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Collective mode and structure factor of a Fermi-Bose mixture at tricriticality

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The collective oscillation frequencies and structure factor in the tricritical region of a mixture of weakly interacting bosons and fermions have been calculated by use of effective boson Hamiltonian derived in a previous publication. The collective mode corresponds to a phonon-type dispersion relation with a damping constant proportional to the magnitude of the wave vector **q**. The structure factor exhibits divergence for $q \to 0$. Experimental investigation at tricriticality of a ^{3}He-^{4}He mixture is worth attempting to confirm the latter.

Most of our present knowledge about dynamic properties near the tricritical point (TCP) in ^{3}He-^{4}He mixtures comes from classical, phenomenological approaches[1-3] based on mode-mode coupling theories and Langevin-type stochastic equations. These approaches are reasonably good to describe experimental data for tricritical indices. An investigation of tricritical dynamics starting from an appropriate quantum-mechanical basis, nevertheless, is desirable, for, unlike statics, dynamics is not completely independent of the microscopic detail in an atomic Hamiltonian.

A theory of tricritical behavior in a mixture of weakly interacting bosons and fermions has been developed recently by Singh and the author.[4,5] The system serves as a model for a ^{3}He-^{4}He mixture. The theory reduces the system to that describable by an effective, low-momentum boson Hamiltonian where momenta involved have magnitude less than or equal to a cutoff p_c small compared to the boson thermal momentum $\lambda_B^{-1}(T)=(m_4/4\pi\beta)^{1/2}$. The order parameter in the theory is the boson field. Since the correlation length ξ of order-parameter fluctuations is singular[5] in the neighborhood of the TCP and therefore large compared to $\lambda_B(T)$, in this theory p_c may be regarded to be of the same order as ξ^{-1} in the tricritical region. A move is initiated to study the tricritical dynamics of the system under this assumption starting with the effective Hamiltonian in Ref. 4.

This article reports two new features of the tricritical behavior of the system. These are, in the neighborhood of TCP, the following: (i) the collective mode of the system corresponds to a phonon-type dispersion relation with damping constant proportional to the magnitude q of the wave vector **q** rather than q^2, and (ii) the boson structure factor $S(q) \sim |T-T_t|^{-1/2}$ for fermion chemical potential $\mu_3=\mu_{3t}$, where (T_t,μ_{3t}) are the coordinates of the TCP in the T-μ_3 plane. These features refer to $q \ll \xi^{-1}$. At first sight, the divergence in $S(q)$ for $q \to 0$ may appear to be inexplicable, for this deviates from the corresponding prediction of classical theories.[3,6] In the lattice-gas model[6] of tricritical behavior, for example, $S(q=0)$ is nonsingular for small $T-T_t$ and $\mu_3=\mu_{3t}$. As will be seen below, these features are consequences of Bose statistics. Their revelation therefore cannot be expected in classical approaches.

The effective boson Hamiltonian (in units such that $\hbar=1$) is

$$H_e=C_0+\sum_q\left[\frac{q^2}{m_4}-\mu_4'\right]b_q^\dagger b_q+\frac{u_4'}{V}\sum_{q_1-q_4}b_{q_1}^\dagger b_{q_2}^\dagger b_{q_3}b_{q_4}\delta_{q_1+q_2,q_3+q_4}+\frac{u_6}{V^2}\sum_{\substack{q_1q_2q_3\\ q_1',q_2',q_3'}}b_{q_1}^\dagger b_{q_1-q_1'}b_{q_2}^\dagger b_{q_2-q_2'}b_{q_3}^\dagger b_{q_3-q_3'}\delta_{q_1'+q_2',-q_3'}\ , \tag{1}$$

$$C_0=V[\tfrac{1}{2}u_3(n_3^F)^2+u_{34}n_3^F n_4'+2u_4(n_4')^2+O(u_{34}^2)]\ , \tag{2}$$

$$\mu_4'=\mu_4-u_{34}n_3^F+O(u_{34}^2)\ , \tag{3}$$

$$u_4'=u_4-\tfrac{1}{2}u_{34}^2\frac{\partial n_3^F}{\partial\mu_3}+O(u_{34}^3)\ , \tag{4}$$

$$u_6=\frac{u_{34}^3}{6}\frac{\partial^2 n_3^F}{\partial\mu_3^2}\ , \tag{5}$$

$$n_3^F=2V^{-1}\sum_k\left[\exp\beta\left[\frac{k^2}{m_3}-\mu_3\right]+1\right]^{-1}, \tag{6}$$

$$n_4'=V^{-1}\sum_{|p|>p_c}\left[\exp\beta\left[\frac{p^2}{m_4}-\mu_4\right]-1\right]^{-1}. \tag{7}$$

Here μ_4 denotes the boson chemical potential and u_3, u_4, and u_{34}, respectively, correspond to fermion-fermion, boson-boson, and boson-fermion interactions. The Hamiltonian was derived under the assumption that u_4 is of $O(u_{34}^2\partial n_3^F/\partial\mu_3)$. The quantity u_6 is positive definite.

Natural oscillation frequencies of the system are given by poles of the density-response function

$$D^R(q,\omega)=-i\int dt\, e^{i\omega t}\langle[\rho_q(t),\rho_q^\dagger(0)]_-\rangle\theta(t)\ , \tag{8}$$

where

$$\rho_q(0)=\sum_{q_1} b_{q_1}^\dagger b_{q+q_1}\ , \tag{9}$$

$$\rho_q(t)=\exp(iH_e t)\rho_q(0)\exp(-iH_e t)\ , \tag{10}$$

and the angular brackets $\langle\ \rangle$ denote thermodynamic average calculated with H_e. To calculate $D^R(q,\omega)$, it is convenient to introduce the temperature function

$$\mathscr{D}(q,\tau)=-\langle T\{\rho_q(\tau)\rho_q^\dagger(0)\}\rangle\ , \tag{11}$$

where

$$\rho_q(\tau)=\exp(H\tau)\rho_q(0)\exp(-H\tau)\ . \tag{12}$$

The Fourier coefficient

$$\mathscr{D}(q,z)=\int_0^\beta d\tau\, e^{z\tau}\mathscr{D}(q,\tau)\ , \tag{13}$$

where $z=2\pi ni/\beta$ with $n=0,\pm1,\pm2,\ldots$, is related to $D^R(q,\omega)$:

$$\mathscr{D}(q,z)\,|_{z=\omega+i0^+}=D^R(q,\omega)\ . \tag{14}$$

In what follows we first calculate $\mathscr{D}(q,z)$ in the normal phase of the system. This will lead to oscillation frequencies in the tricritical region.

The perturbation expansion of $\mathscr{D}(q,\tau)$ in terms of the unperturbed propagator

$$\mathscr{G}_1^0(q,\tau)=-\langle T\{b_q(\tau)b_q^\dagger(\tau)\}\rangle\ , \tag{15}$$

where the angular brackets denote the thermodynamic average defined with $(\sum_q[(q^2/m_4)-\mu_4']b_q^\dagger b_q)$, provides a scheme to calculate $\mathscr{D}(q,z)$. It will be seen that the expansion does not make sense in the critical region. In the tricritical region, however, it does. This is not surprising, for even a mean-field approximation (MFA) has been found inadequate to describe ordinary critical behavior of the system (see second article in Ref. 4). The expansion is also valid when one is far away from critical and tricritical regions.

Contributions to $\mathscr{D}(q,z)$, calculated using the perturbation expansion of $\mathscr{D}(q,\tau)$, are represented graphically in Figs. 1 and 2. The lowest-order contribution, involving the product of the noninteracting Matsubara propagators

$$\mathscr{G}_1^0(q,z)=(z-\varepsilon_q^0+\mu_4')^{-1},\ \ \varepsilon_q^0=q^2/m_4\ , \tag{16}$$

corresponds to the Fig. 1(a) and is typical of a polarization insertion. The contribution may be written as $[-V\Pi_0(q,z)]$, where

$$\Pi_0(q,z)=\beta^{-1}\int\frac{d^3q_1}{(2\pi)^3}\sum_{z_1}\mathscr{G}_1^0(q+q_1,z+z_1)\mathscr{G}_1^0(q_1,z_1)\ . \tag{17}$$

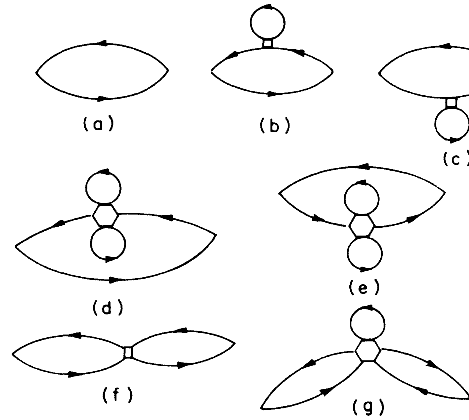

FIG. 1. Graph (a) represents lowest-order contribution to $\mathscr{D}(q,z)$ and graphs (b)–(g) first-order contributions. A square vertex corresponds to u_4' while a hexagonal one to u_6.

A typical term of the frequency sum in (17) is of order $|z_1|^{-2}$ as $|z_1|\to\infty$. The sum therefore converges absolutely. The term $\Pi_0(q,z)$ may also be expressed in the following form:

$$\Pi_0(q,z)=\int\frac{d^3q_1}{(2\pi)^3}\frac{(n_{q+q_i}^0-n_{q_1}^0)}{z-\varepsilon_{q+q_1}^0+\varepsilon_{q_1}^0}\ , \tag{18}$$

where

$$n_q^0=\{\exp[\beta(\varepsilon_q^0-\mu_4')]-1\}^{-1}\ . \tag{19}$$

The first-order contributions to $\mathscr{D}(q,z)$ correspond to Figs. 1(b)–1(g). A square vertex represents u_4' whereas a hexagonal one u_6. We have assumed[7] that u_4' is of same order of smallness as $(u_6 n')$, where $n'=V^{-1}\sum_q n_q^0$. On calculating contributions of these graphs, one finds that $\mathscr{D}(q,z)$, correct to first order, is given by the expression

$$\begin{aligned}-V\Pi_0(q,z)[1-\beta(4u_4'n'+18u_6n'^2)\\ -\beta(4u_4'n'+18u_6n'^2)\Pi_0'(q,z)\Pi_0^{-1}(q,z)]\\ +4V(u_4'+9u_6n')\Pi_0^2(q,z)\ ,\end{aligned} \tag{20}$$

where

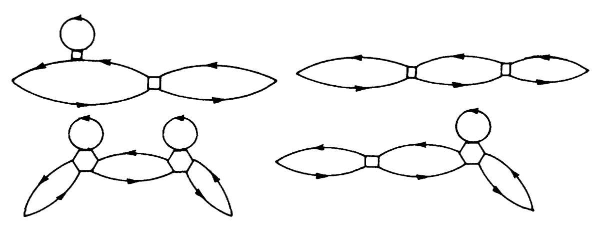

FIG. 2. A few typical graphs representing second-order contributions to $\mathscr{D}(q,z)$.

$$\Pi_0'(q,z)=\int\frac{d^3q_1}{(2\pi)^3}\frac{[(n^0_{q+q_1})^2-(n^0_{q_1})^2]}{z-\varepsilon^0_{q+q_1}+\varepsilon^0_{q_1}}\ . \tag{21}$$

In the limit $|z|\to\infty$, one obtains

$$\Pi_0(q,z)\simeq\frac{2}{z^2}\int\frac{d^3q_1}{(2\pi)^3}n^0_{q_1}(1+n^0_{q+q_1})(\varepsilon^0_{q_1}-\varepsilon^0_{q+q_1})\ , \tag{22}$$

$$\Pi_0'(q,z)\simeq\frac{2}{z^2}\int\frac{d^3q_1}{(2\pi)^3}n^0_{q_1}(1+n^0_{q+q_1})(n^0_{q_1}+n^0_{q+q_1})\times(\varepsilon^0_{q_1}-\varepsilon^0_{q+q_1})\ . \tag{23}$$

Since momenta $\mathbf{q}$ in (1) are such that $0\le q\le p_c$ and $p_c\lambda_B(T)\ll1$, one may use for n^0_q its approximate form

$$n^0_q=4\pi\lambda_B^{-2}(T)(q^2+\xi^{-2})^{-1} \tag{24}$$

in (22) and (23). Here $\xi[=(-m_4\mu_4')^{-1/2}]$ is the correlation length of order parameter fluctuations in the normal phase [cf. Eq. (50) in Ref. 5]. With the help of (22)–(24) we now examine validity of the expansion of $\mathscr{D}(q,\tau)$. This expansion may be regarded satisfactory provided that the third term in the large parentheses in (20), in particular, is small compared to unity.

As already stated, the cutoff momentum p_c may be regarded to be of order ξ^{-1} at criticality. For $p_c\xi\sim1$ and $q/p_c\ll1$, one finds from (22)–(24) that

$$\Pi_0(q,z)\simeq-\frac{m_4q^2\xi}{3\pi^2z^2\beta^2}\{\arctan(p_c\xi)-[(p_c\xi)^{-1}+p_c\xi]^{-1}\}\ , \tag{25}$$

$$\Pi_0'(q,z)\simeq-\frac{4m_4^2q^2}{3\pi^2z^2\beta^3p_c^3}[(p_c\xi)^{-2}+1]^{-3}\ . \tag{26}$$

Upon taking $n'\sim O(\lambda_B^{-3}(t))$, it is clear from (25) and (26) that the third term in the large parentheses in (20) is of order

$$\alpha(T,\mu_3,\mu_4)=\frac{m_4u_4'\xi^2}{\lambda_B^3(T)}\ . \tag{27}$$

The second term is of order $[m_4u_4'/\lambda_B(T)]$ and small compared to the third term. In the high-density fermion limit $\mu_3\gg k_BT$, it has been shown[5] that

$$\xi^{-2}\simeq\frac{m_4u_{34}m_3^2k_B^2T_\lambda}{12(m_3\mu_3)^{1/2}}(T-T_\lambda) \tag{28}$$

for small $T-T_\lambda$ and $T>T_\lambda$. Here T_λ stands for a λ transition temperature of the system. Thus the condition of validity of the perturbation expansion, in the critical region, is

$$\frac{T-T_\lambda}{T_\lambda}\gg\frac{u_4}{u_{34}}\left[\frac{\mu_3}{k_BT_\lambda}\right]^{1/2}\ . \tag{29}$$

This condition is difficult to be satisfied even though $u_4/u_{34}\ll1$ (see Ref. 4). In the tricritical region, for $\mu_3=\mu_{3t}$, ξ^{-2} is obtained from (28) replacing T_λ by T_t and μ_3 by μ_{3t}. The quantity u_4' is of order $T-T_t$ [cf. (A.10) in Ref. 5]:

$$u_4'\simeq\frac{u_{34}^2m_3^2k_B^2T_t}{48(m_3\mu_{3t})^{3/2}}(T-T_t)\ . \tag{30}$$

The condition of validity of the expansion, along the line $\mu_3=\mu_{3t}$, is thus

$$\frac{\mu_{3t}}{k_BT_t}\gg\frac{u_{34}m_3}{\lambda_B(T_t)}\ . \tag{31}$$

This condition is definitely satisfied in the weak-interaction approximation. The conclusion is that whereas in the critical region the expansion of $\mathscr{D}(q,\tau)$ is probably meaningless, in the tricritical region it is not so.

Upon examining also the second- and higher-order contributions to $\mathscr{D}(q,z)$ (see Fig. 2), one finds that $D^R(q,\omega)$ is given by

$$D^R(q,\omega)=-V\epsilon^{-1}(q,\omega)\Pi^*(q,z)\,|_{z=\omega+i0^+}\ , \tag{32}$$

$$\epsilon(q,\omega)=[1+4(u_4'+9u_6n')\Pi^*(q,z)\,|_{z=\omega+i0^+}]\ , \tag{33}$$

$$\Pi^*(q,z)=\Pi_0(q,z)-\beta(4u_4'n'+18u_6n'^2)[\Pi_0(q,z)+\Pi_0'(q,z)]+\cdots\ . \tag{34}$$

The function $\epsilon(q,\omega)$ may be identified as the dielectric function of the system in the normal phase. It is clear that natural oscillation frequencies of the system are determined by zeros of $\epsilon(q,\omega)$. The ellipses in (34) represents higher-order terms. To the lowest order, from (18) and (34), one finds that

$$F_1(q,\omega)\equiv\mathrm{Re}[\Pi^*(q,z)\,|_{z=\omega+i0^+}]=\int\frac{d^3q_1}{(2\pi)^3}n^0_{q_1+(1/2)q}\mathscr{P}\left[\frac{1}{\omega-2\mathbf{q}\cdot\mathbf{q}_1m_4^{-1}}-\frac{1}{\omega+2\mathbf{q}\cdot\mathbf{q}_1m_4^{-1}}\right]\ , \tag{35}$$

$$F_2(q,\omega)\equiv\mathrm{Im}(\Pi^*(q,z)\,|_{z=\omega+i0^+})=\pi\int\frac{d^3q_1}{(2\pi)^3}n^0_{q_1+(1/2)q}[\delta(\omega+2\mathbf{q}\cdot\mathbf{q}_1m_4^{-1})-\delta(\omega-2\mathbf{q}\cdot\mathbf{q}_1m_4^{-1})]\ . \tag{36}$$

Here $\mathscr{P}$ denotes a Cauchy principal value. In view of (24), the right-hand side of (35) can be approximated further. One obtains

$$F_1(q,\omega)\simeq\frac{1}{\pi\lambda_B^2(T)}\int_{|q_1|\le p_c}q_1^2dq_1\int_0^{\pi}d\theta\left[\frac{\sin\theta}{q_1^2+\frac{q^2}{4}+\xi^{-2}+qq_1\cos\theta}\right]\mathscr{P}\left[\frac{1}{\omega-2qq_1\cos\theta m_4^{-1}}-\frac{1}{\omega+2qq_1\cos\theta m_4^{-1}}\right]. \quad (37)$$

The integration relative to θ followed by the introduction of a new variable t, defined by $q_1=\sqrt{t}p_c$, yield

$$F_1(q,\omega)\simeq-\frac{m_4}{\pi\lambda_B^2(T)q}\mathscr{P}[I_1(q,\omega)+I_2(q,\omega)], \quad (38)$$

$$I_1(q,\omega)=\frac{m_4\omega}{2p_c^2}\int_0^1\frac{dt}{\left[\left[t+\frac{y^2}{4}+\lambda^2\right]^2-\frac{m_4^2\omega^2}{4p_c^4}\right]}\times\ln\left|\frac{\frac{m_4\omega}{2p_c^2}+y\sqrt{t}}{\frac{m_4\omega}{2p_c^2}-y\sqrt{t}}\right|, \quad (39)$$

$$I_2(q,\omega)=\int_0^1\frac{dt(t+y^2/4+\lambda^2)}{\left[\left[t+\frac{y^2}{4}+\lambda^2\right]^2-\frac{m_4^2\omega^2}{4p_c^4}\right]}\times\ln\left|\frac{t+y^2/4+\lambda^2-y\sqrt{t}}{t+y^2/4+\lambda^2+y\sqrt{t}}\right|, \quad (40)$$

$$y=\frac{q}{p_c},\quad \lambda=(p_c\xi)^{-1}. \quad (41)$$

It has been seen above that in the limit $|z|\to\infty$, the expansion of $\mathscr{D}(q,\tau)$ is meaningful in the tricritical region. We therefore assume $m_4|\omega|/2p_c^2\gg1$ which yields

$$I_1(q,\omega)\simeq-\frac{16p_c^3q}{3m_4^2\omega^2}, \quad (42)$$

$$I_2(q,\omega)\simeq-\frac{4p_c^4}{m_4^2\omega^2}\left[\left[\frac{1}{2}+\frac{y^2}{4}+\lambda^2\right]\ln\left|\frac{1+\frac{y^2}{4}+\lambda^2-y}{1+\frac{y^2}{4}+\lambda^2+y}\right|+I_2'(q,\omega)\right], \quad (43)$$

$$I_2'(q,\omega)=[J_5(q)-K_5(q)]+\frac{y}{2}[J_4(q)+K_4(q)]+2\left[\frac{y^2}{4}+\lambda^2\right][J_3(q)-K_3(q)]+y\left[\frac{y^2}{4}+\lambda^2\right][J_2(q)+K_2(q)], \quad (44)$$

$$J_m(q)=\int_0^1\frac{x^mdx}{\frac{y^2}{4}+\lambda^2+yx+x^2},\quad m\ge0, \quad (45)$$

$$K_m(q)=\int_0^1\frac{x^mdx}{\frac{y^2}{4}+\lambda^2-yx+x^2},\quad m\ge0. \quad (46)$$

It is quite straightforward to evaluate $I_2'(q,\omega)$ with the help of the relations

$$J_{m+1}(q)=m^{-1}-yJ_m(q)-\left[\frac{y^2}{4}+\lambda^2\right]J_{m-1}(q),\quad m\ge1, \quad (47)$$
$$K_{m+1}(q)=m^{-1}+yK_m(q)-\left[\frac{y^2}{4}+\lambda^2\right]K_{m-1}(q),\quad m\ge1.$$

On assuming $y\ll1$, in view of (38), (42), and (43) one obtains

$$F_1(q,\omega)\simeq-\frac{m_4}{4\pi p_c\lambda_B^2(T)}\left[\frac{2p_c^2}{m_4\omega}\right]^2[y^2f_1(\lambda)-y^4f_2(\lambda)+O(y^6)], \quad (48)$$

where

$$f_1(\lambda)=3+8\lambda\arctan\lambda+\frac{7+15\lambda^2}{3(1+\lambda^2)}, \quad (49)$$

$$f_2(\lambda)=\frac{1}{(1+\lambda^2)^2}\left[\lambda(1-\lambda^2)^2+\frac{1+3\lambda^2+6\lambda^4}{3(1+\lambda^2)}\right]. \quad (50)$$

As long as α in (27) is small compared to unity, one does not need higher-order contributions to $F_1(q,\omega)$.

We now consider the dielectric function $\epsilon(q,\omega)$ in (33). Zeros $(\omega_q-i\Gamma_q)$ of this function are determined by the equation

$$1+4u_4''[F_1(q,\omega_q-i\Gamma_q)+iF_2(q,\omega_q-i\Gamma_q)]=0, \quad (51)$$

where $u_4''=(u_4'+9u_6n')$. We assume that damping constant $\Gamma_q\ll|\omega_q|$. One then obtains

$$\omega_q=\pm cq,\quad c=\frac{2}{\lambda_B(T)}\left[\frac{u_4''p_cf_1(\lambda)}{m_4\pi}\right]^{1/2} \quad (52)$$

and

$$\Gamma_q=2u_4''\omega_qF_2(q,\omega_q) \quad (53)$$

in the first approximation. In MFA,[4,5] the equation of the λ line is $\mu_4'=0$ and the endpoint of the λ line, viz. $\mu_4'=0$, $u_4'\to O^+$, corresponds to TCP. In the region $\mu_4',u_4'<0$ of the T-μ_3 plane, a coexistence of the normal and condensed phases is possible. Obviously enough, when u_4' becomes zero or negative, the term $(9u_6n')$ plays a very significant role—it does not allow c to vanish or become imaginary.

To ascertain whether Γ_q is positive, we calculate $F_2(q,\omega)$. It is clear from (36) that $F_2(q,\omega)$, to the lowest order, is an odd function of ω. The fact that (q,ξ^{-1}) are small compared to $\lambda_B^{-1}(T)$ enables us to write

$$F_2(q,\omega)\simeq\lambda_B^{-1}(T)\int_{|q_1|\le p_c} q_1^2 dq_1 \int_0^\pi d\theta \left[\frac{\sin\theta}{q_1^2+\frac{q^2}{4}+\xi^{-2}+qq_1\cos\theta}\right]\left[\delta\left[\omega+\frac{2qq_1\cos\theta}{m_4}\right]-\delta\left[\omega-\frac{2qq_1\cos\theta}{m_4}\right]\right] . \quad (54)$$

Simple change of variables and use of the definition of the unit step function $\theta(x)$ yield

$$F_2(q,\omega)\simeq\frac{m_4^2\omega}{\lambda_B^2(T)q}\int_{|q_1|\le p_c} q_1 dq_1 \left[\left[q_1^2+\frac{q^2}{4}+\xi^{-2}\right]^2-(\tfrac{1}{2}\omega m_4)^2\right]^{-1}\left[\theta\left[\omega+\frac{2qq_1}{m_4}\right]-\theta\left[\omega-\frac{2qq_1}{m_4}\right]\right] . \quad (55)$$

The integrand in (55) is nonzero only when $q_1 > |\omega| m_4/2q$. Moreover, as already stated, at criticality $p_c\xi\sim 1$ and $\alpha \ll 1$. In view of these facts, for $q/p_c \ll 1$, one obtains

$$F_2(q,\omega_q)\simeq\frac{m_4^2\omega_q(p_c\xi)^2}{2\lambda_B^2(T)q(p_c^2+\xi^{-2})} . \quad (56)$$

Substituting (56) in (53), one may write

$$\Gamma_q=\nu p_c q;\quad \nu=\frac{4f_1(\lambda)\alpha^2}{\pi m_4}\frac{(p_c\lambda_B(T))^2}{[1+(p_c\xi)^2]} . \quad (57)$$

As expected, Γ_q is positive. Since

$$\frac{\Gamma_q}{|\omega_q|}=\frac{2f_1^{1/2}(\lambda)\alpha^{3/2}}{\pi^{1/2}}\frac{(p_c\lambda_B(T))^{3/2}}{[(p_c\xi)^{-1}+p_c\xi]} \ll 1 , \quad (58)$$

the assumption of small damping is fully justified. A proper study of hydrodynamical equations of the present system is necessary to establish a relation between the damping coefficient ν and thermodynamic functions.

We next turn our attention to structure factor $S(q)[=\langle\rho_q^\dagger(0)\rho_q(0)\rangle]$ for the normal phase of the system. Its behavior near TCP will provide the possibility of testing validity of the present approach by means of scattering experiment. It is quite straightforward to show that

$$\langle\rho_q^\dagger(t)\rho_q(0)\rangle = -\int_{-\infty}^{+\infty}\frac{d\omega'}{\pi}\frac{e^{-i\omega' t}}{1-e^{-\beta\omega'}}\mathrm{Im}[\mathscr{D}'(q,z)|_{z=\omega'+i0^+}] , \quad (59)$$

where

$$\mathscr{D}'(q,z)=\int_0^\beta d\tau\, e^{z\tau}\mathscr{D}'(q,\tau) , \quad (60)$$

$$\mathscr{D}'(q,\tau)=-\langle T\{\rho_q^\dagger(\tau)\rho_q(0)\}\rangle . \quad (61)$$

As long as $\alpha \ll 1$, it suffices to consider only lowest-order contribution to $\mathscr{D}'(q,z)$. To this order, (59) yields

$$S(q)=V\int\frac{d^3q_1}{(2\pi)^3}\frac{n^0_{q+q_1}-n^0_{q_1}}{1-e^{\beta(\varepsilon^0_{q+q_1}-\varepsilon^0_{q_1})}} . \quad (62)$$

Using (24) in (62), one finds that for $(q/p_c)\ll 1$, the approximate form of (62) is

$$S(q)\simeq\frac{8V\xi}{\lambda_B^4(T)}[I_0(\lambda)-\tfrac{1}{3}(q/p_c)^2 I_1(\lambda)+\cdots] , \quad (63)$$

$$I_0(\lambda)=\left[\arctan\lambda^{-1}-\frac{\lambda}{\lambda^2+1}\right] , \quad (64)$$

$$I_1(\lambda)=\frac{1}{4\lambda^2}\left[\arctan\lambda^{-1}-\frac{\lambda(\lambda^2-1)}{(\lambda^2+1)^2}\right] . \quad (65)$$

Near TCP $\lambda\sim 1$ and, for $T>T_t$, $\xi\sim(T-T_t)^{-1/2}$ along the line $\mu_3=\mu_{3t}$. It transpires that along this line $S(q\ll\xi^{-1})\sim(T-T_t)^{-1/2}$.

In the condensed phase of this system also, $S(q\to 0)$ diverges along the line $\mu_3=\mu_{3t}$ near TCP. To clarify, we turn to Bogoliubov approximation in Ref. 5 for this phase. In this approximation

$$S(q)\simeq\frac{Vn_0q^2}{m_4E(q)}\coth\frac{\beta E(q)}{2} , \quad (66)$$

where

$$E(q)=\frac{q^2}{m_4}[1+(q\xi')^{-2}]^{1/2} , \quad (67)$$

n_0 is the number density of particles in the condensate and ξ' is the correlation length of order-parameter fluctuations in the condensed phase. The term $(\coth[\beta E(q)/2])$ is a consequence of the finite-temperature formalism used in deriving (66). As $T\to 0$ $\coth[\beta E(q)/2]\to 1$ and the right-hand side of (66) assumes the well-known Feynman-Cohen[8] form provided n_0 is replaced by mean number density of bosons. For finite T, since $q\ll\lambda_B^{-1}(T)$, $(\coth[\beta E(q)/2])$ can be approximated by $2(\beta E(q))^{-1}$. Near TCP one then obtains

$$S(q)\simeq\frac{2Vn_0k_BT_t}{\left[\frac{q^2}{m_4}+\xi'^{-2}\right]} . \quad (68)$$

Since, for $T<T_t$ and $\mu_3=\mu_{3t}$, $n_0\sim(T_t-T)^{1/2}$ and $\xi'\sim(T_t-T)^{-1/2}$, it follows from (68) that $S(q\ll\xi'^{-1})\sim(T_t-T)^{-1/2}$.

In the lattice-gas model[6] of Blume, Emery and Griffiths (BEG) one finds that $S(q=0)$ is nonsingular. To see how this develops, one may proceed as follows. A general relation of classical statistical mechanics is that if Hamiltonian of a classical system contains the parameter $f(r)$ in the combination $[-\int dr\, f(r)p(r)]$ and if we let $f(r)\to f(r)+\delta f(r)$, the increment $\delta f(r)$ brings about a change in $\langle p(r)\rangle$ which is given by

$$\delta\langle p(r)\rangle=\beta\delta f\int dr'\, g(r,r')=\beta\delta f g(q=0) \quad (69)$$

for δf independent of r. Here

$$g(r,r')=\langle[p(r)-\langle p(r)\rangle][p(r')-\langle p(r')\rangle]\rangle . \quad (70)$$

In the theory of BEG, $f(r)$ and $p(r)$ correspond to μ_4 and the number of ^{4}He atoms N_4, respectively. From (69) one finds that

$$S(q=0)=k_B T\frac{\partial N_4}{\partial \mu_4} \quad (71)$$

in this theory. With the help of (71) and Eq. (3.7) of Ref. 6, for the normal phase, one obtains

$$S(q=0)=\frac{2Ne^{\beta\Delta}}{(e^{\beta\Delta}+2)^2}, \quad (72)$$

where N is the total number of sites and $\Delta \simeq \mu_3 - \mu_4$. Within the framework of MFA in Ref. 6 it can now be easily shown that $S(q=0)$ is nonsingular for small $T-T_t$ and $\Delta=\Delta_t$. One finds

$$S(q=0)=\left[\frac{k_B}{4d_4}+\frac{d_2}{6d_4}\right]\left[1-\left[\frac{3}{2T_t}+\frac{d_2}{k_B T_t^2}\right](T-T_t) + O(T-T_t)^2\right]. \quad (73)$$

Here d_2 and d_4, respectively, are the derivatives with respect to temperatures at TCP of coefficients of (order-parameter)^2 and (order-parameter)^4 terms in the Landau expansion of a free energy in Ref. 6.

The regular behavior of $S(q=0)$ near TCP in BEG theory is not quite unexpected, for the divergence in $S(q\to 0)$ predicted above is a consequence of Bose statistics. Experimental investigation in the tricritical region of a ^{3}He-^{4}He mixture alone can decide whether the prediction is meaningful or merely an artifact of the microscopic approach.

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