Effect of spin scattering and magnetic order on the electronic heat capacity of magnetic superconductors

Simanchal Panigrahi
Department of Physics, SMIT, Brundaban Bihar, Ankuspur, Berhampur 761100, Orissa, India

Narayan C. Das
Department of Physics, Berhampur University, Berhampur 760 007, Orissa, India

(Received 30 July 1985; revised manuscript received 22 September 1986)

An expression for the difference in thermodynamic potentials $\Omega_s - \Omega_n$ of the superconducting and normal states is derived by use of thermodynamic Green's functions. The change in the heat capacity $C_s - C_n$ in the normal-to-superconducting phase transition in the presence of a periodic magnetic order and spin fluctuations is obtained. The jump δC in the specific heat at superconducting transition temperature is calculated. The variation of electronic heat capacity C with temperature for $T \gtrless T_c$ is studied. It is noted that the presence of magnetic order reduces the jump in specific heat at T_c.

I. INTRODUCTION

The electromagnetic and thermodynamic properties of rare-earth (R) ternary superconductors have been studied extensively[1,2] in recent years. Ever since the discovery of coexistence of superconductivity and magnetism in ternary rare-earth compounds of the types $R\mathrm{Mo}_6X_8$ (X = S,Se) and $R\mathrm{Rh}_4\mathrm{B}_4$, a large amount of experimental and theoretical work[1-13] has been carried out. The study of upper critical field[3-5] and the heat capacity[6,7] of ternary superconductors has been a problem of considerable interest. Theoretical works[8-13] in these problems have tried to provide understanding of the mechanism involved in the coexistence phase in order to explain some of their interesting properties. The rare-earth ternary compounds exhibit two phase transitions at low temperature. The first phase transition occurs at an upper critical temperature T_c at which the system undergoes transformation from the normal paramagnetic state to a superconducting paramagnetic state. The second phase transition takes place at a lower critical temperature T_M at which the system goes from the superconducting paramagnetic state to either a superconducting antiferromagnetic (AF) state or a normal ferromagnetic state. In the case of ferromagnetic superconductors $\mathrm{ErRh}_4\mathrm{B}_4$ and $\mathrm{HoMo}_6\mathrm{S}_8$, superconductivity and a periodic magnetic order are seen to coexist only in a narrow range of temperature above T_M while in AF superconductors $R\mathrm{Mo}_6\mathrm{S}_8$ (R = Gd,Dy,Tb) and $R\mathrm{Rh}_4\mathrm{B}_4$ (R = Nd,Sm,Tb) the superconducting order coexists with antiferromagnetic order below T_N.

The variation of the heat capacity[1,2] of these compounds at low temperature is marked with characteristic anomalous features. The chief sources of such anomaly are (1) the crystalline electric field effect (CEF) associated with the Schottky anomaly and (2) magnetic order which gives rise to a jump in specific heat at lower critical temperature. In ferromagnetic superconductors $\mathrm{ErRh}_4\mathrm{B}_4$ and $\mathrm{HoMo}_6\mathrm{S}_8$ the jump in heat capacity is observed at the upper critical temperature T_c superimposed on the Schottky anomaly. Near the second phase-transition point a spike-shaped feature is observed superimposed on another anomaly associated with the long-range magnetic order at T_M. The low-temperature specific-heat measurements performed on AF superconductors reveal a pronounced λ-type anomaly at the magnetic phase transition near the lower critical temperature besides the jump at T_c. Considering the heat capacity of a system to be comprised of lattice and electronic contributions a reasonable estimate of the effect of magnetic ions on these contributions is necessary in order to understand the observed anomalies. Recently there have been attempts on the experimental side to separate the CEF effects from the magnetic phenomena. On the other hand, recent theoretical works so far have dealt with the effects of paramagnetic impurity on the specific-heat jump at T_c. Zarate and Carbotte[12] have used Eliashberg equations to study the specific-heat jump in the presence of paramagnetic impurities. They conclude that the jump at T_c decreases in the presence of scattering from impurity spins. Sihota and Nagi[13] have studied the effects of uniform magnetic field on specific-heat jump of a superconducting alloy containing paramagnetic impurities described by the Shiba-Rusinov model. However, there is no theoretical result available to estimate the effects of magnetic order and spin scattering on heat capacity of magnetic superconductors at low temperature.

In this paper we present a theoretical study of the temperature variation of heat capacity of magnetic superconductors in the presence of magnetic order and spin scattering. Our formulation is applicable to systems with $T_c \gtrless T_M$.

We consider a system of localized $4f$ electrons of the rare-earth ions and the superconducting $4d$ electrons of transition metal atoms interacting with each other via an exchange interaction. The Hamiltonian for the system includes the reduced BCS interaction and the exchange interaction between superconducting ($4d$) electrons and the ($4f$) electrons responsible for magnetism. The system is described by Green's-function matrices. We formulate the self-consistent gap equations in terms of normal-state Green's-functions following our earlier work.[9] We obtain the difference in thermodynamic potentials $\Omega_s - \Omega_n$ of the superconducting and normal states. We derive an expression for the change in heat capacity δC in going from normal to superconducting state. The effects of magnetic order and spin fluctuations arising out of the exchange interaction on the heat capacity of magnetic superconductors are discussed in detail. The temperature variation of the electronic specific heat and the jump in the electronic specific heat at T_c are calculated for a few systems using our formulation.

The structure of the paper is as follows. In Sec. II we give a general formulation of the problem in terms of normal state Green's functions. In Sec. III we obtain an expression for the change of heat capacity. In Sec. IV we report numerical calculations an discuss our results.

II. GENERAL FORMULATION

We consider a magnetic superconductor in the presence of a magnetic induction field described by the vector potential $\mathbf{A}(\mathbf{r})$. The Hamiltonian is given by

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{U} , \tag{1}$$

where

$$\mathscr{H}_0 = \sum_\sigma \int d^3r\, \Psi_\sigma^\dagger(\mathbf{r}) \times \left[\frac{1}{2m} \left[-i\hbar\nabla + \frac{e\mathbf{A}}{c} \right]^2 - \mu_\sigma \right] \Psi_\sigma(\mathbf{r}) + \int d^3r [\Delta^*(\mathbf{r})\Psi_\uparrow(\mathbf{r})\Psi_\downarrow(\mathbf{r}) + \Delta(\mathbf{r})\Psi_\downarrow^\dagger(\mathbf{r})\Psi_\uparrow^\dagger(\mathbf{r})] \tag{2}$$

and

$$\mathscr{U} = \sum_{\mu,\nu} \int d^3r\, \Psi_\mu^\dagger(\mathbf{r}) V_{\mu\nu}(\mathbf{r}) \Psi_\nu(\mathbf{r}) . \tag{3}$$

The exchange interaction $V(\mathbf{r})$ between the conduction electron of spin $\boldsymbol{\sigma}$ and local spin $\mathbf{S}_j$ at lattice site $\mathbf{R}_j$ is assumed to have the form

$$V(\mathbf{r}) = -\frac{1}{2N} \sum_j J\mathbf{S}_j \cdot \boldsymbol{\sigma}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{R}_j) . \tag{4}$$

In the above Ψ_σ is the electron field with spin σ, Δ is the gap function, $\mu_\sigma = \mu - \sigma \mid \mu_B \mid H$, $\sigma = \pm 1$, and H is the applied external magnetic field. μ is the chemical potential and μ_B is the Bohr magneton. The differential equations[9,14] for the matrix Green's functions which describe the system are

$$\left[i\hbar\omega_n + \frac{1}{2m} \left[\hbar\nabla + \frac{ie\mathbf{A}}{c} \right]^2 + \mu_\sigma \right] G(\mathbf{r},\mathbf{r}',\omega_n) - V(\mathbf{r}) G(\mathbf{r},\mathbf{r}',\omega_n) + \Delta(\mathbf{r}) \mathscr{F}^\dagger(\mathbf{r},\mathbf{r}',\omega_n) = \hbar\delta(\mathbf{r}-\mathbf{r}') , \tag{5a}$$

$$\left[i\hbar\omega_n - \frac{1}{2m} \left[\hbar\nabla - \frac{ie\mathbf{A}}{c} \right]^2 - \mu_\sigma \right] \mathscr{F}^\dagger(\mathbf{r},\mathbf{r}',\omega_n) + V^t(\mathbf{r}) \mathscr{F}^\dagger(\mathbf{r},\mathbf{r}',\omega_n) + \Delta^*(r) G(\mathbf{r},\mathbf{r}',\omega_n) = 0 , \tag{5b}$$

where ω_n is the Matsubara frequency $\omega_n = (2n+1)\pi/\beta\hbar$. From Eq. (5a) we obtain the equation for the normal-state ($\Delta = 0$) Green's function

$$\left[i\hbar\omega_n + \frac{1}{2m} \left[\hbar\nabla + \frac{ie\mathbf{A}}{c} \right]^2 + \mu_\sigma \right] \mathscr{G}(\mathbf{r},\mathbf{r}',\omega_n) - V(\mathbf{r}) \mathscr{G}(\mathbf{r},\mathbf{r}',\omega_n) = \hbar\delta(\mathbf{r}-\mathbf{r}') . \tag{6}$$

From Eqs. (5a), (5b), and (6) the solutions for the Green's functions G and $\mathscr{F}^\dagger$ are obtained[9,14] in terms of the normal-state Green's function $\mathscr{G}$.

We use the relation

$$\Delta_{\alpha\beta}^*(\mathbf{r}) = g(\beta\hbar)^{-1} \lim_{\eta\to 0} \sum_n e^{-i\omega_n\eta} \mathscr{F}_{\alpha\beta}^\dagger(\mathbf{r},\mathbf{r},\omega_n) , \tag{7}$$

and substitute the solution for $\mathscr{F}_{\alpha\beta}^\dagger(\mathbf{r},\mathbf{r},\omega_n)$, obtained in terms of normal-state Green's function, in Eq. (7). We take the $\downarrow\uparrow$ matrix element of the resulting expression for $\Delta_{\alpha\beta}^*(\mathbf{r})$ and use eikonal approximation for $\mathscr{G}$ to obtain the gap equation[9,14]

$$g^{-1}\Delta^*(\mathbf{r}) = a_1\Delta^*(\mathbf{r}) + \frac{a_2}{6} \left[\nabla - \frac{2ie\mathbf{A}(\mathbf{r})}{\hbar c} \right]^2 \Delta^*(\mathbf{r}) + B(T)\Delta^*(\mathbf{r}) \mid \Delta(\mathbf{r}) \mid^2 , \tag{8}$$

where g is electron-phonon coupling constant,

$$a_1 = \int d^3r\, Q_{\downarrow\uparrow}^0(\mathbf{r}) , \tag{9}$$

$$a_2 = \int d^3r\, r^2 Q_{\downarrow\uparrow}^0(\mathbf{r}) , \tag{10}$$

$$B(T) = \int d^3r_1 d^3r_2 d^3r_3 R_{\downarrow\uparrow}^0(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) , \tag{11}$$

$$Q_{\downarrow\uparrow}^0(\mathbf{r}) = 2(\beta\hbar^2)^{-1} \left[\frac{\pi\hbar N(\mu)}{k_F r} \right]^2 \times \sum_{n=0}^{\infty} \left[1 + \left[\frac{a\mu}{b} + \frac{a^2}{2b} \right] \frac{1}{i\hbar\tilde{\omega}_n - b} - \left[\frac{a\mu}{b} + \frac{a^2}{2b} \right] \frac{1}{i\hbar\tilde{\omega}_n + b} \right] e^{-2\tilde{\omega}_n r / v_F} , \tag{12}$$

$$a = A_Q/2\epsilon_Q \ , \tag{13}$$

$$A_Q = \frac{J^2 M_Q^2}{16} + \frac{J^2 M_Q}{8}(\langle \tilde{S}^z_{-Q}\rangle + \langle \tilde{S}^z_Q\rangle) \ , \tag{14}$$

$b = \mu + a$, $\tilde{S}^z_Q = (S^z_Q - \langle S^z\rangle)$ is the spin fluctuation with wave vector $\mathbf{Q}$, $\tilde{\omega}_n = \omega_n + \delta$, δ is defined by

$$\mp i\hbar\delta = \frac{J^2}{4n\beta g_J^2 \mu_B^2}\sum_{\mathbf{q}} \frac{\chi^{-+}(\mathbf{q}) + \chi^{zz}(\mathbf{q})}{\pm i\hbar\omega_n - \mathscr{E}_{\mathbf{k}-\mathbf{q}}} \ , \tag{15}$$

n is the number of magnetic ions per unit volume, g_J is the Landé g factor, $\chi^{-+}(\mathbf{q}), \chi^{zz}(\mathbf{q})$ are staggered susceptibilities,[9]

$$\mathscr{E}_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m} - \mu, \ \epsilon_Q = \hbar^2 Q^2/2m \ ,$$

$$\begin{aligned} R^0_{\downarrow\uparrow}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) &= -(\beta\hbar^4)^{1-}\sum_n \mathscr{G}^0_{\mu\downarrow}(\mathbf{r}_1,\mathbf{r},-\omega_n)\varepsilon_{\mu\nu}\mathscr{G}^0_{\nu\delta}(\mathbf{r}_1,\mathbf{r}_2,\omega_n) \\ &\quad \times \varepsilon^\dagger_{\delta\sigma}\mathscr{G}^0_{\rho\sigma}(\mathbf{r}_3,\mathbf{r}_2,-\omega_n) \\ &\quad \times \varepsilon_{\rho\lambda}\mathscr{G}^0_{\lambda\uparrow}(\mathbf{r}_3,\mathbf{r},\omega_n) \ , \end{aligned} \tag{16}$$

$$\varepsilon_{\alpha\beta} = -i\sigma_{\alpha\beta} \ .$$

$Q^0_{\downarrow\uparrow}$ is obtained[9] by replacing $\mathscr{G}$ and $\mathscr{G}^0$, the normal-state Green's function, in zero magnetic induction field. The evaluation of $\mathscr{G}^0$ has been done in the presence of periodic magnetic order and spin exchange scattering. The expression for self-energy has been obtained in Ref. 9.

III. ELECTRONIC HEAT CAPACITY OF MAGNETIC SUPERCONDUCTORS

In an external magnetic field $\mathbf{H}$ along the $\hat{\mathbf{z}}$ direction and a periodic magnetic order $M_Q\cos(\mathbf{Q}\cdot\mathbf{r})$ in the system, the effective vector potential which corresponds to the magnetization density is given by[9]

$$\mathbf{A}(\mathbf{r}) = \hat{\mathbf{y}}Bx + \hat{\mathbf{n}}\frac{4\pi M_Q}{Q}\sin(\mathbf{Q}\cdot\mathbf{r}) \ , \tag{17}$$

where $\hat{\mathbf{n}}$ is the unit vector specifying the direction of the periodic part of vector potential and $\mathbf{B} = (1+4\pi\chi_0)\mathbf{H}$, χ_0 is the average uniform susceptibility. Following our earlier work[9] we take the gap function to be

$$\Delta^*(\mathbf{r}) = \Delta^*\exp\left[-\frac{eB}{\hbar c}x^2\right] \ , \tag{18}$$

substitute the expressions for $\Delta^*(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ in Eq. (8), and average over the coherence volume. In the limit of zero external field (i.e., $\mathbf{B}=\mathbf{0}$), we obtain

$$g^{-1} = a_1 + B(T)\Delta^2 - \frac{4\pi^2 a_2}{3\rho_0^2\xi_0^2}\left[\frac{M_Q}{H^0_{c2}(0)}\right]^2[1-j_0(2\rho)] \ , \tag{19}$$

taking Δ to be real. $j_0(2\rho)$ is the spherical Bessel function. In the above ξ is the superconducting coherence length and $\rho = Q\xi$. ρ_0 is the value of ρ for $\xi = \xi_0$, the coherence length at $T=0$. Considering the gap function $\Delta(\mathbf{r})$ to be uniform[15] in space in the absence of an external magnetic field (i.e., $\mathbf{B}=\mathbf{0}$) the difference in the thermodynamic potentials[16] of the superconducting and normal states is given by

$$\Omega_s - \Omega_n = V\int_0^\Delta d\Delta'(\Delta')^2\frac{d}{d\Delta'}\left[\frac{1}{g}\right] \ . \tag{20}$$

From Eqs. (19) and (20) we get

$$\Omega_s - \Omega_n = \tfrac{1}{2}B(T)V\Delta^4 \ . \tag{21}$$

To obtain the expression for Δ^2 we use the values of a_1 and a_2 which have been evaluated from Eqs. (9) and (10). We have

$$\begin{aligned} a_1 = \frac{N^2(\mu)}{N(0)}\Bigg\{ &\ln\left[2\gamma_0\frac{\beta\hbar\omega_D}{\pi}\right] \\ &+ \frac{\beta\hbar\delta}{2\pi}\left[\psi'\left[\frac{\beta\hbar\omega_D}{2\pi}+1\right] - \psi'(\tfrac{1}{2})\right] \\ &- \frac{a}{b}\left[a+\frac{\mu}{b}\right]\mathrm{Re}[\psi(z)-\psi(x)]\Bigg\} \ , \end{aligned} \tag{22}$$

where γ_0 is Euler's constant and ω_D is the Debye cutoff frequency, $x = \frac{1}{2} + \beta\hbar\delta/2\pi$, $z = x + i\beta b/2\pi$, and

$$a_2 = N(\mu)\alpha(T) \ , \tag{23}$$

where

$$\alpha(T) = \frac{N(\mu)}{8\pi^2 N(0)}\frac{\xi_0^2}{p_0^2}\zeta(3,\tfrac{1}{2}+\beta\hbar\delta/2\pi)\frac{T_c^2}{T^2}R(T) \ , \tag{24}$$

$$R(T) = 1 - \frac{2a}{b} + \frac{8\pi^2 a}{b^3\beta^2}\frac{\mathrm{Re}[\psi(z)-\psi(x)]}{\zeta(3,\frac{1}{2}+\beta\hbar\delta/2\pi)} \ . \tag{25}$$

Here $N(\mu)$ and $N(0)$ are the conduction electron density of states at the Fermi level for the system in the presence of local spins and absence of local spins, respectively. δ has been defined in Eq. (15). The value of $p_0 = 0.18$ and ζ is the Riemann zeta function. We substitute the values of a_1 and a_2 in Eq. (19) and simultaneously use the value of the resulting expression at $T = T_c$ to eliminate ω_D. We obtain

$$\begin{aligned} \Delta^2 = \frac{N^2(\mu)}{B(T)N(0)}&\left[\ln\frac{T}{T_c} + \frac{\pi\hbar\delta}{4}(\beta-\beta_c) + (\phi_1 - \phi_1^c)\right] \\ &+ \frac{4\pi^2 N(\mu)}{3\rho_0^2\xi_0^2 B(T)}(\phi_2 - \phi_2^c) \ , \end{aligned} \tag{26a}$$

where

$$\phi_1 = \frac{a}{b}\left[1+\frac{\mu}{b}\right]\mathrm{Re}[\psi(z)-\psi(x)] \ , \tag{26b}$$

$$\phi_2 = \alpha(T)\left[\frac{M_Q}{H^0_{c2}(0)}\right]^2[1-j_0(2\rho)] \ . \tag{26c}$$

ϕ_1^c, ϕ_2^c refer to the values at $T=T_c$ where $\beta=\beta_c=1/k_B T_c$, $a=a_c$, $b=b_c$, $z=z_c$, $x=x_c$, $\alpha(T)=\alpha(T_c)$, $M_Q=M_Q^c$, $\rho=\rho_c$, $H_{c2}^0(0)$ is the upper critical field of spin-field BCS system at $T=0$.

The change in electronic specific heat at a temperature ($T<T_c$) in going from normal to superconducting state is given by

$$\delta C=C_s-C_n=-T\frac{\partial^2}{\partial T^2}\left[\frac{\Omega_s-\Omega_n}{V}\right]. \quad (27)$$

Use of Eqs. (21) and (26) in Eq. (27) yields[17,18]

$$\delta C=-\frac{N^2(\mu)T}{2}\frac{\partial^2}{\partial T^2}\left[\frac{P^2(T)}{B(T)}\right], \quad (28)$$

where

$$P(T)=\frac{N(\mu)}{N(0)}\left[\ln\frac{T}{T_c}+\frac{\pi\hbar\delta}{4}(\beta-\beta_c)+(\phi_1-\phi_1^c)\right]+\frac{4\pi^2}{3\rho_0^2\xi_0^2}(\phi_2-\phi_2^c). \quad (29)$$

$B(T)$ has been evaluated from Eq. (11) using standard techniques of integration. At $T=T_c$, $P(T_c)=0$ and the specific heat jump at T_c is given by

$$\delta C=-\left[T\frac{N^2(\mu)}{B(T)}[P'(T)]^2\right]_{T_c}. \quad (30)$$

IV. RESULTS AND DISCUSSION

Our results for the change in electronic heat capacity δC in Eqs. (28) and (30) include the effects of periodic magnetic order and spin fluctuations. We note that in the absence of local spins in the system (i.e., $M_Q=0=\delta=a$) $N(\mu)=N(0)$,

$$B(T)=-\frac{7\zeta(3)}{8\pi^2}\frac{N(0)}{k_B^2T^2} \quad (31)$$

and $P'(T)=1/T$. In this case Eq. (30) reduces to

$$\delta C\,|_{T=T_c}=\frac{8\pi^2N(0)}{7\zeta(3)}k_B^2T_c$$

which is the well known BCS result for the specific heat jump at $T=T_c$.

We have used our expression in Eq. (28) to calculate the temperature variation of electronic heat capacity C of magnetic superconductors for $T<T_c$. The results for $C/\gamma T_c$ of $TbMo_6S_8$, $GdMo_6S_8$, and $HoMo_6S_8$ are plotted in Figs. 1—4. Figure 1 shows the temperature variation of heat capacity in the absence of magnetic order ($M_Q=0$) but in the presence of spin fluctuation effects ($\delta\neq0$). The jump in electronic heat capacity δC at T_c for all these compounds is found to be near about the BCS value 1.42. Figures 2—4 show the variation of heat capacity in the presence of both magnetic order ($M_Q\neq0$) and spin fluctuation effects ($\delta\neq0$). We find that the periodic magnetic order causes a relatively large reduction in the specific-heat jump in comparison with the reduction due to the

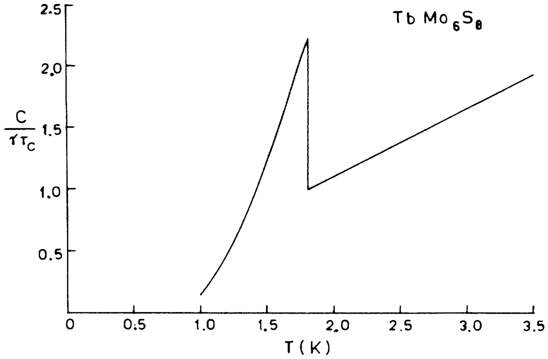

FIG. 2. $C/\gamma T_c$ for $TbMo_6S_8$. $M_Q=20$ G, $\mu=1$ eV, $a=0.001\,312\,32$, $b=1.001\,312\,32$.

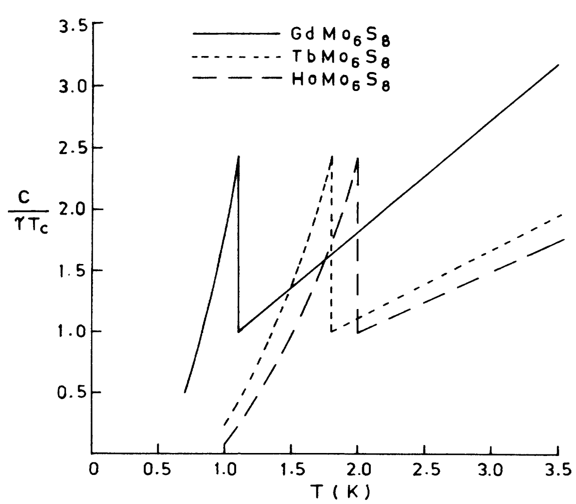

FIG. 1. $C/\gamma T_c$ for $TbMo_6S_8$, $GdMo_6S_8$, and $HoMo_6S_8$ in the absence of magnetic order ($M_Q=0$). $\beta_c\hbar\delta=10^{-3}$, $N(\mu)/N(0)=0.999$. The other parameters are shown in Table I.

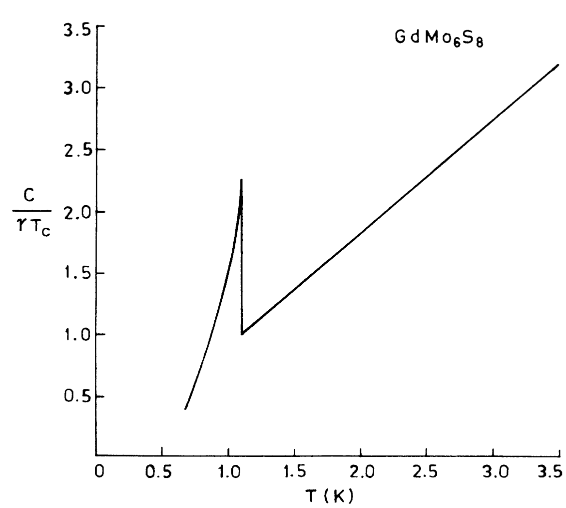

FIG. 3. $C/\gamma T_c$ for $GdMo_6S_8$. $M_Q=20$ G, $\mu=1$ eV, $a=0.001\,312\,32$, $b=1.001\,312\,32$.

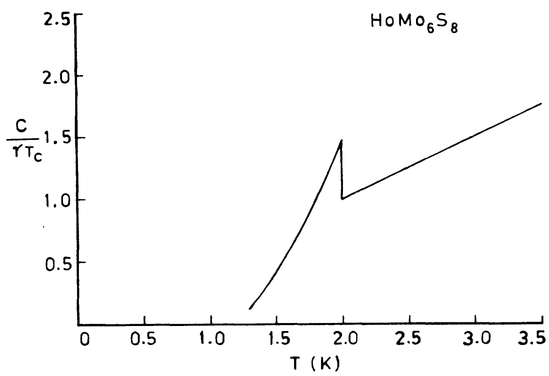

FIG. 4. $C/\gamma T_c$ for $HoMo_6S_8$. $M_Q=40$ G, $a=0.014\,581\,4$, $b=1.014\,581\,4$, $\mu=1$ eV.

spin fluctuation effects. This happens due to a comparatively large modification introduced in the gap function Δ by the periodic magnetic order. In our formulation the two prominent interactions between local spins and conduction electrons considered are the electromagnetic interaction and the exchange interaction. The electromagnetic interaction involves the momenta of superconducting electrons and the staggered magnetization and is produced[2] by the vector potential $\mathbf{A}(\mathbf{r})$. The exchange interaction between local spins and conduction electrons can be decomposed into (1) the molecular field part which depends on the staggered magnetization and (2) the spin fluctuation part. Owing to the involvement of periodic magnetic order in both the interactions it plays a dominant role in the modification of gap function. Earlier works[19,20] on magnetic superconductors have made similar conclusions about the reductions in the strength of pairing interactions due to these two effects. We observe deviation from the usual shape of specific-heat curves of BCS superconductors in Figs. 1–4. This departure is mainly attributable to the effect of interaction between local spins and superconducting electrons. We note that $P(T)$ and $B(T)$ occurring in Eq. (28) for specific heat contain contributions from periodic magnetic order and spin fluctuations. Since these contributions are temperature dependent they introduce deviations, in the specific-heat value of the spin-free BCS superconductor, which vary with temperature. It may be seen from Eqs. (28), (29), and (31) that in the absence of local spins (i.e., $M_Q=\delta=0$) only the first term in $P(T)$ is nonzero, which gives the specific heat of a spin-free BCS superconductor.

Although the electromagnetic interaction, introduced via vector potential $\mathbf{A}(\mathbf{r})$, has been treated in a mean-field approach the scattering effects are treated exactly in our formulation. The exchange scattering effects appear via a_1 and a_2 in the gap equation (8). The parameters a_1 and a_2 involve the normal-state Green's function $\mathscr{G}^0$ which can be exactly calculated[9] in the presence of exchange interaction and periodic magnetic order.

Further, we note that the earlier theoretical works[12,13] mentioned here have been confined to the systems containing magnetic impurities and have studied the scattering effect of impurity spins on the specific heat of BCS superconductors. Our present work differs from these works in the sense that we have studied the specific heat of a system containing a lattice of magnetic ions which undergo magnetic ordering at low temperature. In such a system the effects of periodic magnetic order on the superconducting state is important in addition to the scattering effects. We have included both these effects in our study of the specific heat. In case of the antiferromagnetic superconductors $TbMo_6S_8$ and $GdMo_6S_8$ the reduction in δC is found to be small compared to that of the ferromagnetic superconductor $HoMo_6S_8$.

TABLE I. Parameters used for theoretical calculation.

Compound	T_c (K)	$M_Q/H_{c2}^0(0)$	ρ	T_M (K)
$TbMo_6S_8$	1.8	0.2	12	1.05
$GdMo_6S_8$	1.1	0.2	15	0.85
$HoMo_6S_8$	2.0	0.16	10	0.6

In our calculation we have chosen the parameters[9] from the available data[2] for the systems. For all these compounds we have taken $\beta\hbar\delta=10^{-3}$. The parameters are shown in Table I.

Our expression for δC in Eqs. (28)–(30) is valid for the temperature regime $T\le T_c$. For plotting the graphs we have calculated $C/\gamma T_c$ for the two regimes of temperature, namely, $T>T_c$ and $T\le T_c$, separately. In region $T>T_c$, i.e., for the normal state, we assume the periodic magnetic order to be absent and the effect of spin fluctuations on heat capacity to be small. The value of $C/\gamma T_c$ in this region is taken to be T/T_c which varies linearly with temperature. In the temperature regime $T\le T_c$ we have included the effects of periodic magnetic order characterized by M_Q and the spin fluctuations characterized by δ. In our calculation we have used a constant value of M_Q although in the actual case M_Q varies[9] with temperature. The jump in heat capacity at T_c shows a reduction from the corresponding value for the spin-free BCS superconductor. The variation of $C/\gamma T_c$ below T_c is nonlinear and shows sharper drop for the systems containing magnetic ions.

[1]M. B. Maple, H. C. Hamaker, and L. D. Wolf, in *Superconductivity in Ternary Compounds*, edited by M. B. Maple and Ø. Fischer (Springer-Verlag, Berlin, 1982), Vol. II, pp. 99–141.
[2]M. Ishikawa, Ø. Fischer, and J. Muller, in *Superconductivity in Ternary Compounds*, edited by M. B. Maple and Ø. Fischer (Springer-Verlag, Berlin, 1982), Vol. II, pp. 143–249.
[3]G. W. Crabtree, F. Behroozi, S. A. Campbell, and D. G. Hinks, Phys. Rev. Lett. **49**, 1342 (1982).

[4]U. Rauchschwalbe, W. Lieke, C. D. Bredl, F. Steglich, J. Aarts, K. M. Martini, and A. C. Mota, Phys. Rev. Lett. **49**, 1448 (1982).

[5]H. B. Radousky, A. T. Aldred, G. S. Knapp, and J. S. Kouvel, Phys. Rev. B **27**, 4236 (1983).

[6]H. B. Radousky, B. D. Dunlap, G. S. Knapp, and D. G. Niarchos, Phys. Rev. B **27**, 5526 (1983).

[7]Israel Felner and Israel Nowik, Solid State Commun. **47**, 831 (1983).

[8]C. Ro and K. Levin, Phys. Rev. B **29**, 6155 (1983).

[9]N. C. Das, Phys. Lett. **102A**, 253 (1984); Phys. Rev. B **30**, 2603 (1984), and the references therein.

[10]Liam Coffey, K. A. Muttalib, and K. Levin, Phys. Rev. Lett. **52**, 783 (1984).

[11]K. Machida, J. Low. Temp. Phys. **53**, 405 (1983); M. Fujita, K. Machida, and T. Matsubara, *ibid.* **54**, 535 (1984).

[12]H. G. Zarate and J. P. Carbotte, J. Low. Temp. Phys. **55**, 67 (1984).

[13]B. Sihota and A. D. S. Nagi, J. Low. Temp. Phys. **51**, 347 (1983); Phys. Rev. B **30**, 2917 (1984); **30**, 2659 (1984).

[14]A. A. Abrikosov and L. P. Gorkov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys.—JETP **12**, 1243 (1961)].

[15]E. W. Fenton, Solid State Commun. **50**, 961 (1984).

[16]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), p. 450.

[17]S. Panigrahi and N. C. Das, Proc. Solid State Phys. Symp. (India) **27C**, 145 (1984).

[18]S. Panigrahi and N. C. Das, Ind. J. Phys. **59A**, 454 (1985).

[19]S. Maekawa and M. Tachiki, Phys. Rev. B **18**, 4688 (1978).

[20]J. Ashkenazi, C. G. Kuper, and A. Ron, Phys. Rev. B **28**, 418 (1983).