Theory of cyclotron resonance in an electron-phonon system

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An expression for the cyclotron-resonance power absorption by electrons interacting with phonons is derived with the help of a projection operator that takes into account the effects of exchange. It is shown that the correct evaluation of this expression requires the summation of an infinite number of terms in order to describe the effects of scattering to second order. These results contradict some recently proposed theories of this phenomenon, which are discussed and criticized.

I. INTRODUCTION

There has been recently a renewed interest in the theory of the cyclotron-resonance power absorption line shape[1–24] in solids, since such studies provide useful information about the scattering mechanisms for the charge carriers. Elastic scattering by static impurities as well as inelastic scattering by phonons have been considered.

Among the various theories proposed, there are some (Refs. 1, 2, 10, 11, 13, 15, 16, 19, 22, and 24) that give an *explicit* expression for the shape of the cyclotron-resonance absorption line, which is claimed to be *rigorously* correct to second order in λ, the strength of the scattering interaction. These expressions have been used as sound bases for a number of specific applications (Refs. 7, 9, 12, 14, 17, 18, and 23).

Theories of this type, such as those of Kawabata[1] and of Lodder and Fujita,[2] that deal with the case of scattering by static impurities have been criticized by Argyres and Sigel[6] and shown to be in error. By an introduction of a projection operator they[6] were able to derive an *exact* expression for the line shape in terms of a complex linewidth for each electron state, $\Gamma_\alpha(\omega)$, which to order λ^2 was identical to that of the theories under discussion.[1,2] However, they pointed out[6] that the correct expression for $\Gamma_\alpha(\omega)$ cannot be expanded in powers of λ, because some terms of order λ^n with $n \geq 3$ diverge for frequencies near the cyclotron frequency. In fact, by summing an infinite subset of the most divergent terms, they showed that the line shape can be correctly obtained to order λ^2 only by means of the integral quantum kinetic equations.[25] Recently, Choi and Chung[15,19] have claimed that the work of Argyres and Sigel[6] is not correct, but their claim was proven[20,21] to be based on a misunderstanding of the projection operator and thus, as these authors[22,24] have admitted, their criticism is invalid.

For the case of electrons being scattered inelastically by phonons, theories for the cyclotron-resonance line shape to order λ^2 have been given by Lodder and Fujita[2] and more recently by Choi and Chung[16,19] and others.[13,22,24] Although their techniques differ, they give essentially the same expression for $\Gamma_\alpha(\omega)$ which they claim to be rigorous to order λ^2. Choi and Chung[16,19] have proposed an extension of the Argyres-Sigel[6] projection operator method to the case of electron-phonon scattering. They have claimed[16,19] that their results do not suffer from the Argyres-Sigel criticism for the analogous theory of elastic scattering by impurities.

In this paper we propose a generalization of the Argyres-Sigel method[6] which enables us to arrive at a general expression for the cyclotron-resonance power absorption in an electron-phonon system that takes into account the exchange effects among the electrons and the inelasticity of their collisions with the phonons. This provides the basis for a critique of these theories[2,13,16,19,22,24] of this phenomenon. We point out, first, that these theories ignore all exchange effects among electrons; although it is true that exchange has no effect on the collisions of electrons with static impurities,[26] it has been recognized[27,28] long ago that this is *not* the case for electrons moving in a phonon field. Secondly, we argue that we cannot give a *rigorous explicit* expression for the complex linewidth $\Gamma_\alpha(\omega)$ to order λ^2 for the same reasons as for the case of scattering by static impurities. That is, in the formal expansion of $\Gamma_\alpha(\omega)$ in powers of λ, there are terms of order λ^n with $n \geq 3$ which diverge for frequencies near the resonant cyclotron frequency. Thus, a correct expression for the linewidth to order λ^2 can be obtained correctly within this formalism only by summing the most divergent terms of order λ^n ($n \geq 3$) in the expansion of $\Gamma_\alpha(\omega)$ in powers of λ, which leads to an integral equation, in conformity with the theory[27–29] of quantum kinetic equations. In fact, it is this discrepancy between the modern theories[2,13,16,19,22,24] under investigation and the older theories[27–29] that has provided the motivation of this work.

In the following section we derive a general expression for the cyclotron-resonance power absorption of dynamically independent charged fermions in a phonon field, subject to the same approximations that the theories[2,13,16,19,22,24] under discussion have made. We achieve this by generalizing the method of Argyres and Sigel[6] in a way that treats rigorously the exchange effects of the charge carriers in their interactions with the phonons, while the strength of this interaction can be arbitrary. This general formula forms the basis on which we discuss, in Sec. III, the earlier theories. In Sec. IV we conclude with a brief summary of this discussion.

II. CYCLOTRON-RESONANCE POWER ABSORPTION FOR THE ELECTRON-PHONON SYSTEM

We derive in this section an expression for the conductivity tensor of dynamically independent electrons in a uniform magnetic field interacting with phonons that will provide the basis for a discussion of some theories[2,13,16,19,22,24] for the cyclotron-resonance power absorption. This expression is obtained by generalizing the method of Argyres and Sigel[6] to take full account of the effects due to exchange and the inelasticity of the collisions of the electrons with the phonons.

In contrast to the case of dynamically independent electrons moving in the field of *static* impurities, the case of electrons moving in the field of phonons cannot[27,28] be reduced exactly to that of a single electron. Thus exchange effects for the electrons cannot be eliminated in this case, and in fact it is well known[30] that these effects along with the inelasticity of the collisions with the phonons play an important role for the temperature dependence of the conductivity.

We thus consider an assembly of dynamically independent electrons in a magnetic field B, taken in the z direction, with energy

$$H_e=\sum_\alpha \epsilon_\alpha a_\alpha^\dagger a_\alpha \ . \tag{2.1}$$

Here, $a_\alpha^\dagger$ (a_α) are the creation (destruction) operators for the electrons, obeying the usual anticommutation relations, in the Landau states $|\alpha\rangle$, i.e., the eigenstates of an electron in the magnetic field B, with energy ε_α. We have $|\alpha\rangle=|n\mathbf{k}\rangle$, where $n=0,1,2,\ldots$ and $\mathbf{k}=(k_y,k_z)$, so that

$$\langle \mathbf{r}\,|\,\alpha\rangle=\langle \mathbf{r}\,|\,n\mathbf{k}\rangle \propto \phi_n(x+k_y/m\omega_c)e^{i\mathbf{k}\cdot\mathbf{r}} \ , \tag{2.2}$$

$$\varepsilon_\alpha=\varepsilon_{n\mathbf{k}}=(n+\tfrac{1}{2})\omega_c+k_z^2/2m \ , \tag{2.3}$$

with $\phi_n(x+k_y/m\omega_c)$ being the eigenfunctions of a harmonic oscillator of the cyclotron frequency $\omega_c\equiv|e|B/mc$, centered at $(-k_y/m\omega_c)$, and $\hbar$ is taken equal to 1. In the following we shall adopt the notation $|\alpha+1\rangle$ to denote the state $|n+1,\mathbf{k}\rangle$ if $|\alpha\rangle=|n\mathbf{k}\rangle$. We ignore the spin of the electrons. The electrons interact with a phonon field described by

$$H_p=\sum_q \omega_q(b_q^\dagger b_q+\tfrac{1}{2}) \ , \tag{2.4}$$

where $q\equiv(s,\mathbf{q})$ denotes the polarization index s and the wave vector $\mathbf{q}$ of a phonon of energy ω_q, and $b_q^\dagger$ (b_q) is the creation (annihilation) operator for the phonons, obeying the usual commutation relations. The scattering interaction is then

$$V=\sum_{\alpha,\alpha'}\sum_q c_{\alpha\alpha'}(q)a_\alpha^\dagger a_{\alpha'}(b_q+b_{-q}^\dagger) \ , \tag{2.5}$$

where $c_{\alpha\alpha'}(q)=\langle\alpha\,|\,c(q)\,|\,\alpha'\rangle$ are the matrix elements of the one-electron operator $c(q)$, which describes in a self-consistent approximation scheme the interaction of an electron with the vibrating lattice. Since V is Hermitian, $c(-q)=c^\dagger(q)$, where $-q=(s,-\mathbf{q})$. Finally, the electrons are driven by a weak oscillating electric field E_i ($i=1,2,3$) of frequency ω, so that, in the scalar gauge, the perturbing Hamiltonian is

$$F(t)=-eE_iX_ie^{i\omega t}+\text{H.c.} \ , \tag{2.6}$$

where

$$X_i=\sum_{\alpha,\alpha'}\langle\alpha\,|\,x_i\,|\,\alpha'\rangle a_\alpha^\dagger a_{\alpha'} \tag{2.7}$$

and the summation convention for the Cartesian components X_i,E_i is adopted.

The induced steady-state current density is then $\mathbf{J}(t)=\mathbf{J}(\omega)e^{i\omega t}+\text{c.c.}$ with

$$J_i(\omega)=\mathrm{Tr}\{J_iR(\omega)\} \ , \tag{2.8}$$

if we take for convenience the volume of the sample equal to unity. J_i is the ith component of the current operator for the electrons, i.e.,

$$J_i=\sum_{\alpha,\alpha'}\langle\alpha\,|\,j_i\,|\,\alpha'\rangle a_\alpha^\dagger a_{\alpha'} \ , \tag{2.9}$$

where j_i is the one-electron current operator and $R(\omega)$ is the steady-state density operator for the system of electrons and phonons, linear in the electric field. This satisfies the equation of motion[27]

$$(-\omega^- -L)R(\omega)=-eE_i[X_i,R_0(H')] \ . \tag{2.10}$$

Here $\omega^-\equiv\omega-i\eta$ with η a positive infinitesimal ($\eta\to0^+$),

$$H'=H-\mu N \ , \tag{2.11}$$

$$H=H_e+H_p+V\equiv H_0+V \ , \tag{2.12}$$

μ is the chemical potential, $N=\sum_\alpha c_\alpha^\dagger c_\alpha$ is the total number of electrons operator,

$$R_0(H')=e^{-\beta H'}/\mathrm{Tr}\,e^{-\beta H'} \tag{2.13}$$

is the equilibrium density matrix for a grand canonical ensemble at absolute temperature $T\equiv1/k\beta$, and L denotes the Liouville operator corresponding to H, i.e.,

$$LY\equiv[H,Y] \tag{2.14}$$

for any operator Y. The formal solution of (2.10) is

$$R(\omega)=G(-\omega^-)[R_0(H'),eX_i]E_i \ , \tag{2.15}$$

where we have introduced the propagator $G(z)$ corresponding to L, namely,

$$G(z)\equiv(z-L)^{-1} \tag{2.16}$$

for any complex number z. Thus, the conductivity tensor $\sigma_{ij}(\omega)$ is obtained from (2.8) and (2.15) and is given by

$$\sigma_{ij}(\omega)=\mathrm{Tr}\{eX_j[R_0(H'),G(\omega^-)J_i]\} \tag{2.17}$$

due to the invariance property of the trace under cyclic permutations.

The average power absorbed by the electrons when driven by an electric field E circularly polarized in the xy plane is[2]

$$p(\omega)=\tfrac{1}{2}E^2\mathrm{Re}\sigma_{+-}(\omega) \ , \tag{2.18}$$

where Re stands for the "real part of" and, according to (2.17),

$$\sigma_{+-}(\omega)=\mathrm{Tr}\{R_0(H')[G(\omega^-)J^+,eX^-]\}\ , \tag{2.19}$$

with $J^{\pm}\equiv J_1\pm iJ_2$ and $X^{\pm}\equiv X_1\pm iX_2$. These are exact expressions for $\sigma_{+-}(\omega)$ and the power absorption $p(\omega)$.

We now adopt the approximation made in the theories[2,13,16,19,22,24] under discussion, and considered appropriate for $\omega\cong\omega_c$ and small λ, namely,

$$R_0(H')=R_0(H'_0+V)\cong R_0(H'_0)=R_0(H'_e)R_0(H_p)\ . \tag{2.20}$$

We thus have for consideration the basic expression

$$\sigma_{+-}(\omega)\cong\mathrm{Tr}\{R_0(H'_0)[K(\omega),eX^-]\}\ , \tag{2.21}$$

where

$$K(\omega)\equiv G(\omega^-)J^+ \tag{2.22}$$

is a many-electron—phonon operator. Using (2.7) and the selection rule

$$(ex^-)_{\alpha\alpha'}=(i/\omega_c)(j^+_\alpha)^*\delta_{\alpha',\alpha+1}\ , \tag{2.23}$$

we have for the quantity of interest (2.21)

$$\sigma_{+-}(\omega)=\frac{i}{\omega_c}\sum_\alpha(j^+_\alpha)^*\langle K(\omega)\rangle_\alpha\ . \tag{2.24}$$

Here we have introduced the notation $x^{\pm}\equiv x_1\pm x_2$, $j^{\pm}\equiv j_1\pm ij_2$, and for any one-electron operator y

$$y_\alpha\equiv\langle\alpha+1\,|\,y\,|\,\alpha\rangle\ , \tag{2.25}$$

while for any many-electron—phonon operator Y

$$\langle Y\rangle_\alpha\equiv\mathrm{Tr}\{R_0(H'_0)[Y,a^\dagger_\alpha a_{\alpha+1}]\}\ . \tag{2.26}$$

We now evaluate the quantity $\langle K(\omega)\rangle_\alpha$ in (2.24) by a procedure that is a generalization of that introduced[6] for the case of scattering by static impurities. From (2.22) and (2.16) $K(\omega)$ obeys the equation

$$(\omega^- -L)K(\omega)=J^+\ . \tag{2.27}$$

We introduce the projection operator P defined by

$$PY\equiv J^+\langle Y\rangle_\alpha/\langle J^+\rangle_\alpha\ , \tag{2.28}$$

where $\langle Y\rangle_\alpha,\langle J^+\rangle_\alpha$ are defined by (2.26). Clearly,

$$PJ^+=J^+,\quad P'J^+\equiv(1-P)J^+=0\ , \tag{2.29}$$

while

$$PK(\omega)=J^+\langle K(\omega)\rangle_\alpha/\langle J^+\rangle_\alpha \tag{2.30}$$

and

$$\langle J^+\rangle_\alpha=j^+_\alpha(f_{\alpha+1}-f_\alpha)\ , \tag{2.31}$$

where $f_\alpha\equiv f(\varepsilon_\alpha)\equiv[\exp\beta(\varepsilon_\alpha-\mu)+1]^{-1}$ is the Fermi-Dirac distribution function. By the same method as in Ref. 6, we then find for the quantity of interest $\langle K(\omega)\rangle_\alpha$ the expression

$$\langle K(\omega)\rangle_\alpha=\langle J^+\rangle_\alpha\{\omega^- -(1/\langle J^+\rangle_\alpha)\times\langle[L+LG'(\omega^-)P'L]J^+\rangle_\alpha\}^{-1}\ , \tag{2.32}$$

where we have introduced a new propagator

$$G'(z)\equiv(z-P'L)^{-1}\ . \tag{2.33}$$

Since we are interested in the case of a weak scattering potential V, it is convenient to introduce the Liouville operators $L_0=L_e+L_p$ and L_1 corresponding to $H_0=H_e+H_p$ and V, respectively, i.e., $L=L_0+L_1$ with

$$L_0Y=(L_e+L_p)Y\equiv[H_0,Y]=[H_e,Y]+[H_p,Y]\ ,$$
$$L_1Y\equiv[V,Y]\ . \tag{2.34}$$

We then note that $L_0J^+=\omega_cJ^+$, $P'L_0J^+=0$, $\langle L_0P'Y\rangle_\alpha=0$, and $\langle L_1J^+\rangle_\alpha=0$. We thus can write (2.32), using (2.31), in the form

$$\langle K(\omega)\rangle_\alpha=\frac{ij^+_\alpha(f_{\alpha+1}-f_\alpha)}{i(\omega^- -\omega_c)+\Gamma_\alpha(\omega)}\ , \tag{2.35}$$

where

$$\Gamma_\alpha(\omega)\equiv-\frac{i}{\langle J^+\rangle_\alpha}\langle[L_1+L_1G'(\omega^-)P'L_1]J^+\rangle_\alpha\ . \tag{2.36}$$

Using (2.35) in (2.24) we have finally

$$\sigma_{+-}(\omega)=\frac{1}{\omega_c}\sum_\alpha\frac{[f(\varepsilon_\alpha)-f(\epsilon_\alpha+\omega_c)]\,|\,j^+_\alpha\,|^2}{i(\omega-\omega_c)+\Gamma_\alpha(\omega)}\ . \tag{2.37}$$

Equations (2.35) and (2.36) are exact formal expressions for $\langle K(\omega)\rangle_\alpha$, valid for arbitrarily strong electron-phonon interaction V. Thus (2.37) gives an expression for $\sigma_{+-}(\omega)$ which is exact except for the approximation (2.20), $R_0(H)'\cong R_0(H'_0)$, which was introduced in (2.21). Thus, this expression (2.37) provides a rigorous basis for our discussion of the earlier theories of the cyclotron-resonance power absorption, which also made the approximation (2.20).

Expression (2.37) for $\sigma_{+-}(\omega)$ for the electron-phonon system is identical to expression (2.32) of Ref. 6 for the electron-impurity system, except for the detailed expression for the quantity $\Gamma_\alpha(\omega)$ which determines the line shape. In the present case, $\Gamma_\alpha(\omega)$ is given by (2.36) in terms of many-electron—phonon operators and the projection operator P is given by (2.28), while for the electron-impurity case, $\Gamma_\alpha(\omega)$ is given by (2.31) of Ref. 6 in terms of one-electron operators and the projection operator P there given by (2.21) of Ref. 6. It is easy to recover (2.31) of Ref. 6 from (2.36) above by specializing V to the electron-impurity interaction. To emphasize a trivial point, which however lies at the basis of our criticism of the earlier theories, the basic difference between the impurity and phonon scattering cases is the fact that the static positions of the scattering centers are c numbers while those of the vibrating nuclei are operators.

III. DISCUSSION

The quantity $\Gamma_\alpha(\omega)$, (2.36), clearly determines the line shape of the cyclotron-resonance power absorption through (2.37) and (2.18). We are interested in obtaining an expression for it for a weak scattering potential V, the strength of which will be denoted by λ. It would seem

that it is straightforward to generate a power series in λ for $\Gamma_\alpha(\omega)$ by simply expanding $G'(\omega^-)$ in powers of V. From (2.33) we have

$$G'(z)P' = (z - L_0 - P'L_1)^{-1}P' = G_0(z)\sum_{n=0}[P'L_1G_0(z)]^nP' , \quad (3.1)$$

where we have introduced the unperturbed propagator

$$G_0(z) \equiv (z - L_0)^{-1} , \quad (3.2)$$

and made use of the relation $PL_0P' = 0$, which holds since $\langle L_0P'Y\rangle_\alpha = 0$. Substituting (3.1) in (2.36) we get a formal expansion of $\Gamma_\alpha(\omega)$ in powers of λ

$$\Gamma_\alpha(\omega) = \sum_{n=1}^{\infty}\Gamma_\alpha^{(n)}(\omega) , \quad (3.3)$$

$$i\Gamma_\alpha^{(n)}(\omega) = \langle L_1[G_0(\omega^-)P'L_1]^{n-1}J^+\rangle_\alpha / \langle J^+\rangle_\alpha . \quad (3.4)$$

More explicitly, we find easily for the first term

$$i\Gamma_\alpha^{(1)}(\omega) = \langle L_1J^+\rangle_\alpha / \langle J^+\rangle_\alpha = 0 . \quad (3.5)$$

For the second we get after a lengthy but straightforward calculation

$$\begin{aligned} i\Gamma_\alpha^{(2)}(\omega) = {} & \langle L_1G_0(\omega^-)L_1J^+\rangle_\alpha / \langle J^+\rangle_\alpha \\ = {} & \sum_q\sum_\beta \left[c^\dagger_{\alpha+1,\beta}(c_{\beta,\alpha+1} - c_{\beta-1,\alpha}j^+_{\beta-1}/j^+_\alpha)\left[\frac{1+N_q-f_\beta}{\omega^- - \omega_{\beta\alpha} - \omega_q} + \frac{N_q+f_\beta}{\omega^- - \omega_{\beta\alpha} + \omega_q}\right]\right. \\ & \left. + (c_{\alpha\beta} - c_{\alpha+1,\beta+1}j^+_\beta/j^+_\alpha)c^\dagger_{\beta\alpha}\left[\frac{1+N_q-f_\beta}{\omega^- - \omega_{\alpha+1,\beta} + \omega_q} + \frac{N_q+f_\beta}{\omega^- - \omega_{\alpha+1,\beta} - \omega_q}\right]\right] \\ & + \sum_q\sum_\beta f_\beta\left[(c^\dagger_{\alpha+1,\alpha+1} - c^\dagger_{\alpha\alpha})c_{\beta\beta}\left[\frac{1}{\omega^- - \omega_c - \omega_q} - \frac{1}{\omega^- - \omega_c + \omega_q}\right]\right. \\ & \left. + (c^\dagger_{\alpha+1,\alpha}/j^+_\alpha)(j^+_\beta c_{\beta,\beta+1} - j^+_{\beta-1}c_{\beta-1,\beta})\left[\frac{1}{\omega^- - \omega_q} - \frac{1}{\omega^- + \omega_q}\right]\right] , \end{aligned} \quad (3.6)$$

where for convenience we have written $c_{\alpha\beta}$ for $c_{\alpha\beta}(q)$, the electron-phonon interaction matrix element, $N_q \equiv (e^{\beta\omega_q} - 1)^{-1}$ is the number of phonons q in equilibrium at temperature $T = 1/k\beta$, and $\omega_{\alpha\beta} \equiv \varepsilon_\alpha - \varepsilon_\beta$. In (3.6) the terms associated with the Fermi-Dirac distribution function f_β clearly describe the effects of exchange among the electrons.

If in expression (3.6) for $\Gamma_\alpha^{(2)}(\omega)$ we ignore the effects of electron exchange by putting $f_\beta = 0$, we recover the expression for the same quantity obtained by Lodder and Fujita,[2] Choi and Chung,[16,19] and others.[13,22,24] (The few terms by which the expression of Choi and Chung differs from that of the others clearly vanish in the thermodynamic limit.) These authors arrived at their expression by considering from the start a system of one electron moving in a phonon field, thus ignoring all exchange effects. But, although it is true that such exchange effects disappear[26] for electrons moving in the field of static impurities, it has been recognized[27,28] long ago that this is not the case for electrons moving in the field of vibrating nuclei. The explicit calculation presented here makes this point abundantly clear. One can, of course, treat the electron-impurity system starting with a many-electron system, thus allowing for the exchange effects as in the theory presented here. It is easy, however, to see that one recovers the results of the earlier treatment[6] which starts with the one-electron system, in conformity with the general theorem[26] for such systems. It is worthwhile to remark that in the usual[30] transport theory of conductivity of solids the exchange effects of electrons in their collisions with the phonons, as they are described[30] through the exclusion factors, play an important role in the determination of the temperature dependence of resistivity for low temperatures.

We now show that, even if we take into account the exchange effects as in the calculation of $\Gamma_\alpha^{(2)}(\omega)$ above, we cannot ignore the terms $\Gamma_\alpha^{(n)}(\omega)$ with $n \geqq 3$ in (3.3) even for small λ. Specifically, we show that the expansion of $\Gamma_\alpha(\omega)$ in powers of λ in (3.3) breaks down for $\omega = \omega_c$, i.e., higher-order terms become infinite for $\delta\omega = \omega^- - \omega_c = 0$, and thus this expansion is valid only in the wings of the absorption line. Briefly, the approximation of $\Gamma_\alpha(\omega)$ by terms of order up to λ^2 in (3.6), which these authors[2,13,16,19,22,24] have adopted, cannot describe correctly the shape of the cyclotron-resonance absorption line, especially near the center of the line.

In order to demonstrate that the terms of order λ^3 and higher in the expansion (3.3) of $\Gamma_\alpha(\omega)$ involve terms that diverge for $\omega = \omega_c$ (and in the final limit $\eta \to 0^+$), we note that every $G_0(\omega^-)$ in (3.3) is followed by $P' = 1 - P$, so that there are always terms that involve $G_0(\omega^-)J^+$. But, since $L_0J^+ = \omega_cJ^+$, we have $G_0(\omega^-)J^+ = J^+/\delta\omega$. Thus, these terms diverge for $\delta\omega = 0$, except for the terms in (3.3) of order λ^2, which vanish due to $\langle L_1J^+\rangle_\alpha = 0$. More generally, however, we note that any many-electron–phonon operator Y has a part Y_1 such that $G_0(\omega^-)Y_1 = Y_1/\delta\omega$. For example, the part of Y that is diagonal in the phonon quantum numbers and off-diagonal in the electronic quantum numbers with energy difference equal to ω_c is such a Y_1. By contrast, the part

$Y_2 \equiv Y - Y_1$ of any operator Y gives rise to regular terms when $G_0(\omega^-)$ operates on it, in the thermodynamic limit and for $\eta \to 0^+$. Thus, in the expansion (3.3) of $\Gamma_\alpha(\omega)$ in powers of λ, every factor $G_0(\omega^-)P'$, except for the last one that operates on $L_1 J^+$, brings in terms proportional to $\lambda/\delta\omega$, which clearly diverge for $\omega = \omega_c$ (and $\eta \to 0^+$). Therefore, in (3.3) there are terms proportional to $\lambda^2(\lambda/\delta\omega)^n$ with $n \geqq 1$. It follows that the procedure of keeping terms of order λ^2 and ignoring the others in the expansion (3.3) of $\Gamma_\alpha(\omega)$ is not valid for frequencies ω close to ω_c, the very frequencies for which $\Gamma_\alpha(\omega)$ is evaluated.

Thus, in order to get a correct expression for $\Gamma_\alpha(\omega)$ for small λ, we must sum the infinite subset of terms in the expansion (3.3) of $\Gamma_\alpha(\omega)$ that are of the form $\lambda^2(\lambda/\delta\omega)^n$ $(n \geqq 1)$. These are the dominant terms for sufficiently small λ, and will yield an expression for the linewidth correct to order λ^2. We can isolate and sum these terms by introducing a projection operator Δ that isolates the part Y_1 of any operator Y. Since the method is identical to the one used in Sec. IV of Ref. 6, we shall not indicate it here. In this way we find, as in Ref. 6, that the desired matrix elements can be obtained only through an *integral* equation. For this system of electrons and phonons we have the additional complication of not only projecting out the phonon variables but also reducing the whole scheme to one-electron quantities. Although we shall not pursue this further here, we wish to point out that the method of kinetic equations[27,28] accomplishes this much more efficiently, as it was explicitly demonstrated in Ref. 6 for the simpler system of one electron in the field of random impurities.

IV. CONCLUSION

We conclude that the explicit expressions for the resonance linewidth given by the recent theories[2,13,16,19,22,24] as rigorously valid up to order λ^2 are not correct for two reasons. First they ignore all exchange effects of the electrons, which can be substantial especially at low temperatures. Secondly, their method of calculation of $\Gamma_\alpha(\omega)$ to order λ^2 is strictly not valid for $(\omega - \omega_c)\tau \leqq 1$, where τ is of the order of the linewidth, i.e., at the center of the cyclotron-resonance line. As we have seen, a correct evaluation of $\Gamma_\alpha(\omega)$ up to λ^2 involves the solution of an integral equation, which is obtained much more easily with the method of kinetic equations.

Finally, in all these theories[2,13,16,19,22,24] including the one presented here, we have made the approximation (2.20), $R_0(H') \cong R_0(H'_0)$. This approximation disregards a number of subtle interference effects between the electric field and the scattering, and it thereby renders invalid the results of these theories also for $(\omega - \omega_c)\tau \gg 1$, i.e., in the wings of the cyclotron-resonance line.

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