Magnetoresistance measurements on fractal wire networks

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Measurements of the magnetoresistance have been carried out on aluminum Sierpinski gasket wire networks fabricated using submicrometer *E*-beam lithography. The results are in agreement with the predictions of recent calculations of the localization contribution to the magnetoresistance in the fractal regime where the phase-breaking length, L_ϕ, is greater than L_0, the minimum gasket dimension. All details of the theory are observed, including periodic corrections to scaling due to the discrete dilational invariance of the gasket. Over the temperature range from $T_c=1.192$ to 4.0 K we have quantitatively analyzed the magnetiresistance and determined the phase-breaking length, $L_\phi(T)$. The predominant phase-breaking mechanism inferred from such analysis is found to be one-dimensional electron-electron scattering with small (quasielastic) energy transfers.

I. INTRODUCTION

Fractal objects, by definition, are those with nonintegral geometric dimension. In general, such systems may be fractal over a limited range of length scales. Subsequently, interpretation of measurements made on these systems depends upon whether the fractal or the homogeneous regime is being probed. Determination of which regime is relevant requires a knowledge of several characteristic lengths. The details of the physics involved determines the probe length, Λ. The geometry of the fractal itself determines its intrinsic length scales including the minimum and maximum dimensions, L_0 and L_N. The length L_0 is the lower bound of the fractal regime. Over shorter lengths the object is assumed to have integral dimension. In the case of the Sierpinski gasket L_0 corresponds to the size of the smallest triangles, about 1.5 μm. Over small distances the relevant structures are the wires themselves (see below), definitely Euclidean objects. Similarly, L_N corresponds to the size of the entire gasket network. For reference, a third-order gasket is pictured in Fig. 1.

Two distinct physical regimes are determined by the relative size of the probe length and the characteristic sample lengths: the homogeneous regime where $\Lambda < L_0$ or $\Lambda > L_N$ and the fractal regime where $L_0 < \Lambda < L_N$. Interesting new physics is contained both in the fractal regime where the anomalous dimensionality of the system is probed and in the crossover region dividing Euclidean from fractal geometries.

Probe lengths relevant to studies of fractal systems include optical,[1] x-ray,[2] neutron,[3] and phonon[4] wavelengths as well as diffusion lengths for electrons,[5] electron spins,[6] noise,[7] and Cooper pairs.[8] The corresponding geometrical lengths characterizing these fractals range from fractions of nanometers (nm) in percolation studies and investigations of random alloys, to hundreds of nm in colloids and to micrometers (μm) in state-of-the-art lithography. The measurements described below employ an ideal combination of probe length and characteristic sample lengths for testing theoretical calculations on fractals. The controlled geometry of lithographically defined structures are combined with the macroscopically long inelastic and superconducting-pair diffusion lengths in clean aluminum films.

Specifically, we have measured the temperature- and magnetic-field-dependent conductivity of aluminum Sier-

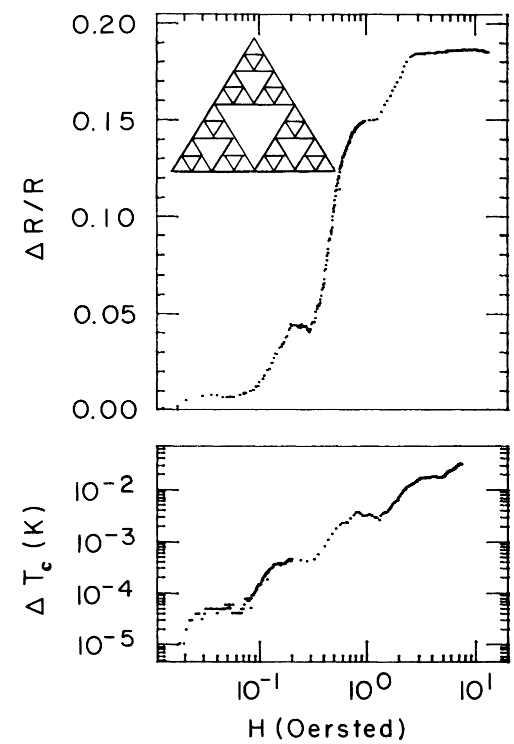

FIG. 1. The lower graph shows the variation of the superconducting transition temperature, T_c, as a function of perpendicular magnetic field. The upper graph, using the same field axis, plots the magnetoresistance at a reduced temperature of $\epsilon=2.5\times10^{-3}$. The field-dependent structure in the MR is analogous to that of the transition temperature. The inset shows a third-order Sierpinski gasket.

pinski gasket (SG) networks at temperatures above the superconducting transition temperature, T_c. The dominant contributions to the magnetoresistance (MR) come from direct superconducting fluctuations, characterized by the pair diffusion (coherence) length, ξ, and localization-related effects which are parametrized by the phase-breaking length, $L_\phi^2 = D\tau_\phi$. At temperatures very close to T_c, where $\xi \gg L_\phi$, direct superconducting fluctuation effects dominate. The MR in this region is characterized by its fine structure and large magnitude while the Aslamazov-Larkin (AL) conductance is expected to exhibit an anomalous dependence on temperature, $G'_{\rm AL} \sim (\ln t)^{(\bar{\bar{d}}/2-2)}$. We introduce the normalized conductance, $t = T/T_c$. Here $\bar{\bar{d}}$ is the fracton dimension, defined in terms of the Hausdorff (fractal) dimension, D, and the anomalous diffusion exponent, θ, via the relation

$$\bar{\bar{d}} = D/(1+\theta/2) = \ln 9/\ln 5 \simeq 1.365 \ ,$$

Ref. 8. In the following we shall make qualitative statements about the MR in this *direct* regime, but we have not been able to study the temperature dependence. This is due to both the small, applicable temperature range (just a few mK) and to small variations in the superconducting properties of the wires.

$\xi(t)$ is a strong function of temperature, however, falling off as $(\ln t)^{-1/2}$. At somewhat higher temperatures, where $\xi < L_\phi$, localization-related effects (weak localization and Maki-Thompson fluctuations), those with characteristic length scale L_ϕ, are predominant. In our aluminum wires the phase-breaking length may be much greater than the minimum fractal length, L_0, over a large temperature range. Because of this, the details of the MR, which are sensitive to L_ϕ, depend upon the fractal nature of the SG. Theoretical calculations of the MR due to weak localization (WL) and Maki-Thompson (MT) effects have recently been carried out for the gasket and are tested in our work.

At temperatures far enough above T_c so that only localization-related effects are important (more than a few mK above T_c) the sole fitting parameter in comparing theory with our data is the phase-breaking length, $L_\phi(T)$. It is interesting to note that L_ϕ is due mainly to electron-electron effects which have their *own* characteristic length scale, $L_T \simeq 0.5(\mu{\rm m})/\sqrt{T({\rm K})} < L_0$. Thus the phase-breaking processes are in the homogeneous regime, unlike the resulting MR curves which depend upon the fractal structure of the network. From an analysis of the measured values of $L_\phi(T)$ we report some of the first evidence for one-dimensional electron-electron scattering.[9] One can envision that with the advances of technology permitting ever-smaller feature sizes that fractal structures might be made such that $L_T \gg L_0$, necessitating the consideration of electron-electron scattering in nonintegral dimensions. Although this has not yet been attempted theoretically it may already be important in understanding weak-localization (WL) data in natural fractal systems (alloys, quench condensed films, composites, etc.) where L_0 may be quite small.[10]

II. EXPERIMENTAL PROCEDURE

Our Sierpinski gaskets were prepared by thermal evaporation of 1000 Å of high-purity Al onto oxidized Si substrates. The networks were formed by prepatterning the substrates with a lift-off mask written into two layers of electron-beam resist by a Cambridge EBMF-2-150 *E*-Beam Microfabricator. The gaskets are of tenth order and are composed of wires with a linewidth of 0.3 μm. All triangles which make up the gasket structure are isosceles due to restrictions inherent in the Cambridge machine. The zero-order triangles have both base and height equal to 1.66 μm and an area of $a = 1.38 \pm 0.01\ \mu{\rm m}^2$.

The full network has a base of nearly 1.7 mm with pairs of contact pads attached at each corner and at the midpoints of each side. A wide film with four contacts was deposited next to the gasket for comparison of resistivities in the bulk and in the wires making up the SG. All of the MR measurements reported below were made with four probes, two at the top of the network and two at the midpoint of the base. Other combinations of the current and voltage probes were tried with no change in the resulting normalized MR $(\Delta R(H)/R = [R(H) - R(0)]/R(0))$. This is to be expected since the probe length in all of our measurements is very small compared to the full gasket dimension, $\Lambda \ll L_{10}$. Here L_{10} is a length characteristic of a tenth-order gasket.

An accurate determination of the sheet resistance, $R_\square$, of the wires which make up the gasket is essential for a proper analysis of the MR data. From measurements of the coevaporated two-dimensional (2D) strip we infer a sheet resistance of $0.040 \pm 0.002\ \Omega$. (Figures are reported for one of the gaskets. Small variations exist from sample to sample.) One might expect, however, that the submicrometer wires are of somewhat higher resistivity. Using scaling arguments we can determine the resistivity of the wires from knowledge of the resistance of the entire gasket. The resistance of the tenth-order gasket is related to the zeroth-order resistance via

$$R(L_{10}) = R(L_0)(L_{10}/L_0)^{-\beta_L} \ .$$

The localization exponent for the SG is $-\beta_L = 0.737$ and $(L_{10}/L_0) = 2.$[10] Using the tenth-order resistance of 43 Ω and the geometry of the zero-order triangles (above) we have calculated the sheet resistance of the wires and find $R_\square = 0.057\ \Omega$. As expected, this slightly exceeds the sheet resistance of the 2D comparison films.

In zero applied magnetic field the superconducting transition temperature was $T_c(H=0) \equiv T_{c0} = 1.192$ K. The high-field dependence of T_c on H (due to the one-dimensional nature of the wires) was used to determine the superconducting-pair diffusion length (coherence length), $\xi(0) = 0.26\ \mu$m.[11] The value of the diffusion constant, $D = 270\ {\rm cm}^2/{\rm s}$, follows from the relationship

$$\xi^2(0) = \pi\hbar D/8k_B T_c \ .$$

III. THEORY

The magnetoresistance of disordered metals with integral dimension has been studied extensively in the past few years. Theoretical and experimental advances have been combined to make MR measurements a convenient tool for investigating electron phase-breaking (often equivalent to inelastic) lifetimes, τ_ϕ, in thin films. Aluminum films, similar in composition to our SG samples, have recently been investigated in both the one-dimensional[11,12] $(L_\phi > W,d)$ and two-dimensional[11] $(W > L_\phi > d)$ weak localization regimes. Measured phase-breaking rates due to inelastic processes are in excellent quantitative agreement with theory.

At most temperatures above T_c the important contributions to the MR come from weak localization and Maki-Thompson effects. A general calculation of the WL magnetoresistance on the SG has recently been carried out by Doucot and Rammal.[13] They find that the MR may be calculated from knowledge of the eigenvalues, λ_α, of the Hermitian matrix, Q, which has diagonal elements $Q_{\alpha\alpha}=z\cosh(\eta)$ and off-diagonal elements $Q_{\alpha\beta}=-e^{-i\gamma_{\alpha\beta}}$. In these expressions z is the node coordination number (4 in the case of the SG) and $\eta=L/L_\phi$, where L is the length of an elementary wire in the gasket. $\gamma_{\alpha\beta}=(2\pi/\phi_0)\int_\alpha^\beta \mathbf{A}\cdot d\mathbf{l}$ is the line integral of the magnetic vector potential along these wires, connecting nodes α and β. The normalized contribution to the MR due to WL effects is

$$\left.\frac{\delta R}{R}\right|_{\mathrm{WL}}=\frac{\kappa}{2}\left[\left(1-\frac{2}{Z}\right)\frac{\eta\cosh(\eta)-\sinh(\eta)}{\eta\sinh(\eta)}+\frac{2}{N}\sinh(\eta)\sum_{\alpha=1}^{N}\lambda_\alpha^{-1}\right]$$
$$\equiv y(H;L_\phi)\ . \tag{1}$$

The prefactor $\kappa/2$ is defined as $[R(L)/(\pi\hbar/e^2)](1/\eta)$ where $R(L)$ is the resistance of a wire of length L (as in the definition of η). In the case of the SG there is currently no simple, closed expression for (1). A numerical method using renormalization techniques has been developed, however, allowing rapid calculation of $\Delta R/R$ to a high precision. Details may be found in Doucot and Rammal.[13]

In order to describe our real gaskets we must explicitly account for the effects of spin-orbit scattering, as well as for the finite width of the wires. In wires with finite width diffusing electrons may follows paths within a single wire (as well as those around loops) which enclose nonzero magnetic flux. This necessitates a redefinition of the phase-breaking length (roughly speaking, the length of a typical closed diffusion path which encloses one quantum of flux)

$$L_\phi(H)=L_\phi\left[1+\frac{\pi^2}{3}\left(\frac{H}{\phi_0/L_\phi W}\right)^2\right]^{-1/2}\ . \tag{2}$$

Spin-orbit (SO) scattering is only slightly more difficult to incorporate into the WL calculation. Formally, one performs the sum in (1) twice, using first the singlet diffusion length, $L_1=L_\phi$, and then the triplet diffusion length, $L_2=[L_\phi^{-2}+(\frac{4}{3})L_{\mathrm{SO}}^{-2}]^{-1/2}$. We assume that there is an insignificant amount of spin-flip scattering, a good approximation in Al films.[14]

The full WL correction to the resistance is

$$\left.\frac{\delta R}{R}\right|_{\mathrm{WL}}=-\tfrac{1}{2}y(H;L_1)+\tfrac{3}{2}y(H;L_2)\ . \tag{3}$$

Finally, for comparison to experiment we define the magnetoresistance

$$\frac{\Delta R(H)}{R}\equiv\frac{\delta R(H)}{R}-\frac{\delta R(0)}{R}\ . \tag{4}$$

The MT contribution to the magnetoresistance is closely related to the WL expression, (1). At most values of field and temperature ($H<\tilde{H}_c$ and $\ln t>\delta$) it is possible to use Larkin's approximation[15]

$$\left.\frac{\Delta R}{R}\right|_{\mathrm{MT}}=-\beta(T/T_{c0})\left.\frac{\Delta R}{R}(H;\tau_{\mathrm{SO}}^{-1}=0)\right|_{\mathrm{WL}}\ . \tag{5}$$

The limits defined in terms of the critical field, $\tilde{H}_c=(2ck_BT/\pi eD)\ln t$, and the pair-breaking parameter, $\delta=\pi\hbar/8k_BT\tau_i$, are discussed in Ref. 11. Larkin's β is a smooth function of the ratio $T/T_{c0}=t$ which diverges as $1/\ln t$ as t approaches unity from above. Since β is field independent the only structure in the MR comes from the renormalization group calculation of the WL magnetoresistance. This is to be compared with the highly structured (fractal) phase boundary, $T_c(H)$, of the SG.

The third contribution to the MR at temperatures above T_c comes from the direct, or Aslamazov-Larkin (AL) (Ref. 16) superconducting fluctuations. Currently there is no calculation of this contribution to the magnetoresistance on the Sierpinski gasket. This poses no serious problem to our analysis, though, since the AL contribution drops off very rapidly above T_c due to the fact that its characteristic length scale, $\xi(t)\sim 1/(\ln t)^{1/2}$, becomes much smaller than L_ϕ just above the transition. For comparison, in two dimensions the AL term becomes insignificant above temperatures such that

$$\ln t>\epsilon_c=[1.52\times10^{-5}R_\square(\Omega)]^{1/2}\simeq10^{-3}\ .$$

This is consistent with our data in which the AL contribution, characterized by its strong temperature dependence and fine structure, is observed only at temperatures within a few millikelvin of T_{c0}.

IV. RESULTS

Before making a detailed analysis of the MR well above T_{c0} we present a brief, qualitative discussion of the region within a few mK of the transition temperature. Here the primary contribution to the magnetoresistance is from direct, or Aslamazov-Larkin, fluctuations. Although the functional form of the AL magnetoresistance on a fractal lattice is not known, it *is* possible to make qualitative predictions. One signature of Aslamazov-Larkin MR is found in the detailed structure of the magnetoresistance curve. In Fig. 1 we have plotted the MR at a temperature $T=1.195$ K such that $(T-T_{c0})/T_{c0}<\epsilon_c$. Again ϵ_c is the size of the AL regime (approximated for 2D above). The fine structure in the resistance as a function of magnetic field is reminiscent of the structure in the superconducting phase boundary, $T_c(H)$, which has been included in the same figure. This is expected since the direct term in the superconducting fluctuations diverges with the factor $1/(\ln T/T_c)$, where $T_c = T_c(H)$.[17] In other words, the AL magnetoresistance depends strongly on the *field-dependent* transition temperature. At higher temperatures, $T \gtrsim 1.200$ K, this structure in $\Delta R/R$ vanishes.

In Fig. 2 we show an example of the MR data at temperature $T = 1.306$ K, or $t = 1.096$. The smooth curve passing through the data points represents a sum of theoretical terms due to weak localization and Maki-Thompson effects. In general, such a fit requires a knowledge of three parameters: $\beta(t)$, $L_\phi(T)$, and L_{SO}. The prefactor, β, has been tabulated by Larkin and is completely defined by the normalized temperature, t. The spin-orbit diffusion length, $L_{SO} = (D\tau_{SO})^{1/2}$, is slightly more difficult to determine. In general L_{SO} may be varied simultaneously with L_ϕ in fitting the experimental MR curves. Because our samples are of very small resistivity, however, we were only able to resolve the low-field MR at temperatures below about 4 K. At these temperatures it is always true that $L_\phi \gg L_{SO}$ and the shape of the magnetoresistance curves are relatively insensitive to the value of the spin-orbit length. It is, however, possible to calculate a value for the spin-orbit length from existing measurements of L_{SO} in aluminum films. From Santhanam[18] we have the empirical expression

$$\tau/\tau_{SO} \simeq 2 \times 10^{-4} .$$

Combining this with the measured diffusivity $D = 270$ $cm^2/s = (\frac{1}{3})v_F^2\tau$ we calculate that $\tau_{SO}^{-1} \simeq 4 \times 10^9$ s^{-1} or $L_{SO} \simeq 2.6$ μm. Thus, we are able to compare our MR data to theory by varying a single parameter, $L_\phi = (D\tau_\phi)^{1/2}$.

At this point we shall make some qualitative comments about the MR curves of Figs. 2 and 3. It is interesting to compare these with measurements of the phase boundary $T_c(H)$ in the same gaskets or in other wire networks. For both the SG and the square network[19] the superconducting phase boundary is a fractal. It follows that their $T_c(H)$ curves will have discontinuities over all scales in the field, H. In addition, the phase boundary of the SG is expected to reflect the discrete dilational invariance of the network. If the MR was due simply to a rigid translation of $R(T)$ with varying T_c we would expect all of the structure of $T_c(H)$ to be present in the magnetoresistance. The striking result lies in the *lack* of structure in the MR curves. The explanation comes from the details of the calculation of the localization component of the magnetoresistance. On a wire network this calculation is closely related to that for $T_c(H)$. The difference lies in the fact that the MR depends upon a *regularizing sum* of the field-dependent eigenvalues of Q (see above) while $T_c(H)$ depends only upon the band-edge energies. In the case of the regular array this regularizing sum completely eliminates all structure in H with period less than one flux quantum per loop. The case of the Sierpinski gasket is a bit different. Here the MR is determined via an exact renormalization-group (RG) calculation and because of the discrete dilational symmetry one expects a small oscillation in $\Delta R/R$ with period $\Delta \ln H = \ln 4$.[5] This oscillato-

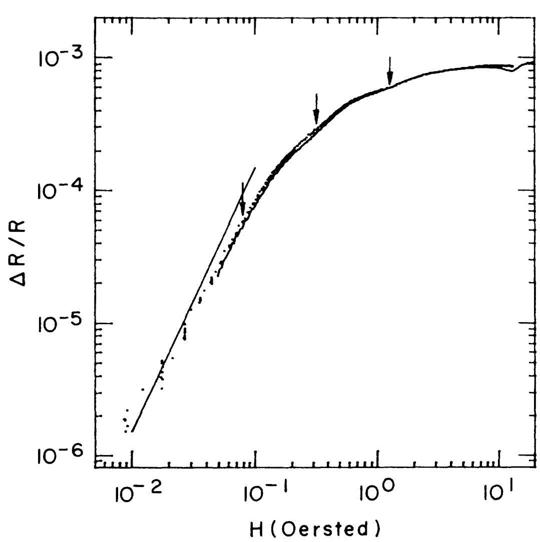

FIG. 2. A plot of the MR at $T = 1.306$ K. The points represent experimental data. The smooth curve passing through the data at higher yields represents a theoretical calculation using the fitting parameter $\eta = L/L_\phi = 0.142$. The straight line indicates the quadratic behavior at low fields. The arrows indicate the positions of the small oscillations due to the discrete symmetry of the gasket.

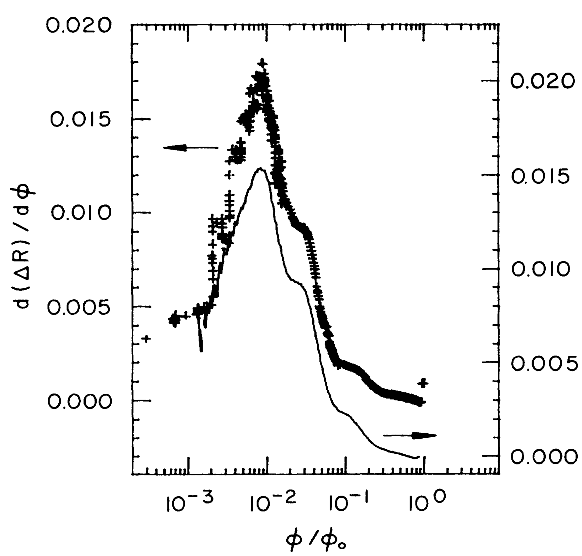

FIG. 3. The derivative of the MR with magnetic field. The field is given in units of flux quanta per elementary triangle. The data of Fig. 2 is used here and represented by the crosses. The theory (smooth curve) has been offset for clarity. The bumps in the curves at fields such that $\Delta \ln H = \ln 4$ are corrections to scaling due to the discrete dilational invariance of the gasket.

ry structure should disappear in random fractal structures such as percolation clusters. Unlike the phase boundary of the gasket the magnetoresistance is expected to be smooth (differentiable) at all fields. The ln4 period oscillations are difficult to see in the raw MR data, Fig. 2, but they are quite evident in the differentiated data in Fig. 3. The oscillations are equally spaced on the logarithmic field axis, in excellent agreement with theory. The limiting quadratic behavior, $\Delta R(H\to 0)\sim H^2$, is also observed (see Fig. 2).

From fits of theory to the MR data we are able to determine the electron phase-breaking rate as a function of temperature. A large body of experimental data exists which identifies contributions to τ_ϕ^{-1} from *inelastic* scattering due to (bulk) electron-phonon (*e*-ph) interactions as well as two- and three-dimensional electron-electron (*e*-*e*) interactions. Although some experiments have been carried out on samples with narrow wire geometries in which one-dimensional WL (and MT) effects are present,[11] only very recently have one-dimensional *e*-*e* processes been observed.[12] This is due to the fact that the characteristic length for *e*-*e* scattering, L_T, is generally much shorter than L_ϕ, W.

In two-dimensional aluminum films and wires it has been demonstrated that the phase-breaking rate has three components: electron-phonon, electron-electron, and electron–superconducting-fluctuation (*e*-f1). The electron-phonon rate in aluminum, both from empirical results[11,12] and from theory,[20] is

$$\tau_{e\text{-ph}}=(1.6\times 10^7)T^3 \ . \tag{6}$$

In two dimensions $(W>L_T=\sqrt{\hbar D/k_BT}>d)$ the electron-electron and electron-fluctuation contributions may be combined and written[21]

$$\tau_{e\text{-}e}^{-1}+\tau_{e\text{-fl}}^{-1}=\frac{k_BT}{\hbar}\frac{R_\square}{(2\pi\hbar/e^2)}\left[\ln\left[\frac{\pi\hbar/e^2}{R_\square}+\frac{2\ln 2}{\ln t+B}\right]\right] . \tag{7}$$

The presence of these contributions to $\tau_\phi^{-1}(T)$ in aluminum films and wide wires is well documented.[11]

In our samples the *e*-*e* and *e*-f1 contributions to the phase-breaking rate are no longer expected to be two-dimensional since $W=0.3\ \mu\text{m}\lesssim L_T\simeq 0.5\ (\mu\text{m})/\sqrt{T(K)}$. The coherent backscattering which results in the MR corrections is characterized by the length $L_\phi\simeq 10\ \mu\text{m}$ (see below) and it samples the fractal nature of the gasket. Electron-electron processes, however, are only sensitive to the 1D nature of the wires since both the width and thickness of the wire are smaller than the characteristic length, L_T.

Electron-electron inelastic processes $(\Delta E\simeq k_BT)$ surely contribute to phase breaking in these narrow wires. Al'tshuler *et al.*,[9] however, have recently pointed out that in the one-dimensional limit multiple scattering events with small, *quasielastic* energy changes $(\Delta E \ll k_BT)$ lead to a large dephasing rate. This rate, which is larger than the inelastic *e*-*e* rate in 1D (this is not true generally), is equivalent to the Nyquist rate, τ_N^{-1}. Thus, for narrow, clean wires we expect a total phase-breaking rate of the form

$$\tau_\phi^{-1}=\tau_{e\text{-ph}}^{-1}+\tau_N^{-1}+\tau_{e\text{-f1}}^{-1}(1\text{D}) \ , \tag{8}$$

where the 1D Nyquist rate is given by

$$\tau_N^{-1}=\left[\frac{R_\square}{\sqrt{2}(\hbar/e^2)}\frac{k_B}{\hbar}\frac{\sqrt{D}}{W}\right]^{2/3}T^{2/3} \ . \tag{9}$$

No calculation exists for the one-dimensional *e*-f1 term, although one might expect it to be comparable in magnitude to the 1D inelastic rate due to electron-electron interactions and to diverge as T approaches T_{c0} from above. We comment below on the omission of this term from our analysis.

Combining the *e*-ph (bulk) and *e*-*e* (quasielastic, one-dimensional) contributions to the phase-breaking rate gives us

$$\tau_\phi^{-1}=(1.6\times 10^7)T^3+(7.8\times 10^7)T^{2/3} \tag{10}$$

for the sample parameters discussed earlier. In Fig. 4 we have plotted our experimentally determined values of $\tau_\phi^{-1}(T)$ for two samples at temperatures from $T_{c0}=1.192$ to 4.0 K. At higher temperatures the MR is too small to reliably infer rates. The solid line in the figure represents the theoretical calculation (7), with *no* free parameters. A two-dimensional *e*-f1 term has been added to suggest the divergent behavior near T_{c0}. One must keep in mind that we probably underestimate the phase-breaking rate due to one-dimensional *e*-f1 effects in this way. Finally, we point out that the good agreement between theory and experiment at higher temperatures depends critically on including spin-orbit effects in the analysis.

Evidence that we do in fact observe 1D (quasielastic) *e*-*e* scattering comes from comparison of the data in Fig. 4 with the dashed line. It has been calculated by replacing the 1D *e*-*e* contribution with that due to 2D electron-

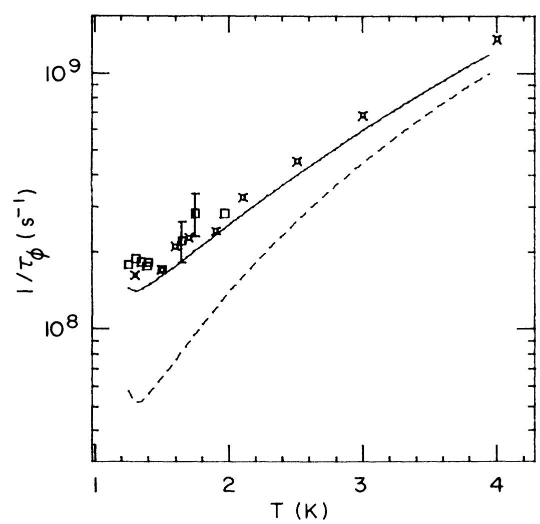

FIG. 4. A plot of the temperature dependence of the phase-breaking rate for two samples. The solid line is the theoretical result of Eq. (10), using no fitting parameters. In calculating the dashed curve we have replaced the predicted 1D electron-electron effects with that expected for 2D.

electron effects, (7). At the lowest temperatures this is roughly a factor of 4 below the experimental results. Although we lack the temperature range to unambiguously identify the $T^{2/3}$ temperature dependence (it is bounded by T_c below and T^3 behavior above), the close absolute magnitude match of τ_ϕ^{-1} to theory offers a compelling consistency argument.

V. SUMMARY

We have made magnetoresistance measurements on aluminum Sierpinski gasket wire networks fabricated with state-of-the-art electron-beam lithography. Because of the submicrometer gasket geometry and the relatively long diffusion lengths in clean Al the magnetoresistance has been found to reflect the large-scale fractal nature of the gaskets. Specifically, we were able to fit recent calculations of the localization-related magnetoresistance to our data. A single fitting parameter, $L_\phi(T)$, was used. All detailed predictions of the theory were observed, including the ln4 period oscillations in the magnetic field due to discrete dilational invariance of the SG.[5,13] The values of the phase-breaking length inferred from these fits are in excellent quantitative agreement with model calculations which include recently suggested one-dimensional, quasielastic electron-electron scattering. These are among the first observations of this novel mechanism.

The theoretical and experimental problems discussed above should be helpful in advancing towards the larger goal of understanding the transport properties of naturally occurring fractals (alloys, composites, colloids, quench-condensed films, etc.). Towards these ends work needs to be done on understanding electron-electron effects in systems of nonintegral dimensions (since the length scales of naturally occurring fractals may be quite small) and the effects of randomness on the physics of fractals.

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