Anomalous time correlations in quenched systems with continuous symmetry

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We continue our exploration of the growth kinetics of the time-dependent Ginsburg-Landau model after a rapid temperature quench in the limit of large N—the number of components of the order parameter. We focus here on time-displaced correlation functions and the approach to the linear-response regime where the dynamics are governed by fluctuations in equilibrium. In the case where one correlates fields at times t and $t+\tau$, with the time after the quench t small, one finds, for typical values of the wave number, that the Fourier transform of the correlation function falls rather rapidly to zero for sufficiently large τ as the fields lose correlation. However, for q near and equal to zero this decay can be extremely slow and serves as a measure of the softness of the restoring force in the system for long times where there are well-developed Nambu-Goldstone modes. For $q=0$ these correlations grow with a power-law dependence on τ and the $q=0$ component is nonergodic.

I. INTRODUCTION

In a previous set of papers,[1,2] we explored the growth kinetics of the time-dependent Ginzburg-Landau (TDGL) model for various types of quenches in the limit of large N—the number of components of the order parameter. Scaling behavior and growth laws were determined and associated with the evolving dominance of a single length in the problem, the average domain size. In the absence of an external ordering field the analysis focused on the correlation function $C(\mathbf{x},\mathbf{x}',t)=\langle\phi(\mathbf{x},t)\phi(\mathbf{x}',t)\rangle$, where $\phi(\mathbf{x},t)$ is the order parameter in the system and t measures the time since the system was taken out of equilibrium. When the system is in equilibrium $C(\mathbf{x},\mathbf{x}',t)$ will be independent of time and it is conventional to study the time-displaced correlation functions $C(\mathbf{x},\mathbf{x}',t,\tau)=\langle\phi(\mathbf{x},t)\phi(\mathbf{x}',t+\tau)\rangle$ which is independent of t. In this paper we study the behavior of this more general quantity in the nonequilibrium situation of a strong temperature quench, again in the limit of large N for a nonconserved order parameter.

In carrying out this study we hoped to understand the approach to the equilibrium dynamics governed by $C(\mathbf{x},\mathbf{x}',\infty,\tau)$. In this case the equilibrium dynamics is relaxational and one is interested in how long it takes before the system is governed by its final equilibrium relaxation rate. When do we enter the linear-response regime? Of course, in a system with a richer dynamics, one would be interested in how collective modes shift or are born as the system evolves from some initial value to some final value. Since there has been very little work[3] on this problem, we were also interested in the structure of the theory.[4] What is the interplay between the two times? Within the context of the simple model we study we can answer both of the questions posed above. As a bonus in this analysis we have found some rather interesting behavior associated with the soft response of the system which is manifested for quenches to below the ordering temperature.

Consider the case of quenching to below the transition temperature and monitoring the Fourier transform $C(q,t,\tau)$ as a function of t and τ. One expects that for large t $C(q,t,\tau)$ will approach its equilibrium form $C(q,\infty,\tau)$ which will be a sum of τ-independent Bragg-peak contribution and a relaxing exponential with a rate proportional to q^2 as a result of the existence of Nambu-Goldstone modes in the final equilibrium state. We indeed verify this evolution quantitatively below. Consider, however, the less obvious situation where t is fixed and τ is taken to be large.[5] Physically this would correspond to taking data for $\phi(\mathbf{q},t)$ as a function of t and then coming back and autocorrelating it with a time displacement τ. What does one expect for t fixed and τ large? Since t may be fixed at some relatively early time after the quench the underlying probability at that time may be very different from that as $t+\tau$. One expects that $C(q,t,\tau)$ would fall rather rapidly to zero for sufficiently large τ as the fields lose correlation. For typical values of the wave number this is true. However, for q near and equal to zero this decay can be extremely slow and serves as a measure of the softness of the restoring force in the system for long times where there are well-developed Nambu-Goldstone modes. Indeed, for $q=0$ these correlations *grow* with a power-law dependence on τ. This reflects a nonergodicity in the system at $q=0$ which is driven by the Nambu-Goldstone modes in the system.

II. FORMALISM AND QUANTITIES OF INTEREST

We begin with a brief review of the model (further details and references are given in Ref. 2). We consider the dynamics described by the time-dependent Ginzburg-Landau model (TDGL)

$$\frac{\partial\boldsymbol{\phi}(\mathbf{x},t)}{\partial t}=-\Gamma\frac{\delta F}{\delta\boldsymbol{\phi}(\mathbf{x},t)}+\boldsymbol{\eta}(\mathbf{x},t)\ , \tag{2.1}$$

where $\boldsymbol{\phi}=(\phi_1,\ldots,\phi_N)$ is an N-component (nonconserved) order parameter, F is the free energy of the model

$$F[\boldsymbol{\phi}]=\tfrac{1}{2}\int d^Dx[(\nabla\boldsymbol{\phi})^2+r\boldsymbol{\phi}^2+u(\boldsymbol{\phi}^2)^2/2N]\ , \tag{2.2}$$

and $\boldsymbol{\eta}(\mathbf{x},t)$ is a Gaussian white noise satisfying

$$\langle\boldsymbol{\eta}(\mathbf{x},t)\rangle=0\ , \tag{2.3a}$$

$$\langle\eta_i(\mathbf{x},t)\eta_j(\mathbf{x}',t')\rangle=2\Gamma\delta_{ij}\delta(t-t')\ , \tag{2.3b}$$

where, for a nonconserved order parameter, Γ is a constant. It is assumed that the system is defined on an underlying lattice which introduces large wave-number cutoff Λ.

The static properties of the model in the large-N limit are well known. For a fixed and positive u there is a critical value of r given by $r_c=-uS_c$, where

$$S_c=\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^2}\ . \tag{2.4}$$

In the disordered phase $(r>r_c)$ the Fourier transform of the order-parameter correlation function is given by

$$C(q)=\frac{1}{q^2+K^2}\ , \tag{2.5}$$

where the inverse correlation length K is defined by

$$K^2=r+uS\ , \tag{2.6}$$

$$S=\int\frac{d^Dq}{(2\pi)^D}C(q)\ . \tag{2.7}$$

In the ordered phase $(r<r_c)$ the continuous $O(N)$ symmetry is broken with the spontaneous magnetization given by

$$m^2=\frac{r_c-r}{u}\ . \tag{2.8}$$

The order-parameter correlation function splits into the longitudinal contribution

$$C_{\parallel}(q)=\frac{1}{q^2+2(r_c-r)} \tag{2.9}$$

and the transverse contribution

$$C_{\perp}(q)=\frac{1}{q^2} \tag{2.10}$$

reflecting the existence of the Nambu-Goldstone modes in the transverse directions.

In the following we consider the time development of the system when it is initially prepared in an equilibrium state above the critical point $(r_I>r_c)$ and, at some time instant t_0, is suddenly quenched to a lower value of $(r_F<r_I)$. The quantity of interest is the time-displaced correlation function

$$C(\mathbf{x}-\mathbf{x}',t,\tau)=\langle\phi_i(\mathbf{x},t)\phi_i(\mathbf{x}',t+\tau)\rangle \tag{2.11}$$

with $t\geq t_0$ and $\tau>0$, whose Fourier transform in the large-N limit obeys the equation of motion

$$\frac{\partial C(q,t,\tau)}{\partial t}=-\tfrac{1}{2}[q^2+\xi(t+\tau)]C(q,t,\tau)\ , \tag{2.12}$$

where

$$\xi(t+\tau)=[r+uS(t+\tau)] \tag{2.13}$$

and

$$S(t+\tau)=\langle\phi_i^2(\mathbf{x},t+\tau)\rangle\ . \tag{2.14}$$

Equation (2.12) is written in dimensionless form measuring lengths in units Λ^{-1}, where Λ is the wave-number cutoff, and time in units $2\Gamma\Lambda^2$.

Integrating Eq. (2.12) with the equal time condition

$$C(q,t,\tau=0)=\tilde{C}(q,t) \tag{2.15}$$

we find

$$C(q,t,\tau)=\tilde{C}(q,t)\exp\left[-\int_0^\tau d\tau'[q^2+\xi(t+\tau')]/2\right]\ , \tag{2.16}$$

where $\tilde{C}(q,t)$ is the Fourier transform of the equal-time correlation function $\langle\phi_i(\mathbf{x},t)\phi_i(\mathbf{x}',t)\rangle$ and the quantity $S(t)$ defined in Eq. (2.14) is given by

$$S(t)=\int\frac{d^Dq}{(2\pi)^D}\tilde{C}(q,t)\ . \tag{2.17}$$

This quantity and $\tilde{C}(q,t)$ were studied in great detail in Ref. 2 (hereafter referred to as I) and the results will be summarized in Sec. III.

III. RESULTS

In this section we analyze the behavior of $C(q,t,\tau)$ for various types of quenches. In all cases we take $r_I=+\infty$. The large value of r_I, as Eqs. (2.5) and (2.6) show, reduces the order parameter fluctuations to zero in the initial state, i.e., $\tilde{C}(q,t_0)=S(t_0)=0$.

A. $r_F>r_c$

When the final equilibrium state is in the disordered phase, the equal-time correlation function $\tilde{C}(q,t)$, as was shown in I, evolves toward the equilibrium form (2.5). Correspondingly the quantity $S(t)$, which gives a measure of the order-parameter fluctuations, grows from the initial zero value to the final equilibrium value given by (2.7) which we denote by $S(\infty)$. The behavior of $S(t)$ is characterized by an initial fast transient followed by a slow approach to the asymptotic value $S(\infty)$.

Let us now consider the behavior of $C(q,t,\tau)$. If the initial time of observation t is chosen very far away in the asymptotic region, then to a good approximation in Eq. (2.15) we can substitute for $\tilde{C}(q,t)$ and $S(t+\tau)$ their time-independent equilibrium values obtaining

$$C(q,t,\tau)=C(q)e^{-[q^2+r_F+uS(\infty)]\tau/2}\ . \tag{3.1}$$

This gives the expected exponential decay of the equilibrium correlations.

The behavior is qualitatively similar when t is chosen in the region of the fast transient, namely very close to the

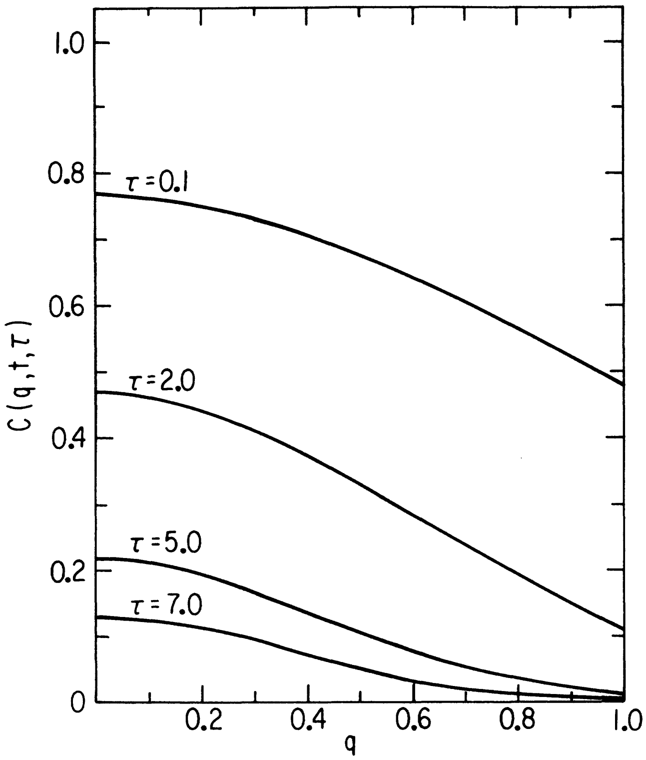

FIG. 1. Decay of $C(q,t,\tau)$ with growing values of τ for a quench above the critical point. $t=1$; $r_F=0.5$; $r_c=-0.051$.

onset of the quench. In this case, $\xi(t+\tau)$ grows through positive values from the initial value $[r_F+uS(t)]$, to the asymptotic value $[r_F+uS(\infty)]$, yielding again the decay of the correlation, as illustrated in Fig. 1.

B. $r_F<r_c$

When the system is quenched below the critical point, the dynamics is quite different and more interesting since it contains the time development of the spontaneous magnetization. The process of ordering is well described by the equal-time structure factor $\tilde{C}(q,t)$ which, as discussed in I, displays the growth of the central Bragg peak associated with the build up of the magnetization, and the growth of the $1/q^2$ behavior associated to the Nambu-Goldstone modes in the transverse directions. The corresponding behavior of $S(t)$ is qualitatively similar to the previous case ($r_F>r_c$), with an initial fast transient followed by a slow approach to the asymptotic value $S(\infty)=m^2+S_c$. In the latter, time region domains ordered in all directions are formed and the characteristic size of the domains becomes the dominant length in the system, producing scaling of the structure factor.

The main difference with respect to the previous case is that r_F is negative and that

$$r_F+uS(\infty)=0 \tag{3.2}$$

as follows from Eq. (2.8). Consequently, $\xi(t+\tau)$ goes to zero through negative values as τ grows. As we show in the following, this produces the unexpected result of the growth of correlations at $q=0$.

The simplest case is obtained taking t so large that the system can be considered very close to equilibrium, then from Eqs. (2.15) and (3.2) we have

$$C(q,t,\tau)=\tilde{C}(q)e^{-q^2\tau/2} . \tag{3.3}$$

Again we have exponential decay of correlations, except at $q=0$. This peculiar behavior is more clearly understood through the evolution of $C(q,t,\tau)$ when t is chosen in the region of the fast transient. Initially ($\tau=0$) we have $\xi(t)<0$. Therefore modes with $q^2>|\xi(t)|$ exhibit a decaying behavior, while correlations tend to grow with τ for modes with $q^2<|\xi(t)|$. However, as the fluctuations increase through the development of instability, $\xi(t+\tau)$ tends to zero and eventually there will be a decay of correlations at all wave vectors, except $q=0$. This type of behavior is illustrated in Fig. 2 which exhibits the initial increase and subsequent decay of correlations at $q>0$ and the monotonous growth of the peak at $q=0$.

In order to extract the growth law at $q=0$, we have plotted in Fig. 3 the behavior of $C(q=0,t,\tau)$ for $t=1$ and $r_F=-10$. When $t+\tau$ is in the scaling region (for $t+\tau>200$), to a very good approximation, we have[6]

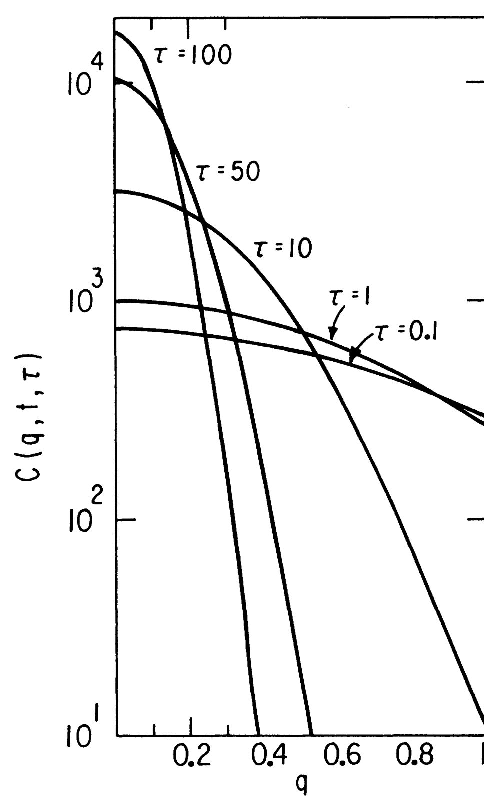

FIG. 2. Evolution of $C(q,t,\tau)$ for a quench below the critical point. $t=1$; $r_F=-10$; $r_c=0.051$.

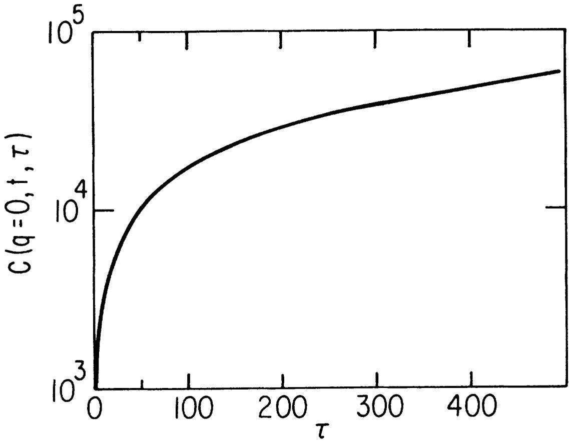

FIG. 3. Growth of the peak at $q=0$ in the quench below the critical point. $t=1$; $r_F=-10$.

$$C(q=0,t,\tau)=545.66(t+\tau)^{3/4} \ . \tag{3.4}$$

The explanation of this kind of behavior is in the gapless nature of the Nambu-Goldstone modes, which build up as the system evolves toward equilibrium. In fact, the quantity $q^2+\xi(t+\tau)$ acts as a restoring force, which at the onset of the quench is negative for the unstable modes. However, the system reacts to the growth of fluctuations induced by the instability, by gradually building up the positive restoring force. As the system equilibrates, it also attempts to break the continuous symmetry. In this process, as order grows, transverse Nambu-Goldstone modes developed. In the case studied here the dominant contribution to the self-consistent restoring force is precisely due to the transverse modes, which, as a consequence of the gapless nature of the Nambu-Goldstone modes, fail to equilibrate the system at $q=0$.

C. $r_F=r_c$

Finally, we comment on the case of the critical quench ($r_F=r_c$). Since $r_c<0$, one again has that $\xi(t+\tau)$ tends to zero and the qualitative behavior is of the type above discussed for the quench below the critical point. Namely there is no restoring force to equilibrate the instability at $q=0$ and the corresponding structure factor grows with the time separation. In Fig. 4 we have plotted the behavior of $C(q=0,t,\tau)$, which for $\tau>200$ is well fitted by the power law $C(q=0,t,\tau)=0.42(t+\tau)^{-0.25}$.

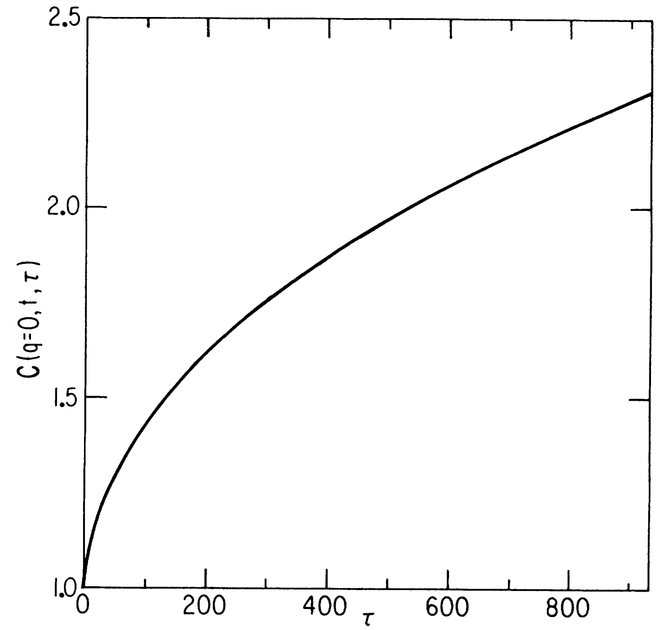

FIG. 4. Growth of the peak at $q=0$ in the critical quench. $t=1$; $r_F=r_c=-0.051$.

IV. COMMENTS

In this paper we began to explore the behavior of the two-time correlation functions which naturally enter into quench problems when one attempts to develop a perturbation-theory approach. We have found, even at the leading order in $1/N$, that these quantities have interesting nontrivial behavior when there is a broken continuous symmetry in the system. In particular, this suggests that one can gain information about the soft-mode structure in a system by doing a time-domain-quenched experiment as opposed to the traditional frequency analysis associated with scattering experiments. The soft-mode structure discussed here should be present in all systems $N>1$ unless there is some defect structure, not present for $N\rightarrow\infty$, which changes the growth kinetics. Investigation of the role of these defects is an interesting but quite challenging task.

This work also sets up a natural extension of the previous work to consider the $O(1/N)$ corrections. This is a technically complicated calculation, and one does expect many of the results of Refs. 1 and 2 to survive, but there may be some interesting new effects not captured in the leading $O(1)$ calculation.

ACKNOWLEDGMENTS

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valid in these models only for early times and leads for longer times to spurious soft modes not present in the $N=1$ case. See our comments in Ref. 2.

[4]The formal structure of field theories evolving under strongly nonequilibrium conditions has been studied by a number of authors: J. Schwinger, J. Math. Phys. **2**, 407 (1961); V. Korenman, Ann. Phys. (N.Y.), **39**, 72 (1966); L. V. Keldysh, Zh. Eksp. Teor. Phys. **47**, 1515 (1964); [Sov. Phys.—JETP **20**, 1018 (1965)]; U. Deker, Phys. Rev. A **19**, 846 (1979). Two time correlation functions of the type discussed here are the main building blocks in such theories. In particular one can not treat $C(\mathbf{x},\mathbf{x}'t,0)$ at higher order in $1/N$ without treating $C(\mathbf{x},\mathbf{x}',t,\tau)$ for $\tau \geq 0$.

[5]Correlation functions of this type cannot be simply treated using the standard Fokker-Planck description. This is discussed in S. Ma and G. Mazenko, Phys. Rev. B **11**, 4077 (1975).

[6]The prefactor and the time $t+\tau$ it takes to enter this regime depend on the choice of the quartic coupling u.