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Direct observation of fluxon reflection in a Josephson transmission line

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We have directly observed the reflection of fluxons in a Josephson transmission line (JTL) with an open end using a Josephson sampler. Depending on the bias current at the end of the JTL, a reflected single fluxon, an antifluxon, and multi-antifluxons are observed with the measurements of the propagating time delay for the reflection process. The observed reflection properties, including the interaction between reflected antifluxons, are in fairly good agreement with numerical simulations taking account of the nonlinear shunting loss and the surface loss.

The dynamics of fluxons in Josephson transmission lines (JTL) has attracted considerable attention to both fields of nonlinear physics and applications,[1,2] since the fluxon in the JTL gives us a typical model of a soliton propagating in the solid state. Fluxon dynamics has been studied extensively on the theoretical side, which has been carried out by perturbation calculations and numerical simulations based on a modified sine-Gordon equation. In the experimental studies, however, static measurements, such as so-called zero-field steps in the current-voltage characteristics, have mainly been investigated, since it is hard to measure directly the wave form of a fluxon with a very small voltage and a narrow time width. Thus, there has been rather limited information on fluxon dynamics so far. Recently, direct observations of fluxons have been performed using minicomputer signal processing[3] or a Josephson sampling technique with very high time resolution and current sensitivity.[4,5] These direct measurements have realized the basic properties of a fluxon propagating in a JTL more clearly.[4-6] In order to understand a fluxon as a soliton more evidently, it is necessary to measure the interaction between fluxons, such as in reflection and collision.

In this Rapid Communication, we report direct observations of the fluxon reflection at the open end of a JTL using a Josephson sampler. The present report shows the first demonstration of the real-time fluxon reflection. It is found that a single fluxon, an antifluxon, and multi-antifluxons are reflected, depending on the bias current at the open end. The propagating time delay of each reflection process is also measured. The observed reflection properties can be satisfactorily explained by numerical simulations using the distributed resistively shunted junction (RSJ) model based on the modified sine-Gordon equation which takes account of the nonlinear quasiparticle resistance and surface rf resistance. Moreover, the interactions between reflected antifluxons are found in both experiments and simulations.

Figure 1 shows a schematic equivalent circuit of a sample used in the present experiments. A Josephson pulse generator (JP), which consists of a two-junction interferometer and a single Josephson junction, generates a single fluxon in the JTL. A direct coupled Josephson sampler (JS), consisting of a two-junction interferometer, resistors, and two single junctions, detects wave forms of fluxons reflected at the open end of the JTL. Both JP and JS are connected at one end of the JTL by resistors. Another end of the JTL is the open end and has a terminal supplying a bias current I_e to the JTL. The details of the Josephson sampling system and fluxon detections have been described in the previous reports.[4,5] The Josephson sampler used in this experiment has the time resolution of less than 6 ps and the current sensitivity of ~ 1 μA. These Josephson circuits and the JTL were fabricated on the same chip by a Nb/Al-oxide/Nb junction technology used for Josephson LSI circuits.[7] In this fabrication process, junctions were made by forming a junction sandwich which consisted of a Nb base electrode, an Al-oxide barrier, and a Nb counter electrode on a whole 2-in. wafer without any vacuum breaking, so that junctions and the JTL had the uniform current density and showed ideal I-V characteristics with a very small subgap leakage. The critical current density of junctions was set to 4 kA/cm^2. Films of Nb and Au-In alloys were used as ground plane and resistors, respectively. The Josephson penetration depth λ_J, plasma frequency ω_J, and characteristic impedance Z_0 of the JTL, were estimated to be 7.2 μm, 1.2×10^{12} rad/sec, and 0.63 Ω, respectively, from the experimental parameters of static I-V characteristics and other workers.[8] The JTL had an inline geometry with a 2.5-μm width and a 150-μm length, corresponding to $\sim 0.35\lambda_J$ and $\sim 21\lambda_J$, respectively.

Figure 2 shows observed wave forms of reflected fluxons for various bias currents I_e at the open end of the JTL. No

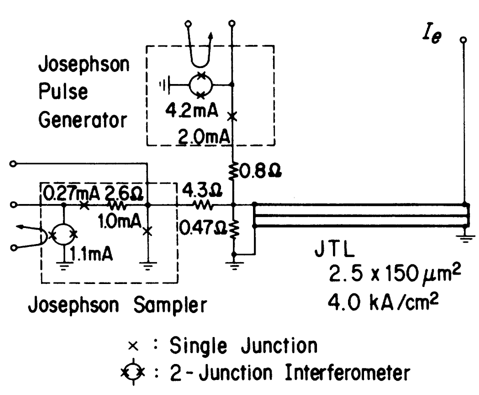

FIG. 1. Schematic equivalent circuit of a sample.

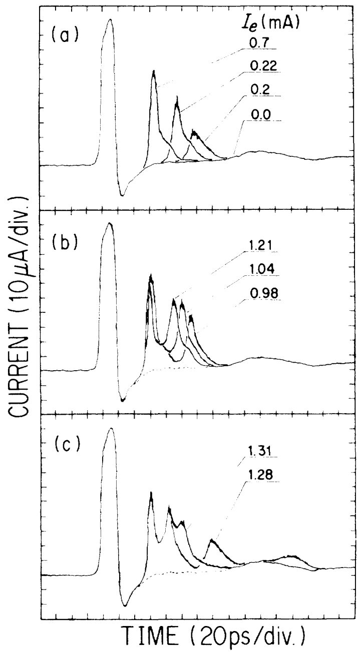

FIG. 2. Observed wave forms of antifluxons reflected at the open end of JTL as a function of positive bias current I_e. (a) Reflected single antifluxon. (b) Reflected two antifluxons. (c) Reflected multi-antifluxons more than two. The broken wave form in (b) and (c) shows the wave form at $I_e=0$ mA as a reference.

reflected fluxons are observed at $I_e=0$ mA as shown in Fig. 2(a). The first signal in the figures indicates a wave form of a pulse from JP. A peak position of this pulse is used as a reference time to determine the propagating time delay of reflected fluxons. For 0 mA $< I_e <$ 0.2 mA, there was no observation of reflected fluxons. A single reflected fluxon is observed for 0.2 mA $\leq I_e \leq$ 0.92 mA in Fig. 2(a). As I_e is increased, it is found that the time delay of the reflected fluxon is decreased, which means that the reflected fluxon returns to the left end of the JTL at a shorter time. Note that the observed reflected fluxon has a wave form with a positive amplitude, indicating that the observed fluxon has a circular current in the counterclockwise direction for the configuration shown in Fig. 1. An incident fluxon generated into the JTL from JP, however, has a circular current in the clockwise direction, contrary to that of the observed one. Therefore, if the incident fluxon is defined as a fluxon, the observed fluxon is an *antifluxon.* There was an anomalous behavior at $I_e=0.93-0.95$ mA, which occurred just before two antifluxons were reflected. The amplitude of the observed wave form was drastically decreased without obviously changing the time position. This anomalous behavior occurred only at $I_e=0.93-0.95$ mA.

For 0.96 mA $\leq I_e \leq$ 1.21 mA, two antifluxons reflected at the end of the JTL are observed, as shown in Fig. 2(b). As I_e is increased, the time delay of the first antifluxon is kept nearly constant, whereas that of the second antifluxon is decreased. Above $I_e=1.21$ mA, more than two antifluxons are reflected. For instance, reflected three and four antifluxons are observed for $I_e=1.28$ and 1.31 mA, respectively, as shown in Fig. 2(c). Multi-antifluxon reflection may physically indicate that the incident fluxon collides with the end in which the energy higher than that of a single antifluxon is supplied by the bias current, and consequently generates multi-antifluxons.

A fluxon, not an antifluxon, reflected at the open end was observed when the negative bias current was supplied at the end. Figure 3 shows the observed wave forms at $I_e=-1.41$ and -1.42 mA. Neither reflected fluxons nor antifluxons were observed for -1.41 mA $\leq I_e \leq 0$ mA. At $I_e=-1.42$ mA, a single fluxon is reflected, accompanied by some noisy signal caused by the latching mode of the JTL. Note that the observed wave form has a negative amplitude, indicating that the signal corresponds to a single *fluxon.* The fact indicates that an incident fluxon is driven back due to the Lorentz force of the bias current at the end. The time delay of a reflected fluxon is measured to be 58 ps. Below -1.42 mA, the multifluxons were reflected.

Note that JP generates a single fluxon in the JTL. It may be confirmed by the fact that the incident fluxon is driven back as a single fluxon when the small negative bias current of -0.24 mA is supplied at the *center* of the JTL.

Figure 4 shows the normalized bias current $\eta=I_e/I_{e0}$ dependence of the time delay T_d, where I_{e0} is the maximum critical current at the end of the JTL ($I_{e0}=1.44$ mA). Here, the time delay T_d is defined as the time interval between the times of the observed Josephson pulse and antifluxons. T_d is used as a measure of the time of a single reflection process consisting of the propagation of an incident fluxon, the reflection at the end, and the propagation of antifluxons. As seen in the figure, a single antifluxon reflected at the end is observed at $\eta=0.14$ ($I_e=0.2$ mA) with $T_d=107$ ps. As η is increased, T_d is decreased approaching the constant time delay of 53 ps. From this value, the average velocity during the single reflection process of a fluxon is estimated to be $\sim 5.7\times 10^6$ m/s. After the anomalous reflection at $\eta=0.65-0.66$, two antifluxons are reflected with further increase of η ($0.67 \leq \eta < 0.85$). T_d of the first antifluxon is nearly kept constant which is, however, smaller than that in a single antifluxon reflection, whereas T_d of the second one is decreased approaching the constant T_d of 83 ps, as η is increased. At $n=0.85$ ($I_e=1.22$ mA), the third reflected

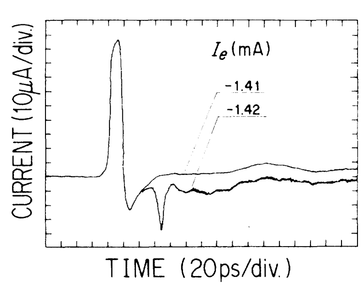

FIG. 3. Observed wave forms of reflected fluxon for negative bias current I_e.

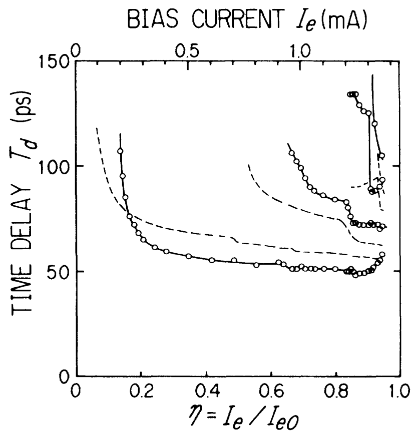

FIG. 4. Normalized bias current η dependence of time delay T_d. The solid lines indicate the experimental results. The broken lines indicate the results of numerical simulations with $\alpha = 0.0091$ and $\beta = 0.045$.

antifluxon appears with $T_d = 134$ ps. As seen in the figure, T_d of the second antifluxon is drastically decreased when the third one begins to appear. This behavior is similar to that of the border between the single- and two-antifluxons reflection. This fact suggests that the third antifluxon could push a second one due to the interaction between the second and third antifluxons. The similar behavior also occurs when the fourth antifluxon is reflected at $\eta = 0.91$ ($I_e = 1.31$ mA). In addition, the decrement of the T_d jump becomes larger as the number of reflected antifluxons is increased. Above $\eta = 0.94$ ($I_e = 1.35$ mA), the bunched antifluxons were reflected.

The fluxon motion for the inline geometry is described by the modified sine-Gordon equation[9]

$$\phi_{xx} - \phi_{tt} - \sin\phi = \alpha\phi_t - \beta\phi_{xxt} \ , \qquad (1)$$

where $\phi(x,t)$ is the space- and time-dependent phase difference between the two superconducting films. The spatial variable x is measured in units of the Josephson penetration depth $\lambda_J = (h/4\pi e\mu_0 dJ_0)^{1/2}$, and the time t in units of the reciprocal plasma frequency ω_J, where $\omega_J = (4\pi eJ_0/hC_J)^{1/2}$. Here, $-e$ is the electronic charge, h is Planck's constant, μ_0 is the permeability of free space, J_0 is the maximum critical current density, d is the magnetic thickness of the barrier ($d = 2\lambda_L + t_0$) where λ_L is the London penetration depth and t_0 the thickness of the barrier, and C_J is the capacitance per unit area. The parameters $\alpha = G(h/2eJ_0C_J)^{1/2}$ and $\beta = \mu_0 d\omega_J/r$ describe dissipative effects due to tunneling of normal electrons across the barrier and due to flow of normal electrons parallel to the barrier, respectively. Here, r is the sheet surface resistance.

In the inline geometry with the asymmetric edge current, the dc bias current enters only through the boundary conditions

$$\phi_x(\rho,t) = 2\eta, \quad \phi_x(0,t) = 0 \ , \qquad (2)$$

where $\eta = I_e/I_{e0} = I_e/2\lambda_J W J_0$ represents the bias current and $\rho = L/\lambda_J$, and L and W are the length and width of the JTL, respectively. Note that in our JTL the edge current flows from the counter electrode to the base electrode only at the same end of the JTL, so that $\eta = 1$ corresponds to the critical current.

A numerical investigation of the fluxon reflection has been carried out by Olsen and Samuelsen.[10] They found the border line between regions of a single reflected fluxon, antifluxon, multifluxons, and multi-antifluxons, depending on the velocity of the incident fluxon and magnitude of the external magnetic field. These various regions of the reflections support our experimental results. Unfortunately, their numerical results cannot directly be compared with our results, since the simulations have been carried out using the pure sine-Grodon equation ($\alpha = \beta = 0$) and for an inline JTL with the symmetrical bias field.

Recently, there have been some reports[6,11-13] in which the surface loss plays an important role in the dynamics of the fluxon propagating in the JTL. Matsuda and Kawakami[6] have explained the propagation of their observed fluxon if a surface loss is larger than the theoretical one. They obtained the value of the surface resistance using Halbritter's method[14] within the framework of the Bardeen-Cooper-Schrieffer theory. According to their discussions, the value of r is estimated to be 4.9 Ω with use of experimental parameters[8] of $C_J = 8$ μF/cm^2, $J_0 = 4$ kA/cm^2, and $\lambda_L = 80$ nm. Thus, the value of β is obtained to be 0.045 for our JTL. The value of α is estimated to be 0.0091 using experimental parameters and under the assumption of G as the conductance below the gap voltage. From the comparison between α and β the surface loss has more dominant damping effect on the fluxon dynamics in our JTL compared to the shunting loss.

Numerical simulations, taking account of the nonlinear shunting resistance and surface resistance, have been performed. The simulations include the circuit of the JP and load resistances. The results of numerical simulations for I_e dependence of T_d are shown in Fig. 4 as broken lines. The experimental data are in fairly good agreement with numerical simulations which show the sudden decrease of the time delay in multi-antifluxon reflections, due to the interaction between antifluxons. If the surface resistance is absent ($\beta = 0$), the simulations show that the second antifluxon begins to be reflected at $\eta = 0.55$, with $T_d = 60$ ps which is much shorter than that of the observed one. The slight discrepancy between the results of experiments and simulations is not yet resolved: It may reflect the additional damping effect of the Al-oxide barrier in the JTL, since a very thin Al-metal layer is considered to remain still on the surface of the Nb base electrode.[15]

From the simulations for the negative bias-current dependence, it is found that a single fluxon is reflected and driven back to the left end with $T_d = 54$ ps at $\eta = -0.95$, which agrees with the observed one ($\eta = -0.98$). The summary of the simulation results for $-0.95 < \eta < 0.05$, in which neither the reflected fluxon nor antifluxon is observed in the experiments, is as follows: For $0.02 \leq \eta < 0.05$, the incident fluxon was reflected into an antifluxon with the extremely small propagating velocity; for $-0.23 < \eta < 0.02$, the incident fluxon was annihilated at the end with a damped oscillating mode; and for

$-0.95 < \eta \leq -0.23$, it ran back as an almost static fluxon.

In summary, we have studied the real-time observation of the fluxon reflection in a Josephson transmission line with the open end using the Josephson sampler. It is found that depending on the bias current at the open end, the incident fluxon is reflected into a single fluxon, into a single antifluxon, and into antifluxons. Numerical simulations with the shunting loss ($\alpha = 0.0091$), the surface loss ($\beta = 0.045$), and the edge bias current are, to a considerable degree, consistent with the experimental data. The interaction between reflected antifluxons is also confirmed in both experiments for the time-delay measurements and simulations.

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