilon_0\kappa q}H_{m'm,n'n}(q)c^\dagger_{m'\mathbf{k}+\mathbf{q}\sigma}c^\dagger_{n'\mathbf{k}_1-\mathbf{q}\sigma_1}c_{n\mathbf{k}_1\sigma_1}c_{n\mathbf{k}\sigma}\ , \tag{22}$$

with

$$H_{m'm,n'n}(q)=\int dz_1\int dz_2\zeta^*_{m'}(z_1)\zeta_m(z_1)\zeta^*_{n'}(z_2)\zeta_n(z_2)e^{-q|z_1-z_2|}\ , \tag{23}$$

and κ is the dielectric constant of the GaAs system. In the following, the envelope functions, $\zeta_n(z)$ are chosen to be real so that

$$H_{m'm,n'n}(q)=H_{mm',n'n}(q)=H_{n'n,m'm}(q)\ .$$

The phonons in a GaAs-AlGaAs heterosystem are reasonably assumed to be the same as those in bulk GaAs. Their normal modes can be described by a 3D wave vector $\mathbf{Q}=(\mathbf{q},q_z)$ and a branch index λ:

$$H_p=\sum_{\mathbf{Q},\lambda}\hbar\Omega_{\mathbf{Q}\lambda}b^\dagger_{\mathbf{Q}\lambda}b_{\mathbf{Q}\lambda}\ , \tag{24}$$

where $b^\dagger_{\mathbf{Q}\lambda}$ ($b_{\mathbf{Q}\lambda}$) are creation (annihilation) operators of the mode $\mathbf{Q},\lambda$. For acoustic phonons, hot phonon effects are believed to be relatively unimportant and their contributions in force and energy balance equations have been examined in Ref. 13 in detail. In this paper we will concentrate on polar optic phonons, which are the dominant lattice scatterers in GaAs-$Al_xGa_{1-x}As$ for electron temperature $T_e\geq 50$ K. Polar-optic phonons are essentially dispersionless

$$H_p=\sum_{\mathbf{q},q_z}\hbar\Omega b^\dagger_{\mathbf{q}q_z}b_{\mathbf{q}q_z}\ , \tag{25}$$

and the electron-polar-optic-phonon coupling takes the form

$$H_{ep}=\left[\frac{1}{SL}\right]^{1/2}\sum_{\substack{\mathbf{q},q_z\\ n',n}}M(\mathbf{q},q_z)I_{n'n}(iq_z)\times e^{-i\mathbf{q}\cdot\mathbf{R}}(b_{\mathbf{q}q_z}+b^\dagger_{-\mathbf{q},-q_z})\rho_{n'n\mathbf{q}}\ , \tag{26}$$

where

$$|M(\mathbf{q},q_z)|^2=\frac{F_r}{Q^2}\equiv\frac{e^2\hbar\Omega}{2\varepsilon_0}\left[\frac{1}{\kappa_\infty}-\frac{1}{\kappa}\right]\frac{1}{Q^2} \tag{27}$$

is the Fröhlich matrix element and κ_∞ is the high-frequency dielectric constant of GaAs. In Eq. (26),

$$\rho_{n'n\mathbf{q}}=\sum_{\mathbf{k},\sigma}c^\dagger_{n'\mathbf{k}+\mathbf{q}\sigma}c_{n\mathbf{k}\sigma} \tag{28}$$

is the electron density operator, and

$$I_{n'n}(iq_z)=\int e^{iq_z z}\zeta^*_{n'}(z)\zeta_n(z)dz \tag{29}$$

is a form factor, L being the size of the system along the z direction.

Needless to say, the electron-phonon coupling is spatially inhomogeneous in nature due to electron localization in the z direction, so that emission and absorption of phonons occur mainly in the region where the electrons have substantial density. If phonon relaxation is very fast this makes no essential difference since the lattice is then always in its equilibrium state everywhere and we can proceed exactly as was done in Ref. 13. However, in the case of finite phonon relaxation, as Cai, Marchetti, and Lax[11] have pointed out, the electron-phonon interaction results in a spatially inhomogeneous phonon distribution in the z direction, and the one-body density matrix of the phonon system is nondiagonal in the z component of momentum in the plane wave representation. Its off-diagonal elements are inversely proportional to the spatial extension over which the electron-phonon interaction is effective. Therefore, it is natural to choose a set of dynamical variables

$$\hat{P}_m=b^\dagger_{\mathbf{q}q_z}b_{\mathbf{q}q_z'}\quad(m=\mathbf{q},q_z,q_z') \tag{30}$$

to describe the phonon system. In order to select an appropriate initial state density matrix, we imagine turning off the electron-phonon interaction and phonon-phonon interaction. The decoupled electron system will quickly go to a thermoequilibrium state described by a Gibbs ensemble with a single temperature T_e. The phonon system, on the other hand, will be much slower to relax (note that we turn off the phonon-phonon interaction also). For a quasi-2D electron distribution of small spatial extension, the nonequilibrium phonon state is rather far removed from plane-wave normal modes. Therefore it is reasonable to assume that the phonon system will first approach a quasiequilibrium state described by a density matrix of the form

$$\rho_p^0=\frac{1}{Z_p}\exp\left[-\sum_m F_m\hat{P}_m\right] \tag{31}$$

before it reaches the ultimate equilibrium state for each normal mode. The parameters F_m here are functions of the expectation values of the operators $\langle\hat{P}_m\rangle\cdots$, which are determined by the condition

$$\langle\hat{P}_m\rangle=\mathrm{Tr}(\rho_p^0\hat{P}_m)\ . \tag{32}$$

This kind of quasiequilibrium statistical operator has been discussed extensively in Refs. 20 and 21. Thus we will use the density matrix

$$\rho^0=\frac{1}{Z_e}\exp(-H_e/k_BT_e)\rho_p^0 \tag{33}$$

as the initial condition of the Liouville equation (7) for the nonequilibrium density matrix $\rho(t)$ instead of using ρ_0 in Eq. (8). This is to say that we expect that the electron-phonon system will approach its steady state (after turning on the interaction and electric field adiabatically) in a shorter time starting from the initial quasiequilibrium statistical state (33) than it would starting from a state (8).

The solution of the Liouville equation for the nonequilibrium density matrix ρ with the initial condition (33) can be obtained following the same procedure described in Refs. 20 and 21 to the lowest order in H_I:

$$\rho(t)=\rho^0+\frac{i}{\hbar}\int_{-\infty}^0 d\tau e^{\varepsilon\tau}[\rho^0,H_I(\tau)]\ , \tag{34}$$

where

$$H_I(\tau)=\exp\left[\frac{i}{\hbar}(H_e+H_p)\tau\right]H_I\exp\left[-\frac{i}{\hbar}(H_e+H_p)\tau\right]\ .$$

In obtaining this result we have made use of the fact that $\mathrm{Tr}\{\rho^0[H_I,\hat{P}_m]\}=0$. The average value of a dynamical variable $\hat{O}$ can therefore be calculated within this framework as

$$\begin{aligned}\langle\hat{O}\rangle&=\mathrm{Tr}[\rho(t)\hat{O}]\\&=\mathrm{Tr}[\rho^0\hat{O}]+\frac{i}{\hbar}\int_{-\infty}^0 d\tau e^{\varepsilon\tau}\mathrm{Tr}\{\rho^0[H_I(\tau),\hat{O})\}.\end{aligned} \tag{35}$$

In the following we define

$$g_{n'n}(\mathbf{q},q_z)=\frac{M(\mathbf{q},\mathbf{q}_z)}{L^{1/2}M_{n'n}(q)}I_{n'n}(iq_z)\ , \tag{36}$$

$$\begin{aligned}M_{n'n}(q)^2&=\frac{1}{L}\sum_{q_z}|M(\mathbf{q},\mathbf{q}_z)|^2|I_{n'n}(iq_z)|^2\\&=\frac{F_r}{2q}H_{n'n,n'n}(q)\ ,\end{aligned} \tag{37}$$

and introduce the creation and annihilation operators of "quasi-2D" phonons

$$\begin{aligned}A_{n'n}(\mathbf{q})&=\sum_{q_z}g_{n'n}(\mathbf{q},q_z)b_{\mathbf{q}q_z}\ ,\\A^\dagger_{n'n}(\mathbf{q})&=\sum_{q_z}g^*_{n'n}(\mathbf{q},q_z)b^\dagger_{\mathbf{q}q_z}\ ,\end{aligned} \tag{38}$$

which satisfy the commutation relations

$$[A_{n'n}(\mathbf{q}),A^\dagger_{m'm}(\mathbf{q})]=\beta_{n'n,m'm}(q)\delta_{\mathbf{q},\mathbf{q}'}\ , \tag{39}$$

with

$$\beta_{n'n,m'm}(q)=\frac{H_{n'n,mm'}(q)}{[H_{n'n,nn'}(q)H_{m'm,mm'}(q)]^{1/2}}\ , \tag{40}$$

whose diagonal part is $\beta_{n'n,n'n}(q)=1$. We also need the operators

$$\hat{N}_{m'm,n'n}(\mathbf{q})=A^\dagger_{m'm}(\mathbf{q})A_{n'n}(\mathbf{q})\ , \tag{41}$$

whose diagonal parts

$$\hat{N}_{n'n,n'n}(\mathbf{q})=A^\dagger_{n'n}(\mathbf{q})A_{n'n}(\mathbf{q}) \tag{42}$$

describe the occupation number operators of the quasi-2D phonons. The physical significance of these phonon operators is elucidated by considering the occupation number operator associated with subband n:

$$\hat{N}_n(\mathbf{q})=\sum_{n'}\hat{N}_{n'n,n'n}(\mathbf{q})=\int dz\,|\,\zeta_n(z)\,|^2 b_{\mathbf{q}}^{\dagger}(z)b_{\mathbf{q}}(z)\ . \tag{43}$$

Here

$$b_{\mathbf{q}}^{\dagger}(z)=\sum_{q_z}e^{iq_z z}b_{\mathbf{q}q_z}^{\dagger}\ ,$$

$$b_{\mathbf{q}}(z)=\sum_{q_z}e^{-iq_z z}b_{\mathbf{q}q_z}$$

are the creation and annihilation operators, respectively, of phonons with wave vector $\mathbf{q}$ which are localized at z. Thus, it is apparent that $\hat{N}_n(\mathbf{q})$ is the occupation number operator for phonons having wave vector $\mathbf{q}$, located within the region where the electron density of subband n is nonzero. It is reasonable to identify these phonons as quasi-2D phonons. In obtaining Eq. (43) we have made use of the completeness of the envelope functions $\sum_n \zeta_n^*(z)\zeta_n(z')=\delta(z-z')$.

In terms of these quasi-2D phonon operators, the electron-phonon interaction [Eq. (26)] can be written as

$$H_{ep}=\frac{1}{S^{1/2}}\sum_{n',n,\mathbf{q}}M_{n'n}(q)e^{i\mathbf{q}\cdot\mathbf{R}}[A_{n'n}(\mathbf{q})+A_{nn'}^{\dagger}(-\mathbf{q})]\rho_{n'n\mathbf{q}}\ , \tag{44}$$

and the force operator acting on the center of mass is

$$\hat{F}_p=-\frac{i}{\hbar}[\mathbf{P},H_{ep}]=-\frac{i}{S^{1/2}}\sum_{n',n,\mathbf{q}}M_{n'n}(q)\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{R}}[A_{n'n}(\mathbf{q})+A_{nn'}^{\dagger}(-\mathbf{q})]\rho_{n'n\mathbf{q}}\ , \tag{45}$$

with the energy-transfer rate operator given by

$$\hat{W}=-\frac{i}{\hbar}[H_p,H_{ep}]=\frac{i}{S^{1/2}}\sum_{n',n,\mathbf{q}}M_{n'n}(q)\Omega e^{i\mathbf{q}\cdot\mathbf{R}}[A_{n'n}(\mathbf{q})-A_{nn'}^{\dagger}(-\mathbf{q})]\rho_{n'n\mathbf{q}}\ . \tag{46}$$

We also identify the rate of change of the generalized occupation number operator for quasi-2D phonons $\hat{N}_{n'n,m'm}$ in terms of two parts, one due to the electron-phonon (ep) interaction and the other due to phonon-phonon (pp) interaction

$$\frac{d}{dt}\hat{N}_{n'n,m'm}(\mathbf{q})=\frac{\partial}{\partial t}\hat{N}_{n'n,m'm}(\mathbf{q})\,|_{ep}+\frac{\partial}{\partial t}\hat{N}_{n'n,m'm}(\mathbf{q})\,|_{pp}\ . \tag{47}$$

The first part is given by

$$\begin{aligned}\frac{\partial}{\partial t}\hat{N}_{n'n,m'm}(\mathbf{q})\,|_{ep}&=-\frac{i}{\hbar}[\hat{N}_{n'n,m',m}(\mathbf{q}),H_{ep}]\\&=\frac{i}{\hbar S^{1/2}}\sum_{l',l}M_{l'l}(q)[\beta_{l'l,n',n}(q)A_{m'm}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{R}}\rho_{l'l\mathbf{q}}-\beta_{m'm,ll'}(q)A_{n'n}^{\dagger}(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{R}}\rho_{l'l,-\mathbf{q}}]\ .\end{aligned} \tag{48}$$

The second part,

$$\frac{\partial}{\partial t}\hat{N}_{n'n,m'm}(\mathbf{q})\,|_{pp}=-\frac{i}{\hbar}[\hat{N}_{n'n,m'm}(\mathbf{q}),H_p+H_{pp}]\ , \tag{49}$$

cannot be explicitly evaluated without a specific expression for H_{pp}. We will employ a relaxation time approximation for it.

The expectation values of $\hat{F}$, $\hat{W}$, and $(\partial\hat{N}_{n'n,m',m}/\partial t)_{ep}$ may now be obtained by a straightforward calculation as

$$\mathbf{F}_p=\langle\hat{F}_p\rangle=-\frac{2}{S}\sum_{n',n,\mathbf{q}}\mathbf{q}M_{n'n}(q)^2\Pi_2(n',n,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[N_{n'n,m'm}(\mathbf{q})-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]\ , \tag{50}$$

$$W=\langle\hat{W}\rangle=\frac{2}{S}\sum_{n',n,\mathbf{q}}\Omega M_{n'n}(q)^2\Pi_2(n',n,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[N_{n'n,m'm}(\mathbf{q})-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]\ , \tag{51}$$

$$\begin{aligned}\frac{\partial}{\partial t}N_{n'n,m'm}(\mathbf{q})\,|_{ep}=&\frac{i}{\hbar S}\sum_{l',l}M_{l'l}(q)^2[\beta_{l'l,n'n}(q)N_{l'l,m'm}(\mathbf{q})\Pi(l',l,\mathbf{q},-\Omega+\mathbf{q}\cdot\mathbf{v})-\beta_{m'm,ll'}(q)N_{n'n,ll'}(\mathbf{q})\Pi(l',l,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})]\\&-\frac{2}{\hbar S}\sum_{l'l}M_{l'l}(q)^2\beta_{l'l,n'n}(q)\beta_{m'm,ll'}(\mathbf{q})\Pi_2(l',l,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\ n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\ .\end{aligned} \tag{52}$$

In these equations, $\Pi=\Pi_1+i\Pi_2$ is the Fourier transform of the diagonal part of the electron density-density correlation function defined by

$$\Pi(n',n,\mathbf{q},t)=-\frac{i}{\hbar}\Theta(t)\langle[\rho_{n'n\mathbf{q}}(t),\rho_{nn',-q}]\rangle \tag{53}$$

[$\Theta(t)=1$ for $t>0$, otherwise $\Theta(t)=0$]; its off-diagonal parts are neglected. Finally,

$$N_{n'n,m'm}(\mathbf{q})=\langle \hat{N}_{n'n,m'm}(\mathbf{q})\rangle=\langle \hat{N}_{n'n,m'm}(\mathbf{q})\rangle_0 \tag{54}$$

is the generalized occupation number of the quasi-2D phonons in the nonequilibrium state. $\langle\cdots\rangle_0$ denotes the average with the initial density matrix ρ^0. The last equality is due to the definition of the quasiequilibrium statistical operator.[20,21] Including the contribution of the last term of Eq. (47) in a relaxation time approximation, we obtain a kinetic equation for the generalized occupation number of quasi-2D phonons

$$\frac{d}{dt}N_{n'n,m'm}(\mathbf{q})=\frac{\partial}{\partial t}N_{n'n,m'm}(\mathbf{q})\,|_{ep}-\frac{1}{\tau_p}[N_{n'n,m'm}(\mathbf{q})-N_{\rm eq}]\ , \tag{55}$$

in which $N_{\rm eq}$ is the equilibrium average of $\hat{N}_{n'n,m'm}(\mathbf{q})$:

$$N_{\rm eq}=\langle \hat{N}_{n'n,m'm}(\mathbf{q})\rangle_{\rm eq}=\langle A^{\dagger}_{n'n}(\mathbf{q})A_{m'm}(\mathbf{q})\rangle_{\rm eq}$$

$$=\sum_{q_z,q_z'} g^{*}_{n'n}(\mathbf{q},q_z)g_{m'm}(\mathbf{q},q_z)\langle b^{\dagger}_{\mathbf{q}q_z}b_{\mathbf{q}q_z'}\rangle_{\rm eq}=\sum_{q_z} g^{*}_{n'n}(\mathbf{q},q_z)g_{m'm}(\mathbf{q},q_z)n\left[\frac{\hbar\Omega}{k_BT}\right] \tag{56}$$

$$=\beta_{n'n,m'm}(q)n\left[\frac{\hbar\Omega}{k_BT}\right]\ . \tag{57}$$

In deriving Eq. (57), we assumed the optic phonon to be dispersionless (Ω is independent of q_z). This limitation does not apply to Eq. (56) for $N_{\rm eq}$. In Eq. (55), the quasi-2D phonon relaxation time τ_p may be $\mathbf{q}$ dependent.

In the steady state $dN_{m'm,m'm}(\mathbf{q})/dt=0$, and the kinetic equation (55) reduces to algebraic equations for occupation numbers $N_{m'm,m'm}(\mathbf{q})$, which, taken jointly with the force- and energy-balance equations $Ne\mathbf{E}+\mathbf{F}=\mathbf{0}$ and $\mathbf{v}\cdot\mathbf{F}+W=0$, form a complete set of equations to determine the steady-state transport of the system.

These equations are greatly simplified if the electron sheet density is low and thermal excitation across subbands is negligible, so that only the lowest subband needs to be taken into account. In this case Eqs. (50) and (51) reduce to [with $M(q)\equiv M_{00}(q)$, $\Pi(\mathbf{q},\omega)\equiv(0,0,\mathbf{q},\omega)$, and $N(\mathbf{q})=N_{00,00}(\mathbf{q})$]

$$\mathbf{F}_p=-\sum_{\mathbf{q}}\hbar\mathbf{q}\left[\frac{\partial N(\mathbf{q})}{\partial t}\right]_{ep}\ , \tag{58}$$

$$W=\sum_{\mathbf{q}}\hbar\Omega\left[\frac{\partial N(\mathbf{q})}{\partial t}\right]_{ep}\ , \tag{59}$$

with

$$\left[\frac{\partial N(\mathbf{q})}{\partial t}\right]_{ep}=\frac{2}{\hbar S}M(q)^2\Pi_2(\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\times\left[N(\mathbf{q})-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_bT_e}\right]\right]\ , \tag{60}$$

and the steady-state nonequilibrium phonon occupation number $N(\mathbf{q})$ can be solved from Eq. (55):

$$\frac{2}{\hbar S}M(q)^2\Pi_2(\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[N(\mathbf{q})-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]-\frac{1}{\tau_p}\left[N(\mathbf{q})-n\left[\frac{\hbar\Omega}{k_BT}\right]\right]=0\ . \tag{61}$$

These result in simple expressions for the steady-state force- and energy-transfer rate:

$$\mathbf{F}_p=\sum_{\mathbf{q}}\hbar\mathbf{q}\frac{\Gamma(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}\times\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right] \tag{62}$$

and

$$W=\sum_{\mathbf{q}}\hbar\Omega\frac{\Gamma(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}\times\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_bT}\right]\right]\ , \tag{63}$$

with

$$\Gamma(\mathbf{q},\omega)=-\frac{2}{\hbar S}M(q)^2\Pi_2(\mathbf{q},\omega)\ . \tag{64}$$

It should be noted that the nonequilibrium phonon occupation number $N(\mathbf{q})$ does not appear in the electron force- and energy-balance equations. All that is required is a knowledge of the optic phonon relaxation time τ_p.

Equations (58) and (59) and the commutation relation $[A(\mathbf{q}),A^{\dagger}(\mathbf{q}')]=\delta_{\mathbf{q}\mathbf{q}'}$ ($A(\mathbf{q})=A_{00,00}(\mathbf{q})$) indicate that the phonon system is two dimensional in nature (with normal modes labeled by 2D wave vector $\mathbf{q}$) in its coupling with the 2D electrons. In fact, if we assume that the Hamiltonian of this 2D phonon system is $H_p=\sum_{\mathbf{q}}\hbar\Omega A^{\dagger}(\mathbf{q})A(\mathbf{q})$, then, instead of a quasiequilibrium statistical operator, we have a Gibbs ensemble as the starting point. Exactly the same results could be obtained by a derivation based on the procedure described in Sec. I.

IV. SUPERLATTICE SYSTEMS

The superlattice model which we will use to represent multilayer heterostructure systems consists of an infinite

number of periodically arranged quantum wells of width a, and d is the spatial period. We assume that the electrons occupy only the lowest subband in each well and that excitation to higher subbands and tunneling to neighboring wells are unimportant. In the effective mass approximation the electron wave function can be written as

$$\psi_{l\mathbf{k}}(\mathbf{r},z)=\frac{1}{S^{1/2}}e^{i\mathbf{k}\cdot\mathbf{r}}\zeta(z-ld)\quad(l=0,\pm1,\ldots)\ , \tag{65}$$

where $\zeta(z)$ is the envelope wave function for the lowest subband. The corresponding energy $\varepsilon_k=\hbar^2k^2/2m$ is degenerate with respect to the layer indices. The Hamiltonian H_e for the electron system has been given in Ref. 14. The coupling between electrons and a specific branch of phonons $(b_{\mathbf{Q}}^{\dagger},b_{\mathbf{Q}})$ can be written as

$$H_{ep}=\left[\frac{1}{SL}\right]^{1/2}\sum_{\mathbf{q},q_z,l}M(\mathbf{q},q_z)I(iq_z)\times e^{i\mathbf{q}\cdot\mathbf{R}}(b_{\mathbf{Q}}+b_{-\mathbf{Q}}^{\dagger})e^{-iq_zld}\rho_{l\mathbf{q}}\ , \tag{66}$$

with

$$\rho_{l\mathbf{q}}=\sum_{\mathbf{k},\sigma}c_{l\mathbf{k}+\mathbf{q}\sigma}^{\dagger}c_{l\mathbf{k}\sigma} \tag{67}$$

as the density operator for electrons in the lth quantum well and

$$I(iq_z)=\int e^{iq_zz}\,|\,\zeta(z)\,|^2dz\ . \tag{68}$$

Here $L=N_Ld$ is the superlattice extension in the z direction, and N_L the total number of quantum wells.

Balance equations for electron transport in a uniform parallel electric field $\mathbf{E}$ have been discussed in detail in the case of fast phonon relaxation.[14] Our present interest is mainly in the interplay of electrons with polar optic phonons, for which the hot-phonon effect is important.

A. 3D phonon model

In contrast to the case of a single heterojunction, where the electron density vanishes everywhere except in a thin layer near the interface, the case of a closely packed multilayer system is characterized by an electron density which exists periodically over the full space. This periodic density distribution of electrons seems not to be severely mismatched with 3D phonons. In other words, an infinite periodic wave packet may decay relatively rapidly to spatially uniform plane wave normal modes. Therefore, it seems plausible to use a 3D phonon model for a close-packed superlattice, just as for the 3D bulk system. In this model, the nonequilibrium phonon effects are taken into account by a mode-dependent phonon temperature $T_{\mathbf{Q}}$ such that the initial condition of the Liouville equation for the density matrix takes the form of Eq. (8). We can obtain the frictional force, the energy transfer rate and the rate of change of phonon occupation number $N_{\mathbf{q}q_z}$ by closely following the derivations in Sec. II and in Ref. 14. Thus we find

$$\mathbf{F}_p=-\frac{2}{SL}\sum_{\mathbf{q},q_z,\lambda}|\,M(\mathbf{q},q_z)\,|^2\,|\,I(iq_z)\,|^2\mathbf{q}\sum_{l,l'}e^{-iq_z(l-l')d}\Pi_2(l,l',\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar\Omega}{k_BT_{\mathbf{Q}}}\right]-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_bT_e}\right]\right]\ , \tag{69}$$

where $\Pi_2(l,l',\mathbf{q},\omega)$ is the imaginary part of the Fourier transform of the electron density-density correlation function:

$$\Pi(l,l',\mathbf{q},t)=-\frac{i}{\hbar}\Theta(t)\langle\rho_{l\mathbf{q}}(t),\rho_{l',-\mathbf{q}}(0)]\rangle\ . \tag{70}$$

For an infinite periodic system $\Pi(l,l',\mathbf{q},t)=\Pi(l-l',\mathbf{q},t)$ is a function of the difference $l-l'$ alone, so we can introduce

$$\Pi(q_z,\mathbf{q},\omega)=\sum_l e^{-iq_zld}\Pi(l,\mathbf{q},\omega) \tag{71}$$

and write the per-layer force due to phonons as

$$\mathbf{f}_p=-\frac{2}{S}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\mathbf{q}\,|\,M(\mathbf{q},q_z)\,|^2\,|\,I(iq_z)\,|^2\Pi_2(q_z,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar\Omega}{k_BT_{\mathbf{Q}}}\right]-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]\ . \tag{72}$$

A similar derivation leads to the per-layer energy transfer rate as

$$w=\frac{2}{S}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\Omega\,|\,M(\mathbf{q},q_z)\,|^2\,|\,I(iq_z)\,|^2\Pi_2(q_z,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar\Omega}{k_BT_{\mathbf{Q}}}\right]-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]\ , \tag{73}$$

and the rate of change of the phonon occupation number due to electron-phonon coupling is obtained as

$$\left[\frac{\partial}{\partial t}N_{\mathbf{q}q_z}\right]_{ep}=\frac{2}{\hbar Sd}\,|\,M(\mathbf{q},q_z)\,|^2\,|\,I(iq_z)\,|^2\Pi_2(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar\Omega}{k_BT_{\mathbf{Q}}}\right]-n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right]\ . \tag{74}$$

Obviously, for total force $\mathbf{F}_p$ and energy transfer rate W, Eq. (16) still applies, and the per-layer quantities $\mathbf{f}_p$ and w are alternatively expressed as

$$\mathbf{f}_p=-\sum_{\mathbf{q}}\frac{d}{2\pi}\int_{-\infty}^{\infty}dq_z\hbar\mathbf{q}\left[\frac{\partial}{\partial t}N_{\mathbf{q}q_z}\right]_{ep},$$
$$w=\sum_{\mathbf{q}}\frac{d}{2\pi}\int_{-\infty}^{\infty}dq_z\hbar\Omega\left[\frac{\partial}{\partial t}N_{\mathbf{q}q_z}\right]_{ep}. \tag{75}$$

The kinetic equation which results by using a relaxation time approximation for phonon-phonon scattering,

$$\frac{d}{dt}N_{\mathbf{q}q_z}=\left[\frac{\partial}{\partial t}N_{\mathbf{q}q_z}\right]_{ep}-\frac{1}{\tau_p}\left[n\left[\frac{\hbar\Omega}{k_BT_{\mathrm{Q}}}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right], \tag{76}$$

leads to the steady-state phonon occupation number

$$n\left[\frac{\hbar\Omega}{k_BT_{\mathrm{Q}}}\right]=n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]+[1+\tau_p\Gamma(q_z,\mathbf{q},\Omega-\mathbf{q}\cdot\mathbf{v})]^{-1}\times\left[n\left[\frac{\hbar\Omega}{k_BT}\right]-n\left[\frac{\hbar(\Omega-\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]\right] \tag{77}$$

and the following expressions for $\mathbf{f}_p$ and w:

$$\mathbf{f}_p=\sum_{\mathbf{q}}\hbar\mathbf{q}K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\times\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right], \tag{78}$$

$$w=\sum_{\mathbf{q}}\hbar\Omega K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\times\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right], \tag{79}$$

with

$$K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=\frac{d}{2\pi}\int_{-\infty}^{\infty}dq_z\frac{\Gamma(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(\mathbf{q}_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})} \tag{80}$$

and

$$\Gamma(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=-\frac{2}{\hbar Sd}|M(\mathbf{q},q_z)|^2|I(iq_z)|^2\Pi_2(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v}). \tag{81}$$

B. Quasi-2D phonon model

The discussion in Sec. III for a single heterojunction can also be applied to a superlattice. We introduce creation and annihilation operators for quasi-2D phonons:

$$A_l(\mathbf{q})=\sum_{q_z}g_l(\mathbf{q},q_z)b_{\mathbf{q}q_z},$$
$$A_l^{\dagger}(\mathbf{q})=\sum_{q_z}g_l^{*}(\mathbf{q},q_z)b_{\mathbf{q}q_z}^{\dagger}, \tag{82}$$

where

$$g_l(\mathbf{q},q_z)\equiv\frac{M(\mathbf{q},q_z)}{L^{1/2}M(q)}I(iq_z)e^{-iq_zld} \tag{83}$$

and

$$M(q)^2=\frac{1}{L}\sum_{q_z}|M(\mathbf{q},q_z)|^2|I(iq_z)|^2=\frac{F_r}{2\pi}\int\frac{|I(iq_z)|^2}{(q^2+q_z^2)}dq_z. \tag{84}$$

(This last equality is for polar optic phonons only.) They satisfy the commutation relations

$$[A_l(\mathbf{q}),A_{l'}^{\dagger}(\mathbf{q}')]=\beta_{ll'}(q)\delta_{\mathbf{q},\mathbf{q}'}, \tag{85}$$

with

$$\beta_{ll'}(q)=\beta_{l-l'}(q)=\frac{1}{LM(q)^2}\sum_{q_z}|M(\mathbf{q},q_z)|^2|I(iq_z)|^2\times e^{i(l'-l)q_zd}. \tag{86}$$

In terms of these quasi-2D phonon operators, the electron-phonon interaction (2) can be written as

$$H_{ep}=\frac{1}{S^{1/2}}\sum_{l,\mathbf{q}}M(q)e^{i\mathbf{q}\cdot\mathbf{R}}[A_l(\mathbf{q})+A_l^{\dagger}(-\mathbf{q})]\rho_{l\mathbf{q}}. \tag{87}$$

The operators for the phonon-induced frictional force $\hat{\mathbf{F}}_p$, the energy-transfer rate $\hat{W}$, and the rate of change of the generalized occupation number of quasi-2D phonons $\hat{N}_{lm}(\mathbf{q})$, defined by

$$\hat{N}_{lm}(\mathbf{q})=A_l^{\dagger}(\mathbf{q})A_m(\mathbf{q}), \tag{88}$$

are given by

$$\hat{\mathbf{F}}_p=-\frac{i}{\hbar}[\mathbf{P},H_{ep}]=-\frac{i}{S^{1/2}}\sum_{l,\mathbf{q}}M(q)\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{R}}[A_l(\mathbf{q})+A_l^{\dagger}(-\mathbf{q})]\rho_{l\mathbf{q}}, \tag{89}$$

$$\hat{W}=-\frac{i}{\hbar}[H_p,H_{ep}]=\frac{i}{S^{1/2}}\sum_{l,\mathbf{q}}M(q)\Omega e^{i\mathbf{q}\cdot\mathbf{R}}[A_l(\mathbf{q})-A_l^{\dagger}(-\mathbf{q})]\rho_{l\mathbf{q}}, \tag{90}$$

$$\frac{\partial}{\partial t}\hat{N}_{lm}(\mathbf{q})\mid_{ep}=-\frac{i}{\hbar}[\hat{N}_{lm}(\mathbf{q}),H_{ep}]$$
$$=\frac{i}{\hbar S^{1/2}}M(q)\sum_{s}[\beta_{sl}(q)A_m(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{R}}\rho_{s\mathbf{q}}-\beta_{ms}(q)A_l^{\dagger}(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{R}}\rho_{s,-\mathbf{q}}]\ . \tag{91}$$

The average of the above dynamical variables with respect to the nonequilibrium density matrix to lowest order in the interaction [Eq. (34)] yields the following expressions ($\tilde{\omega}\equiv\Omega-\mathbf{q}\cdot\mathbf{v}$):

$$\mathbf{F}_p=-\frac{2}{S}\sum_{l,l',\mathbf{q}}M(q)^2\mathbf{q}\Pi_2(l',l,\mathbf{q},\tilde{\omega})\left[N_{l'l}(\mathbf{q})-\beta_{l'l}(q)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right], \tag{92}$$

$$W=\frac{2}{S}\sum_{l,l',\mathbf{q}}M(q)^2\mathbf{q}\Pi_2(l',l,\mathbf{q},\tilde{\omega})\left[N_{l'l}(\mathbf{q})-\beta_{l'l}(q)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right], \tag{93}$$

and

$$\frac{\partial}{\partial t}N_{l'l}(\mathbf{q})\mid_{ep}=\frac{i}{\hbar S}\sum_{s',s}M(q)^2[\beta_{l's'}(q)N_{ls}(\mathbf{q})\Pi(s',s,\mathbf{q},-\tilde{\omega})-\beta_{ls'}(q)N_{l's}(q)\Pi(s',s,\mathbf{q},\tilde{\omega})]$$
$$-\frac{2}{\hbar S}\sum_{s',s}M(q)^2\beta_{ls}(q)\beta_{l's'}(q)\Pi_2(s',s,\mathbf{q},\tilde{\omega})n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]. \tag{94}$$

In these equations $N_{lm}(\mathbf{q})$ is the generalized occupation number of the quasi-2D phonons:

$$N_{lm}(\mathbf{q})=\langle\hat{N}_{lm}(\mathbf{q})\rangle=\langle A_l^{\dagger}(\mathbf{q})A_m(\mathbf{q})\rangle\ , \tag{95}$$

which, similar to $\beta_{lm}(q)$ and $\Pi(l,m,\mathbf{q},\omega)$, depends only on $\mathbf{q}=\mid q\mid$ and $l-m$ due to translational invariance and periodicity of the superlattice. Therefore it is convenient to use the Fourier representations:

$$N(\mathbf{q},q_z)=\sum_m N_{lm}(\mathbf{q})e^{-i(l-m)q_zd}\ ,$$
$$\beta(q,q_z)=\sum_m\beta_{lm}(q)e^{-i(l-m)q_zd}\ . \tag{96}$$

With these, Eqs. (92) and (93) can be written as the per-layer force and energy transfer rate given by

$$\mathbf{f}_p=-\frac{2}{S}\sum_{\mathbf{q}}\frac{d}{2\pi}\int_{-\pi/d}^{\pi/d}dq_zM(q)^2\mathbf{q}\Pi_2(q_z,\mathbf{q},\tilde{\omega})$$
$$\times\left[N(\mathbf{q},q_z)-\beta(q,q_z)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right], \tag{97}$$

$$W=\frac{2}{S}\sum_{\mathbf{q}}\frac{d}{2\pi}\int_{-\pi/d}^{\pi/d}dq_zM(q)^2\Omega\Pi_2(q_z,\mathbf{q},\tilde{\omega})$$
$$\times\left[N(\mathbf{q},q_z)-\beta(q,q_z)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right], \tag{98}$$

and the expression (94) reduces to

$$\frac{\partial}{\partial t}N(\mathbf{q},q_z)\mid_{ep}=\frac{2}{\hbar S}M(q)^2\beta(q,q_z)\Pi_2(q_z,q,\tilde{\omega})$$
$$\times\left[N(\mathbf{q},q_z)-\beta(q,q_z)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right]. \tag{99}$$

The contribution to the rate of change of $N(\mathbf{q},q_z)$ from other effects than the electron-phonon interaction is assumed to be represented by a relaxation time τ_p, such that the kinetic equation for $N(q,q_z)$ takes the form

$$\frac{d}{dt}N(\mathbf{q},q_z)=\frac{\partial}{\partial t}N(\mathbf{q},q_z)\mid_{ep}$$
$$-\frac{1}{\tau_p}[N(\mathbf{q},q_z)-N_{\rm eq}(\mathbf{q},q_z)]\ . \tag{100}$$

Here $N_{\rm eq}(\mathbf{q},q_z)$ is the equilibrium value of $N(q,q_z)$, which can be shown to be

$$N_{\rm eq}(\mathbf{q},q_z)=\beta(q,q_z)n\left[\frac{\hbar\Omega}{k_BT}\right], \tag{101}$$

and

$$\beta(q,q_z)=\frac{1}{dM(q)^2}\sum_{K_d}\mid M(\mathbf{q},q_z+K_d)\mid^2$$
$$\times\mid I(iq_z+iK_d)\mid^2\ . \tag{102}$$

The summation is over all possible values of $K_d=2\pi n/d$, $n=0,\pm1,\ldots$. Substituting Eq. (101) into Eq. (100) we have for the steady state

$$0=\frac{d}{dt}N(\mathbf{q},q_z)=\frac{2}{\hbar S}M(q)^2\beta(q,q_z)\Pi_2(q_z,\mathbf{q},\tilde{\omega})\left[N(\mathbf{q},q_z)-\beta(q,q_z)n\left[\frac{\hbar\tilde{\omega}}{k_BT_e}\right]\right]-\frac{1}{\tau_p}\left[N(\mathbf{q},q_z)-\beta(\mathbf{q},q_z)n\left[\frac{\hbar\Omega}{k_BT}\right]\right]. \tag{103}$$

This immediately gives the steady-state solution of $N(q,q_z)$ and the corresponding steady-state expressions for per-layer force and energy-transfer rate as

$$\mathbf{f}_p=\sum_{\mathbf{q}}\hbar\mathbf{q}K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right], \tag{104}$$

$$w=\sum_{\mathbf{q}}\hbar\Omega K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\left[n\left[\frac{\hbar(\Omega+\mathbf{q}\cdot\mathbf{v})}{k_BT_e}\right]-n\left[\frac{\hbar\Omega}{k_BT}\right]\right], \tag{105}$$

with

$$K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=\frac{d}{2\pi}\int_{-\pi/d}^{\pi/d}dq_z\frac{\Gamma(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}, \tag{106}$$

$$\Gamma(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=\sum_{K_d}\Gamma_{K_d}(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v}), \tag{107}$$

and Γ_{K_d} depends on K_d in accordance with

$$\Gamma_{K_d}(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=-\frac{2}{\hbar Sd}|M(\mathbf{q},q_z+K_d)|^2|I(iq_z+iK_d)|^2\Pi_2(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v}), \tag{108}$$

where $K_d=2\pi n/d$, $n=0,\pm1,\ldots$.

It is worth noting that the corresponding expression (80) for $K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})$ in the 3D phonon model can be rewritten as

$$K(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})=\frac{d}{2\pi}\int_{-\pi/d}^{\pi/d}dq_z\sum_{K_d}\frac{\Gamma_{K_d}(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}{1+\tau_p\Gamma_{K_d}(q_z,\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})}. \tag{109}$$

Therefore, if $\tau_p=0$ the results of the 3D phonon and the quasi-2D phonon models are identical and reduce exactly to those given in Ref. 14.

V. NUMERICAL RESULTS AND DISCUSSION

From the structure of our balance equations in steady state, one can expect that the finite nature of the phonon relaxation time τ_p has a significant impact on carrier transport in GaAs heterosystems. It is particularly important for the energy-transfer rate induced by the electron—polar-optic-phonon interaction. This effect may be understood from the fact that scatterings with small momentum transfer have a considerable weight in contributing to the electron energy loss, and in the case of a Fröhlich coupling between electrons and polar optic phonons, the effective scattering matrix element $M(q)^2$ and consequently $\Gamma(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})\sim M(q)^2\Pi_2(\mathbf{q},\Omega+\mathbf{q}\cdot\mathbf{v})$ are large at small q. The effects of such large values of Γ are sharply reduced when taken in conjunction with the finite-phonon relaxation time. Of course, the phonon-induced frictional force is also modified by the finite τ_p. However, this effect is not as pronounced as in the energy transfer rate because of the relatively small weight of small q scattering in the constitution of the force, in contrast to the energy loss.

We have calculated the electron nonlinear mobility μ and carrier temperature T_e as functions of drift velocity v based on the force and energy balance equations (62) and (63) for two different n-type GaAs heterosystems, assuming the electron—polar-optic-phonon coupling to be the dominant mechanism of inelastic scattering. One of the systems is a heterojunction with electron sheet density $N_s=3.9\times10^{11}$ cm^{-2} and zero-temperature ohmic mobility $\mu_0=2.2\times10^5$ cm^2/V s. The calculation is carried out at lattice temperature $T=10$ K. The Fang-Howard-Stern variational wave function[19] is used as the electron lowest-subband envelope function: $\zeta(z)\sim z\exp(-bz/2)$, and b is determined by N_s and the depletion-layer charge density N_d with $N_d=0.5\times10^{11}$ cm^{-2}. The second system is a quantum well of width $a=250$ Å with electron sheet density $N_s=3.9\times10^{11}$ cm^{-2} and zero-temperature ohmic mobility $\mu_0=7.9\times10^4$ cm^2/V s. The well potential is assumed deep enough so that $\zeta(z)\sim\cos(\pi z/a)$ adequately represents the lowest-subband envelope function. In GaAs heterosystems various kinds of elastic scattering mechanisms are needed to account for these relatively low experimental μ_0 values.[22] The detailed estimation of the contributions due to different kinds of elastic scatterings is a complicated task and beyond the scope of the present paper. What we require here is the dependence of μ_i (due to elastic scattering) as a function of electron temperature T_e and drift velocity v. For this we use the results given in Refs. 13 and 14, with remote and background impurity scatterings included, and assume equality of the contributions of these two kinds of impurities to the ohmic mobility at $T=0$ K. The remote impurities are assumed to lie on sheets at a distance of 100 Å from the interfaces. The relevant parameters used in the calculation are electron effective mass $m_e=0.07m_0$ (m_0 is the free electron mass), longitudinal optic phonon energy $\hbar\Omega_0=35.4$ meV, GaAs low-frequency dielectric constant $\kappa=12.9$, and optic dielectric constant $\kappa_\infty=10.8$. In all our calculations in this paper, the electron density-density correlation functions employed are those of the random phase approximation with full temperature, wave-vector, and frequency dependencies intact.

The calculated results for nonlinear mobility μ (defined

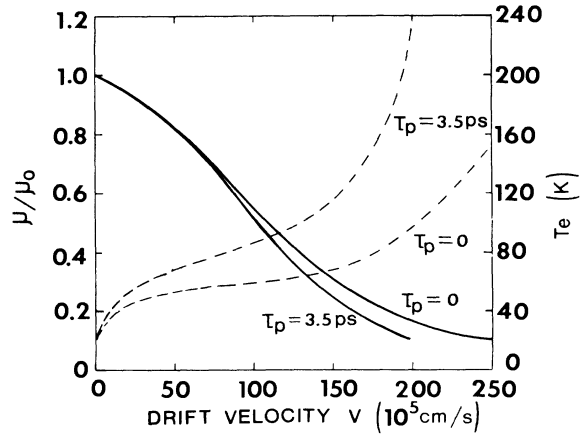

FIG. 1. Nonlinear mobilities normalized to the ohmic limit μ/μ_0 (solid curves) and electron temperatures T_e (dashed curves) are shown as functions of drift velocity v for phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps at lattice temperature $T=10$ K for a n-GaAs heterojunction with carrier sheet density $N_s=3.9\times10^{11}$ cm^{-2} and $\mu_0=2.2\times10^5$ cm^2/V s.

as the ratio of the drift velocity to electric field $\mu=v/E$), normalized to μ_0, and for carrier temperature T_e, are shown in Figs. 1 and 2 as functions of drift velocity v for phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps. The most striking effect of the finite-phonon relaxation time is that for a given drift velocity the electron temperature is much higher than in the case of $\tau_p=0$. Such an increase of electron temperature tends to raise the frictional force in a measure which outweighs the tendency toward dimunition due to finite τ_p, and the normalized mobility μ/μ_0 is lower for finite τ_p than for $\tau_p=0$.

In order to gain direct insight into the behavior of the electron energy-loss rate and to compare with experiments, we replot the above results as inverse carrier temperature $1/T_e$ versus energy-transfer rate per carrier in Fig. 3, together with the experimental data of Shah *et al.*[5] and Yang *et al.*[6] For given T_e, the energy-loss rates due to polar-optic-phonon scattering for $\tau_p=3.5$ ps do show a factor of 5 to 10 times smaller than those for $\tau_p=0$.

We have also carried out a similar calculation for a GaAs quantum-well hole system. For holes both polar optic phonon and nonpolar-optic-phonon scatterings, as well as acoustic phonon scattering, contribute to transport.[23] For simplicity we still use the parabolic band approximation with effective mass $m_h=0.5m_0$ to take account of the effect of light holes. The p-like behavior of the valence-band wave function is accounted for by use of the semiempirical modified coefficient for the electron-phonon coupling as in the earlier analysis of 3D systems.[23] We take (a) $E_1=5.8$ eV for the acoustic phonon deformational potential (without hot-phonon effects), (b) $D=9.0\times10^8$ eV/cm for the optic-phonon deformation potential, (c) a correction factor of 0.455 to the Fröhlich expression for the polar-optic coupling. The longitudinal sound velocity $v_{sl}=5.29\times10^5$ cm/s and $\hbar\Omega_0$, κ and κ_∞ are given above. The calculated hole energy-loss rates for a p-GaAs quantum-well system of width $a=95$ Å, carrier sheet density $N_s=3.5\times10^{11}$ cm^{-2} and zero temperature ohmic mobility $\mu_0=3.6\times10^4$ cm^2/V s are shown in Fig. 4 for lattice temperature $T=2$ K and phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps together with the experimental points of Shah *et al.*[5] The reduction of the hole energy-loss rate due to nonequilibrium phonons is much smaller than for electrons, in agreement with experiments. This is a consequence of the fact that large wave-vector transfer contributions become important in acoustic and optic deformation potential scatterings, as pointed out by Shah *et al.*[5]

In the following, we shall briefly discuss the hot-phonon effect on linear mobility. From the structure of our force- and energy-balance equations in steady state, it

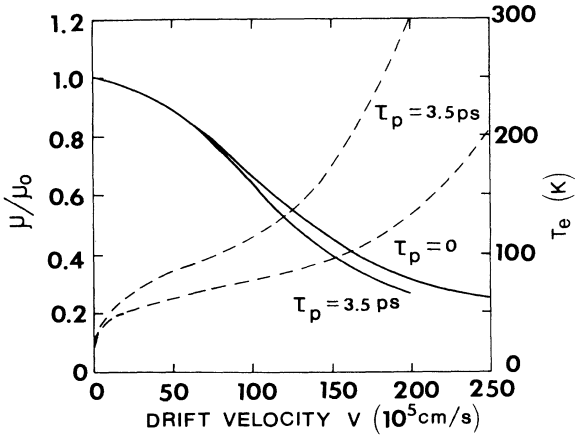

FIG. 2. Nonlinear mobilities normalized to the ohmic limit μ/μ_0 (solid curves) and electron temperatures T_e (dashed curves) are shown as functions of drift velocity v for phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps at lattice temperature $T=2$ K for a n-GaAs quantum well of width $a=250$ Å, $N_s=3.9\times10^{11}$ cm^{-2}, and $\mu_0=7.9\times10^4$ cm^2/V s.

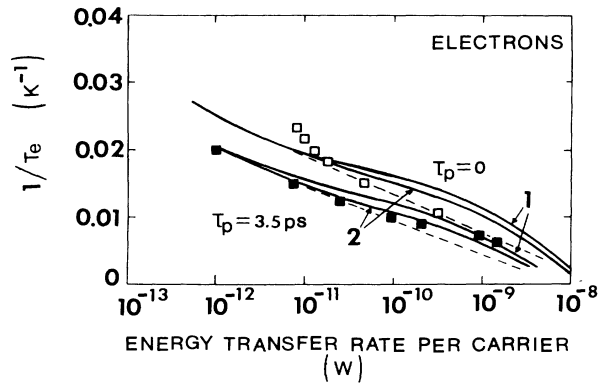

FIG. 3. Inverse electron temperature $1/T_e$ vs electron-energy-transfer rate per carrier for two n-GaAs heterosystems (1, for the system described in Fig. 1; 2, for the system described in Fig. 2) at phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps. The dots are the experimental data from Ref. 5 and the rectangles are those from Ref. 6. The dashed lines are calculated according to Eq. (111) for the system 2.

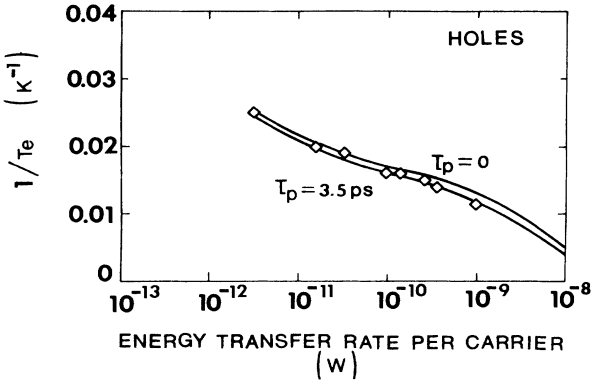

FIG. 4. Inverse hole temperature $1/T_e$ vs hole-energy-transfer rate per carrier for a p-GaAs quantum-well system of width $a=95$ Å, carrier sheet density $N_s=3.5\times10^{11}$ cm^{-2} and 0 K ohmic mobility $\mu_0=3.6\times10^4$ cm^2/V s at lattice temperature $T=2$ K with two different values of phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps. The rhombuses are experimental data from Ref. 5.

is easily seen that in the weak current limit the energy balance equation gives $T_e=T$ up to the first order in drift velocity. Therefore the ohmic mobility μ_p due to optic-phonon scattering is obtained directly from the force-balance equation by setting $T_e=T$:

$$\frac{1}{\mu_p}=\frac{\hbar^2}{N_s e k_B T}\left[-n'\left[\frac{\hbar\Omega}{k_B T}\right]\right]\frac{1}{4\pi}\int_0^\infty K(\mathbf{q},\Omega)q^3\,dq\ . \tag{110}$$

Obviously, finite phonon-relaxation time effects will show up in the optic-phonon-induced ohmic mobility. In Fig. 5

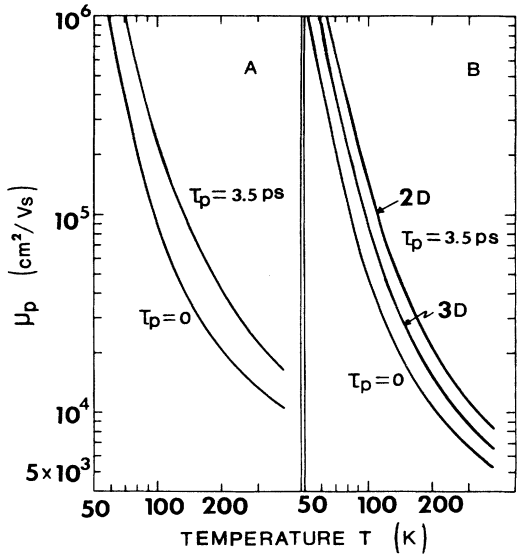

FIG. 5. Electron ohmic mobilities μ_p due to polar-optic-phonon scattering are shown as functions of lattice temperature T with different phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps: (a) for a single heterojunction as described in Fig. 1, quasi-2D phonon model; and (b) for a superlattice of $a=100$ Å, $d=200$ Å, and $N_s=2.3\times10^{11}$ cm^{-2} in the quasi-2D phonon model (2D) and the 3D phonon model (3D).

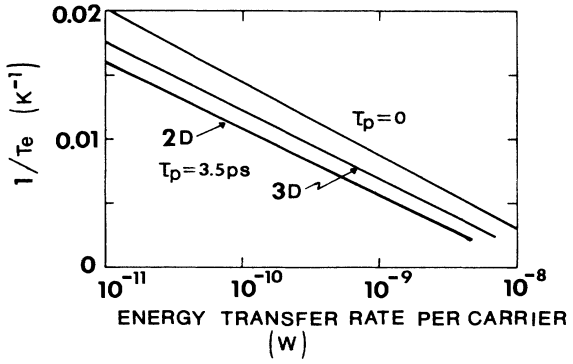

FIG. 6. Electron-energy-transfer rate per carrier vs $1/T_e$ as calculated from Eq. (111) at $T=0$ K with $\tau_p=0$ and $\tau_p=3.5$ ps for a superlattice of $a=100$ Å, $d=200$ Å, and $N_s=2.3\times10^{11}$ cm^{-2} in quasi-2D and 3D phonon models.

we plot the calculated values of ohmic mobility due to polar optic-phonon scattering (a) for a n-GaAs single heterojunction with $N_s=3.9\times10^{11}$ cm^{-2} and $N_d=5\times10^{10}$ cm^{-2}, using the quasi-2D phonon model and (b) for an n-GaAs superlattice of width $a=100$ Å and $d=200$ Å with $N_s=2.3\times10^{11}$ cm^{-2}, using both quasi-2D and 3D phonon models.

Considering the energy-transfer rate w given by the expressions (79) or (105), the neglect of $\mathbf{q}\cdot\mathbf{v}$ in the argument $\Omega+\mathbf{q}\cdot\mathbf{v}$ directly yields the carrier energy-loss rate with no need to solve the balance equation. This corresponds to the usual procedures of a carrier temperature model,[5,6,11] and it leads to

$$w=\hbar\Omega\left[n\left[\frac{\hbar\Omega}{k_B T_e}\right]-n\left[\frac{\hbar\Omega}{k_B T}\right]\right]\eta\ , \tag{111}$$

with

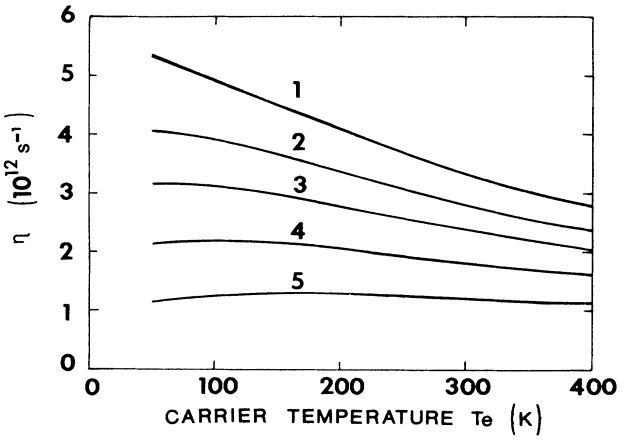

FIG. 7. Calculated values of η as defined in Eqs. (111) and (112) vs electron temperature T_e at $\tau_p=0$ and $\tau_p=3.5$ ps for a superlattice of width $a=200$ Å, but for different values of d, using quasi-2D and 3D phonon models. $1-\tau_p=0$; $2-d=2000$ Å, $\tau_p=3.5$ ps, 3D model; $3-d=800$ Å, $\tau_p=3.5$ ps, 3D model; $4-d=300$ Å, $\tau_p=3.5$ ps, 3D model; $5-\tau_p=3.6$ ps, quasi-2D model.

$$\eta = \sum_{\mathbf{q}} K(\mathbf{q},\Omega) . \quad (112)$$

To examine the difference between the 3D phonon model and the quasi-2D phonon model we plot in Fig. 6 the energy-loss rate versus $1/T_e$ as obtained from Eq. (111) at zero lattice temperature for a n-type GaAs superlattice of $a=100$ Å, $d=200$ Å, and $N_s=2.3\times10^{11}$ cm^{-2}, for phonon relaxation times $\tau_p=0$ and $\tau_p=3.5$ ps. We also plot μ as a function of electron temperature T_e in Fig. 7 for a series of superlattices of width $a=200$ Å, but having different values of $d=300$, 800, and 2000 Å. For the cases of $\tau_p\neq0$ and/or for the quasi-2D model, the d dependence is too small to exhibit. The resulting energy-loss rate in the 3D phonon model, however, does show a d dependence at $\tau_p=0$. The energy-loss rates obtained from the quasi-2D phonon model are always smaller than those of the 3D phonon model for a given phonon relaxation time τ_p. However, further reduced rates can be achieved in the 3D phonon model if one uses a larger τ_p.

It is worth mentioning that, although the carrier-temperature-model-type formula for energy-loss rate is attractive and widely used in the literature due to its simplicity, the error arising from the neglect of $\mathbf{q}\cdot\mathbf{v}$ is significant for $T_e>70$ K. To see this we plot in Fig. 3 as dashed lines the results of $1/T_e$ versus the per carrier energy-transfer rate calculated from Eq. (111) for the same quantum-well system of $a=250$ Å as described in Fig. 1 at $\tau_p=0$ and $\tau_p=3.5$ ps. The difference may be as large as a factor of 3. Therefore, the balance equation analysis carried out here is requisite for a correct determination of the functional dependence of the energy-loss rate in steady-state hot-carrier transport in the presence of an electric field.

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[1]J. F. Ryan, R. A. Taylor, A. J. Turberfield, A. Macid, J. M. Warlock, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **53**, 1841 (1984).

[2]J. Shah, A. Pinczuk, H. L. Stomer, A. C. Gossard, and W. Wiegmann, Appl. Phys. Lett. **44**, 322 (1984).

[3]Z. Y. Xu and C. L. Tang, Appl. Phys. Lett. **44**, 692 (1984).

[4]R. A. Höpfel and G. Weiman, Appl. Phys. Lett. **46**, 291 (1985).

[5]J. Shah, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 2045 (1985).

[6]C. H. Yang, J. M. Carlson-Swindle, S. A. Lyon, and J. M. Worlock, Phys. Rev. Lett. **55**, 2359 (1985).

[7]P. K. Basu and S. Kunda, Appl. Phys. Lett. **47**, 264 (1985).

[8]P. J. Price, Physica, **134B**, 164 (1985).

[9]P. Kocevar, Physica, **134B**, 155 (1985).

[10]S. M. Goodnik and P. Lugli (unpublished).

[11]W. Cai, M. C. Marchetti, and M. Lax, Phys. Rev. B **34**, 8573 (1986).

[12]X. L. Lei and C. S. Ting, Phys. Rev. B **32**, 1112 (1985).

[13]X. L. Lei, J. L. Birman, and C. S. Ting, J. Appl. Phys. **58**, 2270 (1985).

[14]X. L. Lei, N. J. M. Horing, and J. Q. Zhang, Phys. Rev. B **34**, 1139 (1986).

[15]X. L. Lei, N. J. M. Horing, and J. Q. Zhang, Phys. Rev. B **35**, 2834 (1987).

[16]C. S. Ting and T. D. Nee, Phys. Rev. B **33**, 7056 (1986); D. Y. Xing and C. S. Ting, Phys. Rev. B **35**, 3971 (1987).

[17]P. Hu and C. S. Ting, Phys. Rev. B **34**, 7003 (1986).

[18]X. L. Lei, W. Cai, and C. S. Ting, J. Phys. C **18**, 4315 (1985).

[19]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[20]D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Consultants Bureau, New York, 1974).

[21]S. Peletminskii and A. Yatsenko, Zh. Eksp. Teor. Fiz. **53**, 1327 (1967) [Sov. Phys.—JETP **26**, 773 (1968)].

[22]W. Walukiewicz, H. E. Ruda, J. Lagowskii, and H. C. Gatos, Phys. Rev. B **30**, 4571 (1984).

[23]J. D. Wiley, in *Semiconductors and Semimetals* (Academic, New York, 1975), Vol. 10, Chap. 2.