Quasiparticle interaction in the Fermi liquid ^{3}He

M. Pfitzner and P. Wölfle*
Technische Universität München, D-8046 Garching, Federal Republic of Germany
(Received 22 April 1986)

The Fermi-liquid interaction and the quasiparticle scattering amplitude on the Fermi surface for liquid ^{3}He are calculated by solving the pair of coupled Bethe-Salpeter equations in the two particle-hole channels. The method of solution preserves exchange symmetry by employing the eigenfunctions of the exchange operator as a basis set. The quasiparticle effective mass and the totally particle-hole irreducible direct interaction needed as an input are taken from available microscopic calculations. It is found that a small nonlocal part is necessary in the direct interaction in order to obtain a large backflow parameter F_1^s, as observed in ^{3}He. The origin of this contribution is traced back to screening effects on the interaction in the particle-particle channel. Upon including a phenomenological nonlocal part into the direct interaction, good agreement is found with available thermodynamic and transport data.

I. INTRODUCTION

According to Landau[1] the low-energy properties of a Fermi system well below the degeneracy temperature T_F are those of a gas of interacting quasiparticles. This is true at least in the so-called normal state, defined as that state of the interacting system in which the quasiparticle states are in one-to-one correspondence with the single-particle states of the corresponding noninteracting system. In many cases the normal state is not the ground state, as the system condenses into a state with macroscopic order below some temperature T_c. But even then the quasiparticle states may form a basis for the description of the condensed state, provided the transition temperature is well below the Fermi temperature T_F.

The quasiparticles are characterized (i) by their single-particle energy $\varepsilon_{p\sigma}$ as a function of momentum $\mathbf{p}$ and spin σ, parametrized by the effective mass m_p^*, such that $\partial\varepsilon_{p\sigma}/\partial p = p/m_p^*$, (ii) by the quasiparticle scattering amplitude (QSA) $a(12;34)$ describing a collision process of two quasiparticles in initial states $\mathbf{p}_1\sigma_2$ and $\mathbf{p}_2\sigma_2$ scattering into final states $\mathbf{p}_3\sigma_3$ and $\mathbf{p}_4\sigma_4$. The so-called Landau parameters, which account for the screening of external fields by the molecular field, the pair interaction responsible for the formation of Cooper pairs and even the effective mass at the Fermi surface are all derivable from the QSA.[2,3] In fact any other type of macroscopic order that happens to occur in the system below a temperature $T_c \ll T_F$ must relate to some part of the QSA, provided it is a two-particle-correlation effect.

Therefore the QSA, or equivalently as we shall see, the Fermi-liquid interaction must be the central target of any microscopic theory of interacting Fermi systems. Ideally one would like to start with the bare interaction. The interaction between two isolated helium atoms is fairly well known from atomic-beam scattering experiments and thermodynamic data on the gas phase. A suitable analytic form is provided by the HFDHE2 potential of Aziz *et al.*,[4] which reproduces the data and also meets the known theoretical limiting forms at both short and long distances. Further support for the correctness of this potential in the liquid phase comes from the results of Green's-function Monte Carlo calculations,[5] which provide a very accurate account of the equation of state. Like the well-known Lennard-Jones potential, the more refined potential forms possess a strongly repulsive hard core of radius $r_c \simeq 2.6$ Å (which amounts to about one-half of the average interparticle distance), and a weakly attractive van der Waals part (see Fig. 1). The fact that the hard core is so extended is responsible for the very effective screening of density fluctuations and current fluctuations, which in turn lead to a large effective mass: The particles are almost localized due to the blocking effect of the voluminous hard cores, in spite of the large zero-point motion and the weak-binding forces. On the other hand, the exchange forces are relatively weak.

Early attempts to calculate the Landau interaction concentrated on the proper treatment of the hard core. In

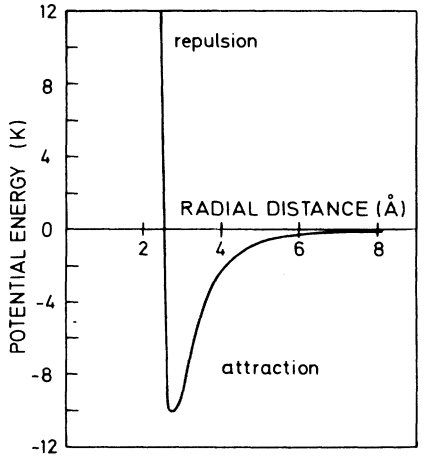


FIG. 1. Interaction potential of two ^{3}He atoms in vacuum.

the presence of a strong potential multiple-scattering processes are important. These may be summed in the ladder approximation, taking into account the presence of the filled Fermi sea. This first partial summation of perturbation theory leads to the replacement of the bare interaction potential by the G matrix[6–9] (for a recent calculation of the G matrix see Ref. 10). However, for liquid ^{3}He the G matrix bears no resemblance with the Landau interaction function. For instance, extracting the Landau parameters from the G matrix results in weak antiscreening of density fluctuations ($F_0^s < 0$), where actually strong screening is observed.

It was later shown by Babu and Brown[11] in a seminal work (for a similar calculation for nuclear matter see Ref. 12) that particle-hole (p-h) excitation processes in the crossed p-h channel, describing density- and spin-density fluctuations for the main part, contribute to the Landau interaction in an essential way. These contributions are not sufficient, however, to account for the experimentally observed Fermi-liquid interaction. It appears that the screening of the interaction in the particle-particle channel is equally important. One is thus left with the problem of summing all two-particle excitation processes consistently.

Such a program has been carried out in a somewhat different formulation using a combination of a variational theory of the ground state and a perturbation theory employing correlated basis functions (CBF) derived from the correlated ground state.[13–15] Within this framework it has been possible to sum large classes of diagrams in perturbation theory approximately.[14,15] The most serious approximation one is forced to introduce in those theories consists of replacing the particle-hole propagator by a so-called collective propagator, which lacks the proper analytic structure near the Fermi surface. While global properties such as the static structure factors and the density molecular field are well described by this approach, the effective interaction of quasiparticles at the Fermi surface cannot be obtained yet in detail.

On the phenomenological side a new kind of effective quasiparticle interaction has been introduced by Aldrich and Pines.[16,17] These so-called polarization potentials represent a generalization of the Landau molecular field to wave numbers up to several Fermi wave numbers. Starting from a plausible pseudopotential form in position space combined with sum rules and Landau limits these authors have constructed what has turned out, to date, to be the only theory capable of describing the density fluctuation excitation in detail. One of the great successes of this theory was the accurate prediction of the high-frequency zero-sound mode in advance of experiment. Bedell and Pines[18,19] used the polarization potentials to construct QSA's for ^{3}He. As the QSA initially calculated from the Landau integral equation [see Eq. (16)] does not satisfy the requirements of exchange symmetry, Bedell and Pines multiplied the symmetry-violating part by a symmetrizing factor and added it onto the exchange symmetric part. Using the QSA so constructed they calculated the transport coefficients, the transition temperature to the superfluid state and the free-energy parameters in the superfluid state. By adjusting a single free parameter [the difference in range between the configuration-space pseudopotentials, $f^{\uparrow\uparrow}(r)$ and $f^{\uparrow\downarrow}(r)$] they obtained excellent agreement with existing experimental data. Attempts to improve on the ad hoc symmetrization procedure used by Bedell and Pines, by including higher angular momentum components such as to restore full exchange symmetry have been only partially successful.[20,21] Apparently the q dependence of the polarization potentials is not known with sufficient accuracy.

Finally, a number of attempts have been made to parametrize the QSA and to determine the coefficients by fitting to experimental data. The so-derived approximate QSA may then be used to calculate other properties. The first of these was the s-p approximation by Dy and Pethick.[22] It was later generalized to an arbitrary number of partial waves in the so-called effective potential approximation.[3,23,24] A completely general parametrization of the QSA respecting exchange symmetry has only recently been proposed.[25]

In this paper we calculate the effective quasiparticle interaction in liquid ^{3}He on a level which is intermediate between a fully microscopic theory, using the bare interaction as the only input, and a phenomenological theory such as the polarization potential theory,[16–19] for example. We exploit the fact that much of the renormalization of the interaction occurs through quasiparticle-quasihole excitation processes near the Fermi surface. These processes may be treated in a relatively controlled fashion by observing that recent microscopic calculations[26–30] imply that the quasiparticle picture is approximately valid in a rather large momentum range $0 \lesssim p \lesssim 2k_F$. Integral equations for the QSA based on this premise are derived in Sec. II. The uncertainty in the quasiparticle assumption (which grows with increasing momentum transfer in the scattering process) is confined to some extent by the exchange symmetry of the QSA, which relates the values of the QSA at small- and large-momentum transfer. Our calculation respects the exchange symmetry property exactly, which is made possible by employing the eigenfunctions of the exchange operator as a basis set.[31] The input needed in this calculation is the quasiparticle energy and the completely particle-hole irreducible interaction $I_{pp'}(q)$, called the direct interaction, i.e., that part of the total scattering amplitude which does not contain any quasiparticle-quasihole intermediate states. Our theory is similar in spirit to recent calculations by Ainsworth *et al.*,[32] Bedell and Ainsworth,[33] and Bedell and Quader[34] (see also the work of Quader and Bedell[35] on the spin-polarized system) based on the Babu-Brown equations as well as generalization of this approach by Ainsworth and Bedell.[36] As a first step I is considered to be a phenomenological quantity in Sec. IV. It is hoped that the direct interaction may be obtained from a fully microscopic theory. The results discussed in Sec. IV show indeed that the effective interaction derived from CBF–hypernetted-chain theory[14,37] provide a fairly good amount of the local part of I. The nonlocal part, which turns out to be necessary to obtain agreement with experiment is addressed in Sec. V, where a rough estimate is made. A few concluding remarks are added in Sec. VI. A brief account of this work has been published.[38]

II. INTEGRAL EQUATIONS FOR THE EFFECTIVE INTERACTIONS

The Landau theory of Fermi liquid may be justified using the methods of quantum field theory.[39–41] We shall need an extension of Landau's theory from the regime of small wave vectors q to the complete range from $q=0$ to $q=2k_F$. In the following we therefore present a brief review of the microscopic derivation.

The single-particle Green's function of the interacting system is diagonal in the energy-momentum representation and given by

$$G_{\alpha\beta}(\mathbf{p},\omega)=\frac{\delta_{\alpha\beta}}{\omega-\varepsilon_p^0-\mu-\Sigma(\mathbf{p},\omega)} \ . \tag{1}$$

Here $\varepsilon_p^0=p^2/2m$ is the bare energy and $\Sigma(\mathbf{p},\omega)$ is the self-energy. We consider the limit of zero temperature throughout this paper. According to a general theorem[39–41] the imaginary part of Σ vanishes at $\omega=\mu$, so that in the vicinity of the Fermi surface the Green's function shows a pole structure, sitting on top of a broad regular background $G^{\rm inc}$:

$$G(\mathbf{p},\omega)=\frac{z_p}{\omega-\varepsilon_p+i\gamma_p\,\mathrm{sgn}(\omega-\mu)}+G^{\rm inc}(\mathbf{p},\omega) \ . \tag{2}$$

The pole term describes quasiparticles of energy ε_p, lifetime γ_p^{-1}, and spectral weight z_p. In terms of the self-energy these parameters are given by

$$z_p^{-1}=1-\frac{\partial}{\partial\omega}\Sigma(\mathbf{p},\omega)\,|_{\omega=\varepsilon_p} \ , \tag{3}$$

$$\varepsilon_p=\varepsilon_p^0+\mathrm{Re}\Sigma(p,\varepsilon_p) \ , \tag{4}$$

$$\gamma_p=z_p\,\mathrm{Im}\Sigma(p,\varepsilon_p) \ , \tag{5}$$

with ε_p defined implicitly as the solution of (4). It is convenient to define a momentum-dependent effective mass by

$$\frac{\partial}{\partial p}\varepsilon_p=\frac{p}{m_p^*} \ . \tag{6}$$

From the dispersion relation (4) one obtains at once

$$\frac{m_p^*}{m}=\frac{z_p^{-1}}{1+\dfrac{m}{p}\left[\dfrac{\partial}{\partial p}\Sigma(p,\omega)\right]_{\omega=\varepsilon_p}} \ , \tag{7}$$

which is interpreted as the product of the "E mass" given by z_p^{-1} and the "p mass,"[42] related to the energy (E) and momentum (p) dependence of Σ, respectively. Detailed calculations for the dilute Fermi gas[43] and models of nuclear matter[44,45] show that the E mass is greater than m and causes a peak of m_p^* at the Fermi energy, whereas the p mass is fairly structureless and is slightly less than the bare mass. Calculations of m^* for ^{3}He in the framework of CBF theory[26,27,30] and those using an extension of Landau theory[28,29] yield the same qualitative behavior of the E mass. Little is known about the p mass, except for the fact that it must be rather large, too. This may be inferred from the fact that the compressibility of liquid ^{3}He is renormalized by a factor $(m^*/m)/(1+F_0^s)$ substantially smaller than unity. As shown by Prange and Kadanoff[46] for the electron-phonon system, a momentum-independent self-energy implies that the compressibility is unrenormalized, i.e., $m^*/m=1+F_0^s$. Inverting the argument and applying it to the interaction fermion system ^{3}He one is led to conclude that the strong renormalization of the compressibility observed there is indicative of a sizable momentum dependence of the self-energy and consequently a fairly large p mass.

If one neglects this contribution to the effective mass for lack of a reliable theory, the single-particle spectrum is found to be given by the bare energy except for a wiggle near the Fermi surface of width $\sim 0.3k_F$, giving rise to the enhanced effective mass.[26–30] The imaginary part of the self-energy is smaller than the real part in the regime $0\le p\lesssim 2.2p_F$, which comprises the set of momentum values $p<2p_F$ accessible in a scattering process on the Fermi surface.[27] We will therefore assume that the quasiparticle description is approximately valid in the complete range of interest here.

The interaction between the quasiparticles is contained in the two-particle Green's function $K(1,2;3,4)$, or more precisely, in the vertex part Γ defined by

$$\begin{aligned}K(12;34)=&(2\pi)^4G(1)G(2)\\&\times[\delta^4(1-3)\delta^4(2-4)-\delta^4(1-4)\delta^4(2-3)]\\&+iG(1)G(2)G(3)G(4)\Gamma(12;34)\\&\times(2\pi)^4\delta^4(1+2-3-4) \ .\end{aligned} \tag{8}$$

Here $1,2,\ldots$ denote the set of momentum, energy, and spin variables, $1=(\mathbf{p}_1,\omega_1;\sigma_1)$, etc. In terms of perturbation theory Γ is given by the sum of all connected diagrams with two incoming and two outgoing lines. The Green's function K as well as Γ has the symmetry of a two-particle wave function under exchange of two particles, i.e.,

$$\Gamma(12;34)=-\Gamma(12;43)=-\Gamma(21;34) \ . \tag{9}$$

It is well known that two-particle intermediate states are of particular interest in the perturbation theory for Γ, as they may give rise to singular behavior of Γ as a function of the transferred momentum and energy.[1] Accordingly the diagrams are classified into two-particle reducible and irreducible ones. This can be done in three possible ways, leading to the Bethe-Salpeter equations shown in Fig. 2. A convenient parametrization of the four-momenta p_i is given in terms of the total momenta in the three channels q,r,k defined by

$$\begin{aligned}q&=p_1-p_3=p_4-p_2 \ ,\\ r&=p_3-p_2=p_1-p_4 \ ,\\ k&=p_1+p_2=p_3+p_4 \ .\end{aligned} \tag{10}$$

The internal momentum p'' in Fig. 2 is summed over. The two particle-hole equations [Fig. 2(a)] and [Fig. 2(b)] are transformed into each other by the exchange operation ($3\leftrightarrow4$ or $1\leftrightarrow2$), whereas the particle-particle equation [Fig. 2(c)] transforms into itself. The kernels Γ^{ph}, $\bar{\Gamma}^{ph}$, and Γ^{pp} defined as the complete set of diagrams irreducible with respect to cutting a pair of lines in the (1,3), the (1,4), and

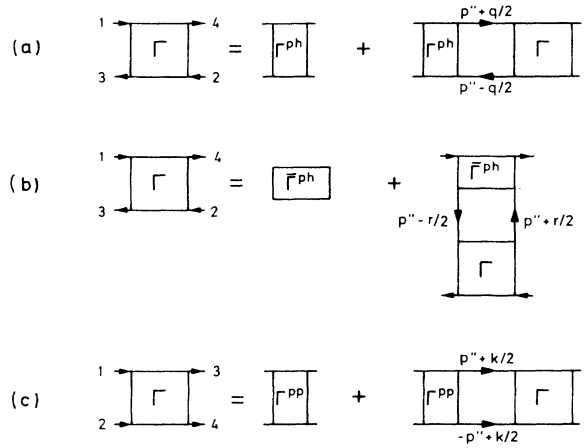


FIG. 2. Bethe-Salpeter equations for the two-particle vertex function Γ in (a),(b) the particle-hole and (c) particle-particle channels.

the (1,2) channel, respectively, may be further decomposed[47] into a kernel Λ^{ph}, which consists of diagrams particle-hole irreducible in both channels and the totally two-particle irreducible vertex part $\Lambda^{\rm irr}$ (see Fig. 3). Note that the lowest correction in $\Lambda^{\rm irr}$ to the bare interaction part is already of fourth order in the interaction potential.

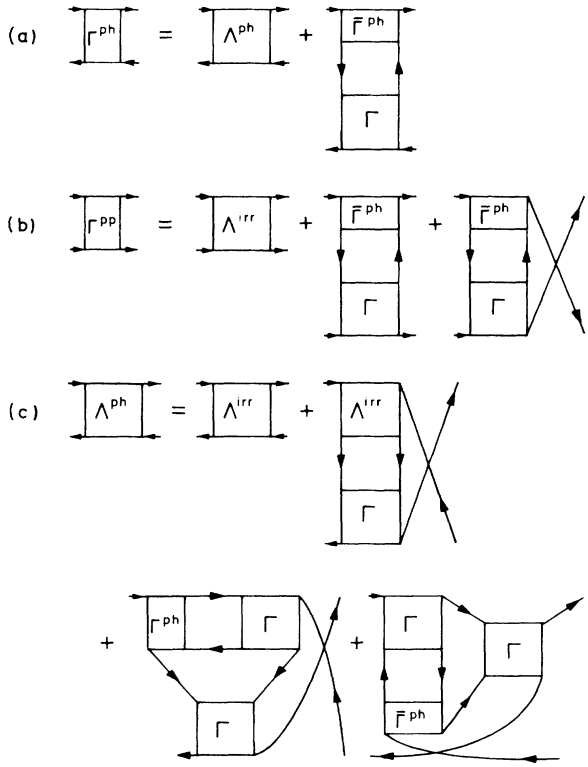


FIG. 3. Representations in terms of irreducible and reducible parts of the partially irreducible vertex functions Γ^{ph} (p-h irreducible) and Λpp (p-h irreducible in both channels). The totally two-particle irreducible vertex function is called $\Lambda^{\rm irr}$.

The various irreducible vertices defined above may be linearly combined to give the following two relations

$$\Gamma=\Gamma^{ph}+\bar{\Gamma}^{ph}-\Lambda^{ph}\ , \tag{11a}$$

$$\Gamma=\Gamma^{pp}+\Lambda^{ph}-\Lambda^{\rm irr}\ . \tag{11b}$$

The vertices Γ^{pp}, Λ^{ph}, and $\Lambda^{\rm irr}$ possess the full exchange symmetry, whereas Γ^{ph} and $\bar{\Gamma}^{ph}$ are connected by

$$\Gamma^{ph}(12;34)=\bar{\Gamma}^{ph}(12;43)=\bar{\Gamma}^{ph}(21;34)\ . \tag{12}$$

The approximation where $\Lambda^{\rm irr}$ is replaced by the bare vertex is the so-called parquet approximation[48,49] (the name describes the two-dimensional, area-covering character of the diagrams summed in this class). It is very likely that it would give a satisfactory description of ^{3}He if one were able to solve the three coupled nonlinear integral equations with sufficient accuracy. This remains to be done.

Even if the complete "parquet" equations could be solved, the solution would be purely numerical and the mechanisms at work in a many-fermion system would not reveal themselves easily. It seems therefore worthwhile to try to connect the microscopic description to the phenomenological Landau theory. As shown by Landau[1,39–41] the dimensionless quasiparticle scattering amplitude on the Fermi surface A is given by the vertex function Γ in the limit of zero excitation frequency $\omega_i=0$ and momenta $|\mathbf{p}_i|=p_F$, renormalized by the quasiparticle spectral weight factors z_i as

$$A(\mathbf{p}_1\sigma_1,\mathbf{p}_2\sigma_2;\mathbf{p}_3\sigma_3,\mathbf{p}_4\sigma_4) =N_F(z_1z_2z_3z_4)^{1/2}\Gamma(12;34)\big|_{\substack{\omega_i=0\\ |\mathbf{p}_i|=p_F}}\ . \tag{13}$$

Here $N_F=m^*p_F/\pi^2$ is the density of states. (We employ units where $\hbar=1$.) In order to derive an integral equation for A, which displays explicitly the effect of quasiparticle-quasihole excitation processes (later referred to as p-h processes), one must isolate the quasiparticle contribution of the full particle-hole propagator

$$G_{p''+q/2}G_{p''-q/2}=(GG)^{\rm qp}+(GG)^{\rm inc} \tag{14}$$

in the p-h Bethe-Salpeter equations in Fig. 2. The quasiparticle term has the explicit form

$$(GG)^{\rm qp}_{p''}(q)=\frac{1}{q_0-\varepsilon''_{p+}+\varepsilon''_{p-}} \times\left[(f''_{p+}-\tfrac{1}{2})\delta\left(\omega''+\frac{q_0}{2}-\varepsilon''_{p+}\right)\right. -\left.(f''_{p-}-\tfrac{1}{2})\delta\left(\omega''-\frac{q_0}{2}-\varepsilon''_{p-}\right)\right]\ , \tag{15}$$

where $f_p\equiv f(\varepsilon_p)$ is the Fermi function. In the limit $q_0=0$ considered here (q_0 is the frequency component of the four vector q), performing the ω'' integration in the intermediate state of the Bethe-Salpeter equation [Fig. 2(a)] amounts to replacing ω'' by

$$\varepsilon_{\mathbf{p}''\pm\mathbf{q}/2}\simeq\pm\frac{\mathbf{p}''\cdot\mathbf{q}}{2m}$$

in the arguments of the vertex functions Γ^{ph} and Γ. In

terms of the two particle variables q,r,k one therefore obtains, for example,

$$\Gamma^{ph}\left[\mathbf{q},q=0;\mathbf{r}=\mathbf{p}-\mathbf{p}'',r_0=\pm\frac{\mathbf{p}''\cdot\mathbf{q}}{2m}\ ;\ \mathbf{k}=\mathbf{p}+\mathbf{p}'',k_0=\pm\frac{\mathbf{p}''\cdot\mathbf{q}}{2m}\right].$$

It is seen that the singularities of Γ^{ph} and Γ for small $|\mathbf{r}|, r_0$ are not involved here, because the ratio $r_0/v_F|\mathbf{r}|$ is less than unity, approaching unity only for $|\mathbf{q}|\sim 2k_F$. We shall therefore approximate Γ^{ph} and Γ by their zero-frequency limits. The first p-h Bethe-Salpeter equation may then be written as

$$A^{s,a}_{\mathbf{pp}'}(q)=F^{s,a}_{\mathbf{pp}'}(q)+\int d\varepsilon_{p''}\int\frac{d\Omega_{p''}}{4\pi}F^{s,a}_{\mathbf{pp}''}(q)\chi_{\mathbf{p}''}(q)A^{s,a}_{\mathbf{p}''\mathbf{p}}(q) \tag{16}$$

where $\chi_{\mathbf{p}}(q)$ is defined by

$$\chi_{\mathbf{p}}(q)=\frac{f_{\mathbf{p}+\mathbf{q}/2}-f_{\mathbf{p}-\mathbf{q}/2}}{\varepsilon_{\mathbf{p}+\mathbf{q}/2}-\varepsilon_{\mathbf{p}-\mathbf{q}/2}}\ . \tag{17}$$

It is convenient here to introduce the momentum variables $\mathbf{p}=\frac{1}{2}(\mathbf{p}_1-\mathbf{p}_3)$ and $\mathbf{p}'=\frac{1}{2}(\mathbf{p}_4-\mathbf{p}_2)$, which have the fixed magnitude $|\mathbf{p}|=|\mathbf{p}'|=p_F(1-Q^2/4)^{1/2}$, where $Q=q/p_F$, provided $|\mathbf{p}_i|=p_F$. Thus A and F depend only on two momentum variables, the angle between $\mathbf{p}$ and $\mathbf{p}'$, and the momentum q. This parametrization of A is more convenient than the one in terms of the angles θ between $\mathbf{p}_1$ and $\mathbf{p}_2$ and Φ between $\mathbf{p}_1\times\mathbf{p}_2$ and $\mathbf{p}_3\times\mathbf{p}_4$.[2,3] The dependence on spin is characterized by two amplitudes, which may be chosen as the singlet and triplet amplitudes in either of the three channels. A^s and A^a are the spin-symmetric and spin-antisymmetric amplitudes in the p-h channel, defined by

$$A(\sigma_1\sigma_2;\sigma_3\sigma_4)=A^s\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_3}+A^a\boldsymbol{\tau}_{\sigma_1\sigma_3}\cdot\boldsymbol{\tau}_{\sigma_2\sigma_4}\ , \tag{18}$$

where $\boldsymbol{\tau}$ is the vector of Pauli matrices. The function F that takes the place of the irreducible vertex part Γ^{ph}, but now is irreducible with respect to cutting a quasiparticle-quasihole pair of lines, is equal to the Fermi-liquid interaction in the limit $q\to 0$. The Landau parameters $F_l^{s,a}$ follow by expanding $F^{s,a}_{\mathbf{pp}'}(q)$ in terms of Legendre polynomials P_l

$$F^{s,a}_{\mathbf{pp}'}(q)=\sum_{l=0}^{\infty}F_l^{s,a}(q)P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') \tag{19}$$

as $F_l^{s,a}=F_l^{s,a}(q=0)$.

The particle-hole propagator $\chi_{\mathbf{p}''}(q)$, considered as a function of $|\mathbf{p}''|$ is peaked at the value of $|\mathbf{p}''|=p_F(1-Q^2/4)^{1/2}$ corresponding to the internal momenta $|\mathbf{p}''\pm\mathbf{q}/2|$ lying on the Fermi sphere [see Fig. 4 for a plot of the angle-averaged $\chi_p(q)$]. This suggests the approximation of taking A and F out of the $\varepsilon_{p''}$-integral in [Fig. 2(a)] and setting $|\mathbf{p}''\pm\mathbf{q}/2|=p_F$. The remaining integral of $\chi_{\mathbf{p}''}(q)$ over $|\varepsilon_{p''}|$ turns out to be a weak function of $\hat{\mathbf{p}}''\cdot\hat{\mathbf{q}}$. As shown in Fig. 5 the coefficients $\chi_n(Q)$ defined by

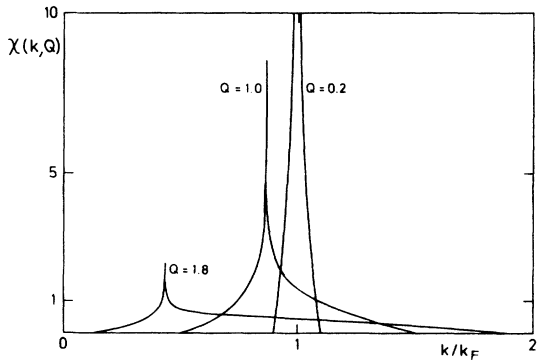


FIG. 4. Particle-hole susceptibility $\chi_{\mathbf{k}}(q)$ [defined in Eq. (17)] averaged over the directions of $\mathbf{k}$ as a function of $k=|\mathbf{k}|$ for various values of $Q=q/k_F$.

$$\chi_n(Q)=-\frac{1}{m^*p_F}\int_0^{\infty}dp\,p^2\int_0^1 d(\cos\Phi)(\sin^{2n}\Phi)\times\frac{f(\varepsilon_{p+q/2})-f(\varepsilon_{p-q/2})}{\varepsilon_{p+q/2}-\varepsilon_{p-q/2}}, \tag{20}$$

where $\cos\Phi=\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}$, vary little with n, in particular so when

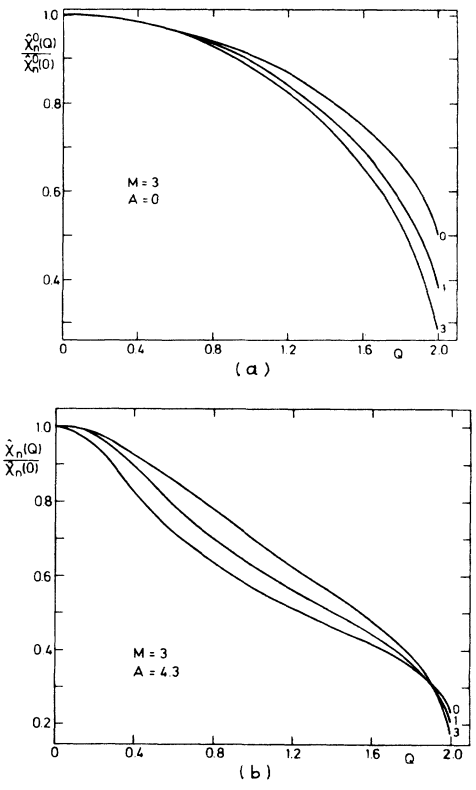


FIG. 5. (a) and (b) Different angular averages of the energy integrated p-h susceptibility $\chi_{\mathbf{p}}(q)$ as defined in Eq. (20) using the energy expression (21) with $M\equiv m^*/m=3$ and $A\equiv\varepsilon_F^0/\varepsilon_c=0$ (a) and $A=4.3$ (b).

the wiggle in the quasiparticle spectrum is taken into account. We may therefore approximate this integral by the lowest coefficient $\chi_0(Q)$. In evaluating $\chi_n(Q)$ we have used the following parametrization of the quasiparticle energy[50,28–30]

$$\varepsilon_p=\frac{p^2-p_F^2}{2m}+\varepsilon_c\{[u_p+(u_p^2+v_p^3)^{1/2}]^{1/3}+[u_p-(u_p^2+v_p^3)^{1/2}]^{1/3}-2z_p\}\ , \tag{21}$$

where

$$u_p=z_p^3-\frac{1}{2}z_p\left[\frac{m^*}{m}-1\right]\ ,$$
$$v_p=\frac{1}{3}\frac{m^*}{m}-z_p^2\ , \tag{22}$$
$$z_p=(\varepsilon_p^0-\varepsilon_F)/3\varepsilon_c\ .$$

The form (21) follows from a model of the imaginary part of the self-energy given by

$$\mathrm{Im}\Sigma(E)=\left[\frac{m^*}{m}-1\right]\frac{E^2}{E^2+\varepsilon_c^2}\varepsilon_c\ , \tag{23}$$

where m^* is the effective mass at the Fermi surface and ε_c is an energy cutoff. Plots of ε_k for typical values of m^*/m and ε_c are shown in Fig. 6.

Employing the above simplifications, the integral equation (16) for A may be solved with the result

$$A_l^{s,a}(q)=\frac{F_l^{s,a}(q)}{1+\chi_0(q)F_l^{s,a}(q)/(2l+1)}\ . \tag{24}$$

This reduces to the well-known relation of the quasiparticle scattering amplitude to the Landau parameters in the limit $q\to 0$, where $\chi_0\to 1$. For quadratic energy spectrum,

$$\varepsilon_k=\frac{k^2}{2m^*}-\mu\ ,$$

the p-h susceptibility $\chi_0(q)$ is identical to the normalized static Lindhard function, which is given by

$$\chi_0(Q)=\tfrac{1}{2}\left[1+\frac{1}{Q}(1-Q^2/4)\ln\left|\frac{1+Q/2}{1-Q/2}\right|\right]\ . \tag{25}$$

Equation (24) might make one believe that any chosen model form for $F_l^{s,a}(q)$ would lead to a suitable QSA. This point of view was to some extent adopted by Bedell and Pines[18,19] who chose to identify $F_0^{s,a}(q)$ and $F_1^s(q)$ with the density, spin density, and backflow polarization potentials of Aldrich and Pines.[16,17] Such an approach is problematic, however, because the property of exchange symmetry obeyed by A imposes strong conditions on the possible q dependence of $F_l(q)$. This is clearly seen if one writes down the explicit expressions for the exchanged quantities, in the representation of the variables $Q=q/p_F$ and $\mu=(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$:

$$\bar{F}^{s,a}(\mu,Q)=\tfrac{1}{2}[F^s(\bar{\mu},\bar{Q})+m^{s,a}F^a(\bar{\mu},\bar{Q})]\ , \tag{26}$$

where $m^s=3$, $m^a=-1$, and the transformed variables are given by

$$\bar{Q}=[2(1-Q^2/4)(1-\mu)]^{1/2}\ ,$$
$$\bar{\mu}=[(1-Q^2/4)(1+\mu)-Q^2/2]/[(1-Q^2/4)\times(1+\mu)+Q^2/2]\ . \tag{27}$$

Here and in the following we denote exchange transformed quantities by an overbar.

From the exchange symmetry of the spin singlet and triplet components of the QSA, A^0 and A^1,

$$A^{0,1}(\mu,Q)=\pm A^{0,1}(\bar{\mu},\bar{Q}) \tag{28}$$

and the relations among the different spin representations

$$A^0=A^s-3A^a\ ,$$
$$A^1=A^s+A^a\ , \tag{29}$$
$$A^s=\tfrac{1}{4}(A^0+3A^1),\quad A^a=\tfrac{1}{4}(A^1-A^0)$$

one obtains the following relation among A^s and A^a

$$A^{s,a}(\mu,Q)=-\tfrac{1}{2}[A^s(\bar{\mu},\bar{Q})+m^{s,a}A^a(\bar{\mu},\bar{Q})]=-\bar{A}^{s,a}(\mu,Q)\ . \tag{30}$$

In the forward scattering limit $\mathbf{p}_1=\mathbf{p}_2=\mathbf{p}_3=\mathbf{p}_4$ and consequently $q=0$, $\mu=1$, the triplet QSA vanishes by virtue of the Pauli principle. This gives rise to the sum rule

$$A^1(\mu=1,Q=0)=\sum_{l=0}^{\infty}(A_l^s+A_l^a)\equiv 0\ . \tag{31}$$

Also, the μ dependence of A at $Q=0$ and the Q dependence at $\mu=1$ are related by

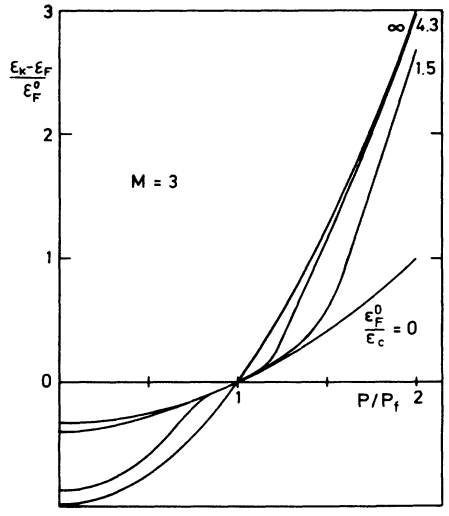


FIG. 6. Quasiparticle energy ε_p as a function of momentum according to (21) for $M\equiv m^*/m=3$ and different values of the energy cutoff ε_c.

$$A^{s,a}(\mu,Q=0)=-\tfrac{1}{2}[A^s(\mu=1,\bar{Q})+m^{s,a}A^a(\mu=1,\bar{Q})]\ , \tag{32}$$

where $\bar{Q}=\sqrt{2(1-\mu)}$.

The above relations show that the dependence on μ and Q is strongly correlated. In particular, for any given ansatz for the Landau interaction $F^{s,a}(\mu,Q)$ in the integral equation (16), the resulting QSA will in general not be exchange symmetric. It is clear that the exact property of exchange symmetry should be exploited to determine A more precisely. This can only by done by calculating A and F simultaneously in an exchange symmetry conserving fashion, and by introducing approximations to a less sensitive quantity. The ideal quantity to approximate is the totally irreducible vertex $\Lambda^{\rm irr}$. This would amount to solving the parquet equations, which is outside the range of our present capability. On a less complicated level one may try to parametrize the particle-hole irreducible vertex Λ^{ph}, or rather its quasiparticle-quasihole irreducible counterpart (including the renormalization factors z_i). This we call the direct interaction $I^{s,a}_{\rm pp'}(q)$, following Babu and Brown,[11] because it does not contain quasiparticle polarization contributions. The direct interaction is related to A and F by the following linear relation derived from Fig. 3(a)

$$A^{s,a}(\mu,Q)=F^{s,a}(\mu,Q)-\bar{F}^{s,a}(\mu,Q)-I^{s,a}(\mu,Q)\ . \tag{33}$$

The exchange transformed quantity $\bar{F}$ is given by (25). Note that I is exchange symmetric. Any exchange symmetric approximation to I yields automatically an exchange symmetric QSA.

III. DIRECT INTERACTION MODEL: ANALYTIC RESULTS

The direct interaction $I^{s,a}(\mu,Q)$ does not contain any of the singular contributions (in the limit $\omega,q\to 0$) caused by quasiparticle-quasihole excitation processes near the Fermi surface. It has been argued that I^s may be identified with the effective interaction derived in CBF calculations.[14] There it has been found[14] that the effective interaction is predominantly a local interaction. The corresponding local potential $v^s(Q)$ is shown in Fig. 7. The nonlocal contribution, part of which is an exchange interaction, has not been calculated yet. Before presenting the results of a numerical solution of (16) and (33) using the potentials $V^s(Q)\equiv N_F v^s(Q)$ to approximate $I(\mu,Q)$, it is instructive to solve the equations in the case where $I(\mu,Q)$ is considered to be a phenomenological potential. This will be considered in the following for the simplest case of pure s-wave interaction and in the somewhat more realistic case of s- and p-wave components.

The solution of (16) and (33) is greatly facilitated by using the eigenfunctions of the exchange operator derived recently.[31] While the transformation laws (27) describing the behavior of the variables μ and Q under exchange appear to be prohibitively complicated, one may in fact use a different set of variables z and y, defined by

$$\begin{aligned} z&=Q^2/2-1=-\hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_3\ ,\\ y&=\mu(1-z)=(\hat{\mathbf{p}}_1+\hat{\mathbf{p}}_3)\cdot\hat{\mathbf{p}}_4\ , \end{aligned} \tag{34}$$

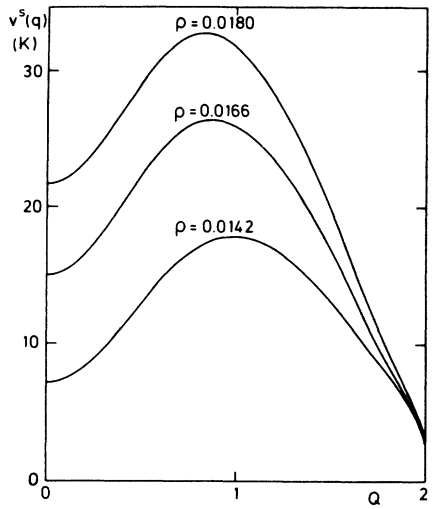


FIG. 7. Local direct interaction potential $v^s(q)$ vs $Q=q/k_F$ as calculated by Krotscheck (Ref. 14), for three values of particle density ρ (in Å^{-3}).

in terms of which the transformation laws become linear:

$$\begin{bmatrix}\bar{z}\\ \bar{y}\end{bmatrix}=\frac{1}{2}\begin{bmatrix}-1 & -1\\ -3 & 1\end{bmatrix}\begin{bmatrix}z\\ y\end{bmatrix}+\frac{1}{2}\begin{bmatrix}-1\\ -1\end{bmatrix}\ . \tag{35}$$

This fact may be exploited to construct polynomial eigenfunctions of the exchange operator $\hat{X}$, defined by

$$\hat{X}X_{lk}(\mu,Q)=X_{lk}(\bar{\mu},\bar{Q})=(-1)^l X_{lk}(\mu,Q)\ , \tag{36}$$

for $l=0,1,\ldots,k$ and $k=0,1,\ldots$. The eigenfunctions are explicitly given by

$$X_{lk}(\mu,Q)=\sum_{l'=0}^{k} D^k_{ll'}Y_{l'k}(\mu,Q)(-1)^{l+l'} \tag{37}$$

where the $D^k_{ll'}$ are known coefficients[31] and

$$Y_{lk}(\mu,Q)=P_l(\mu)Z_{lk}(Q)\ . \tag{38}$$

Here $P_l(\mu)$ are the Legendre polynomials and Z_{lk} is given by

$$Z_{lk}(Q)=\sqrt{k+1}\sqrt{2l+1}(1-Q^2/4)^l P^{(2l+1,0)}_{k-l}(Q^2/2-1)\ . \tag{39}$$

The $P^{(a,b)}_n(x)$ are the Jacobi polynomials of degree n and indices a,b.[51] The first few polynomials sufficient for $l,k\le 3$ are given by

$$\begin{aligned} P^{(a,0)}_0(x)&=1\ ,\\ P^{(a,0)}_1(x)&=\tfrac{1}{2}[(a+2)x+a]\ ,\\ P^{(1,0)}_2(x)&=\tfrac{1}{2}(5x^2+2x-1)\ ,\\ P^{(1,0)}_3&=\tfrac{1}{8}(35x^3+15x^2-15x-3)\ ,\\ P^{(3,0)}_2&=\tfrac{1}{4}(21x^2+18x+1)\ . \end{aligned} \tag{40}$$

Later we will need the property

$$P_n^{(a,0)}(-1)=(-1)^n \ . \tag{41}$$

The coefficients $D_{ll'}^k$ for fixed k may be considered as elements of real, symmetric and orthogonal matrices $\underline{D}^k$, i.e.,

$$\sum_{l''=0}^{k} D_{ll''}^k D_{l''l'}^k = \delta_{ll'} \ . \tag{42}$$

The first few D's are given by

$$D^0=1 \ ,$$

$$D^1=\frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix} , \tag{43}$$

$$D^2=\frac{1}{3}\begin{bmatrix} 1 & \sqrt{3} & \sqrt{5} \\ \sqrt{3} & \frac{3}{2} & -\frac{1}{2}\sqrt{15} \\ \sqrt{5} & -\frac{1}{2}\sqrt{15} & \frac{1}{2} \end{bmatrix} .$$

The functions X_{lk} as well as the functions Y_{lk} are orthogonal with respect to integration over z and y, i.e.,

$$\frac{1}{4}\int_0^2 dQ^2 \int_{-1}^{1} d\mu (1-Q^2/4) X_{lk}(\mu,Q) X_{l'k'}(\mu,Q) = \delta_{kk'}\delta_{ll'} \tag{44}$$

and analogously for Y_{lk}. Both sets of functions, $\{X_{lk}\}$ and $\{Y_{lk}\}$, form a complete basis in (z,y) space or equivalently in (μ,Q) space.

We may therefore expand $A^{s,a}(\mu,Q)$, $F^{s,a}(\mu,Q)$, and $I^{s,a}(\mu,Q)$ in terms of these basis functions. Expansion in Y_{lk} has the advantage that equation (16), which is nonlinear in A and F, becomes diagonal in the index l. On the other hand, expansion in X_{lk} renders equation (33) trivial, and guarantees exchange symmetry of the QSA at all levels of approximation. For the following we choose the expansion in the basis set $\{Y_{lk}\}$, i.e.,

$$A^{s,a}(\mu,Q)=\sum_{l,k} A_{lk}^{s,a} Y_{lk}(\mu,Q) \tag{45}$$

and analogously for $F^{s,a}(\mu,Q)$ and $I^{s,a}(\mu,Q)$. Substituting these expansions into (16) and (33), employing the orthogonality property (44) of the Y_{lk}'s and the behavior of Y_{lk} under exchange

$$Y_{lk}(\bar{\mu},\bar{Q})=\sum_{l'=0}^{k} D_{ll'}^k Y_{l'k}(\mu,Q) \ , \tag{46}$$

one finds the following set of equations for the expansion coefficients

$$\begin{aligned} A_{lk}^{s,a} &= F_{lk}^{s,a} - \sum_{k',k''=l}^{\infty} F_{lk'}^{s,a} \chi_{kk'k''}^{l} A_{lk''}^{s,a} \ , \\ A_{lk}^{s,a} &= F_{lk}^{s,a} - \frac{1}{2}\sum_{l'=0}^{k} D_{ll'}^k (F_{l'k}^{s} + m^{s,a} F_{l'k}^{a}) - I_{lk}^{s,a} \ . \end{aligned} \tag{47}$$

Here the matrix elements of the p-h propagator are defined by

$$\chi_{kk'k''}^{l}=\frac{1}{(2l+1)^2}\int_0^2 dQ\, Q(1-Q^2/4)\chi_0(Q) \times Z_{lk}(Q) Z_{lk'}(Q) Z_{lk''}(Q) \ . \tag{48}$$

For quadratic energy spectrum the expression (25) for $\chi_0(Q)$ may be inserted. The first few matrix elements ($k=0,1$) are then given analytically by

$$\begin{aligned} \chi_{000}^0 &= (2+16\ln 2)/15 \ , \\ \chi_{001}^0 &= (\sqrt{2}/105)(37-64\ln 2) \ , \\ \chi_{011}^0 &= (-4+128\ln 2)/105 \ , \\ \chi_{111}^0 &= (\sqrt{2}/1155)(907-1024\ln 2) \ , \\ \chi_{111}^1 &= (\sqrt{6}/6237)(-565+3072\ln 2) \ . \end{aligned} \tag{49}$$

The Landau parameters follow from the coefficients $F_{lk}^{s,a}$ in straightforward way as

$$F_l^{s,a}=\sum_{k=l}^{\infty} F_{lk}^{s,a} Z_{lk}(0) \ , \tag{50}$$

where

$$Z_{lk}(0)=\sqrt{k+1}\sqrt{2l+1}(-1)^l \ ,$$

using (39).

As seen in Fig. 7 the local part of the direct interaction $V^s(Q)$ is large and positive over the whole range of momentum values of interest here. As a first approximation we may therefore put $V^s(Q)$ equal to a constant $V^s(Q)\simeq V_0$ and neglect the nonlocal part altogether. This leads to only a $k=0$ component of the direct interaction, or $I_{00}^s=-I_{00}^a=\frac{1}{2}V_0$. In the limit $I_0\to\infty$ some of the coefficients are bound to scale with V_0, whereas all the others are approximately independent of V_0 and scale to limiting values of order unity. This is seen from the $l=0$, $k=0$ components of (33), which are explicitly given by

$$A_{00}^s=\tfrac{1}{2}F_{00}^s-\tfrac{3}{2}F_{00}^a-\tfrac{1}{2}V_0=-A_{00}^a \ . \tag{51}$$

A large negative contribution from V_0 on the right-hand side can only be compensated by a corresponding large value of F_{00}^s:

$$F_{00}^s\to V_0, \quad V_0\to\infty \ . \tag{52}$$

This is so because the scattering amplitude is bounded from above by unitarity, which does not allow A_{00}^s to become much larger than unity (unless other components A_{lk} grow large as well, which may be excluded here) and the Landau parameters $F_l^{s,a}$ are bounded from below by the Landau stability criterion $F_l^{s,a}>-(2l+1)$, which excludes large negative F_{00}^a. In fact in the approximation where components with $k,l\neq 0$ are neglected, F_{00}^a is found as

$$F_{00}^a\to -\frac{1}{2\chi_{000}^0}, \quad V_0\to\infty \ . \tag{53}$$

This is an important result. It implies that for strongly repulsive direct interaction ("Hubbard repulsion") the spin-symmetric part of the Landau interaction scales with

the interaction, implying strong screening of density fluctuations. The exchange part of the Landau interaction, however, tends to a negative value of order unity, independent of the strength of the direct interaction. This implies a moderate enhancement of spin fluctuations. The precise value of the Landau parameter F_0^a in our model is determined by the single-particle spectrum through the quantity χ^0_{000}. There are also corrections to F_0^a via (50) from higher k channels as seen from (47a). This in turn implies the generation of higher l components by equation (47b), which is diagonal in k, but mixes different l's, where $l \leq k$. All of these contributions are independent of V_0 in the limit $V_0 \to \infty$ and decrease rapidly with increasing k,l.

A first indication of this universal behavior of a Fermi system with strongly repulsive direct interaction can be found already in the work of Babu and Brown.[11] A recent evaluation of the Babu-Brown equations appears to give results for F_0^s and F_0^a compatible with ours.[32–34] Very similar results[52,53] are obtained within a lattice-gas description of liquid ^{3}He based on a Hubbard model with strong on-site repulsion U. There the parameter U appears to play the role of the direct interaction.

It is clear that the large values of the Landau parameter F_1^s for liquid ^{3}He cannot be obtained in an s-wave model. This parameter describes the effect of backflow, which requires at least p waves in the model. We therefore study as an extension of the above model an s-p-wave model for the direct interaction. This introduces three interaction parameters $V_0^s = -V_0^a, V_1^s, V_1^a$, which we choose to define as

$$I^{s,a}(\mu,Q) = V^{s,a}(Q) - \bar{V}^{s,a}(\mu,Q) \ , \tag{54a}$$

where

$$V^{s,a}(Q) = \sum_{k=0}^{k_{\max}} V_k^{s,a} Y_{0k}(0,Q) \ , \tag{54b}$$

and $k_{\max}=1$ in this case. In other words, $V^{s,a}(Q)$ is a linear expression in Q^2 and $I^{s,a}$ is a bipolynomial of degree one in Q^2 and μ. Equations (54a) and (54b) give the most general parametrization of $I(\mu,Q)$ in first order in μ and Q^2, but not in higher order.

Substituting (54a) and (54b) with $k_{\max}=1$ into (47a) and (47b) one finds in the limit $V_0^s \gg (\,|V_1^s|\,), V_1^a \gg 1$ that F^s_{00}, F^s_{01}, and F^s_{11} scale to large values as

$$F^s_{00} = V_0^s; F^s_{01} = V_1^s; F^s_{11} = \sqrt{3} V_1^a \ . \tag{55}$$

The remaining coefficients saturate at values of order unity. In the approximation where one keeps only the eigenfunctions

$$Y_{00} = 1, Y_0 = [1 - 3(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_3)]\sqrt{2} \tag{56}$$

and

$$Y_{11} = \sqrt{3/2}(\hat{\mathbf{p}}_1 + \hat{\mathbf{p}}_3)\cdot\hat{\mathbf{p}}_4 \ ,$$

and for quadratic energy spectrum the values of the exchange parameters are obtained as $F_0^a = -0.9$ and $F_1^a = -1.65$. These values are subject to corrections from higher (l,k) channels.

The remarkable feature of this result is that the Landau parameter F_1^s is driven by the exchange part, or more generally, the nonlocal part of the effective interaction, i.e.,

$$F_1^s = \sqrt{6} F^s_{11} = 3\sqrt{2} V_1^a; V_1^a \gg 1 \tag{57}$$

and not by the p-wave part of the local potential, or V_1^s, as one might have guessed. The importance of finite range interactions for obtaining large F_1^s was demonstrated earlier in Refs. 32 and 33. The above discussion shows, however, that a local potential of finite range (corresponding to the parameter V_1^s) is not sufficient. Rather, the direct interaction must contain a velocity-dependent contribution (which is equivalent to a spin-exchange term, expressed by the parameter V_1^a, in the s-p approximation). In this respect we disagree with the conclusions of Bedell and Quader,[34] who find F_1^s as well as F_1^a to be proportional to V_1^s and hence large. Their result is an artifact of the assumed independence of F and A on q in the crossed p-h channel. In its original form the Babu-Brown approach is not capable of dealing properly with exchange symmetry. Even the qualitative behavior of the scattering amplitude is given incorrectly in cases where exchange symmetry for large q is important, as for instance in the vicinity of a phase transition or instability.

As already mentioned, the nonlocal part of the direct interaction has not been calculated in microscopic theory so far. The values of V_1^a required to bring F_1^s in agreement with experiment are such that $V_1^a \approx 0.1 V_0^s$, implying that the exchange part of I would be 10–20 % of the local part. This is well inside estimates based on CBF calculations.[37]

A reasonable idea as to where these nonlocal contributions to I, which we now call I^{nonloc}, will have to come from in a microscopic theory can be obtained from the representation of I in terms of p-p irreducible and reducible diagrams given in Fig. 3(c). It is very likely that I^{nonloc} originates from the quasiparticle contribution of the last two terms in Fig. 3(c). We may evaluate those approximately by replacing Γ^{ph} by F, Γ by A, and the Green's functions by their quasiparticle parts [see Fig. 3(c)]. The dominant contribution is coming from the $l=0$ channel of $F^s(\mu,Q)$. For simplicity we replace A by its s-wave part A_0^s as well. Then the frequency sums over the four Green's functions may be performed with the result

$$\begin{aligned}\Lambda_{\mathbf{p}_1\mathbf{p}'}(q) &= \sum_{p'',k} G_{p''} G_{p''+k} G_{p+q+k} G_{p'-k} \\ &= \frac{2}{N_F} \sum_{\mathbf{k}} [(1-f_1)\Pi(-\varepsilon_1,k) - f_1 \Pi(\varepsilon_1,k) \\ &\quad + (1-f_2)\Pi(-\varepsilon_2,k) \\ &\quad - f_2 \Pi(\varepsilon_2,k)]/(\varepsilon_1+\varepsilon_2)\end{aligned} \tag{58a}$$

where $\varepsilon_1 \equiv \varepsilon_{\mathbf{p}+\mathbf{q}+\mathbf{k}}$, $\varepsilon_2 = \varepsilon_{\mathbf{p}'-\mathbf{k}}$, and $f_{1,2} = f(\varepsilon_{1,2})$ is the Fermi function. Here $\Pi(\omega,k)$ is the particle-hole correlation function defined by

$$\Pi(\omega,k) = \frac{2}{N_F} \sum_{\mathbf{p}} f_p (1 - f_{\mathbf{p}+\mathbf{k}})/(\omega - \varepsilon_{\mathbf{p}+k} + \varepsilon_{\mathbf{p}}) \ . \tag{58b}$$

It is remarkable that the ultraviolet divergence in the p-p channel is cut off by the p-h susceptibility π in (57), and not by the $|\mathbf{p}|$ dependence of the scattering matrix as usual. This fact allows us to evaluate the diagram in the quasiparticle picture in the first place. It also gives confidence to the above treatment, which may be viewed as a perturbative evaluation of the contribution in Fig. 3(c) rather than an attempt to solve the complete integral equation Fig. 3(c).

A rough estimate of the quantity Λ defined in (57) may be obtained by replacing $\Pi(\pm\varepsilon_{1,2},k)$ by $\Pi(0,k)$, using the fact that the average of $\varepsilon_{1,2}$ in a region about the Fermi surface is small, and hence:

$$\Lambda_{pp'}(q)\simeq\Lambda^0_{pp'}(k)\equiv\frac{2}{N_F}\sum_{\mathbf{k}}\frac{1-f_1-f_2}{\varepsilon_1+\varepsilon_2}\Pi(0,k)\ . \tag{59}$$

The quantity Λ^0 is the p-p susceptibility, with the k-integral cutoff at $k\gtrsim 2k_F$ by the p-h structure factor $\pi(0,k)$. Nonetheless, the k summation in Λ^0 extends over regions far from the Fermi surface due to the combination of Fermi factors $(1-f_1-f_2)$ in contrast to the $\mathbf{p}$ sum in the p-h quantities $\pi(\omega,k)$ or $\chi_0(Q)$ defined in (58) and (20). There the region in $\mathbf{p}$ space around the Fermi surface is always included in the $\mathbf{p}$ summation. The effective-mass enhancement in the vicinity of the Fermi surface will therefore be less important in the p-p quantity Λ than in Π or χ. We may estimate this effect by replacing the enhanced mass spectrum $\varepsilon_{\mathbf{p}}\cong p^2/2m_p^*$ by the bare spectrum $\varepsilon_p^0=p^2/2m$ in evaluating Λ^0. This produces a factor m/m^* due to the density of states prefactor N_F^{-1} in the definition of Λ Eq. (59).

The contribution of I^{nonloc} to the parameter V_1^a is now readily estimated using the relation

$$I^s_{11}-I^a_{11}\equiv I^{\uparrow\downarrow}_{11}=\sqrt{3}V_1^a\ . \tag{60}$$

Here $I^{s,a}_{11}$ is the coefficient of $Y_{11}(\mu,Q)$ in the expansion (45) of the direct interaction. One finds

$$V_1^a=\frac{1}{\sqrt{3}}\frac{m}{m^*}F_0^s\bar{A}^{\,2}\tilde{\Lambda}^{\,0}_{11}\ , \tag{61}$$

where

$$\tilde{\Lambda}^{\,0}_{11}=4\int_0^2 dQ^2(1-Q^2/4)\int_{-1}^{1}d\mu\,A_{11}(\mu,Q)\tilde{\Lambda}^{\,0}(\mu,Q) \tag{62}$$

and $\tilde{\Lambda}^{\,0}$ is given by the expression (59) for $\tilde{\Lambda}^{\,0}$ evaluated for the Fermi gas, i.e., setting $\varepsilon_p\to\varepsilon_p^0$ and $N_F\to N_F^0$. A numerical evaluation yields $\Lambda^0_{11}\simeq 0.8$. The quantity $\bar{A}$ in (61) denotes an average value of the scattering amplitude, which is of order unity. Thus it follows from (61) that V_1^a is proportional to $m/m^*F_0^s$, and hence, using (56),

$$F_1^s\propto\frac{m}{m^*}F_0^s\ . \tag{63}$$

This relation is obeyed fairly well by the experimental values for liquid ^{3}He, except at pressures below about 8 bar, when F_1^s becomes less than ~ 6. This would imply $V_1^a\sim 1$ and it is not surprising that the above expansion in terms of $1/V_1^a$ ceases to be valid in this case.

A relation similar to (63) has been found within a lattice-gas description of liquid ^{3}He based on the Gutzwiller solution to the Hubbard model for a half-filled band.[53] One may show that the approximate relation $F_0^s\propto(m^*/m)^2$ obtained there depends sensitively on the filling of the band. Even a slight deviation from exact half filling tends to modify the relation to $F_0^s\propto(m^*/m)$. In our case the relation (63) is a consequence of the assumption that the momentum-dependent effective mass decreases rapidly as one goes away from the Fermi surface. The interrelationship of the two models is not clear at present.

What can be said, however, is that in our theory a large value of the effective-mass ratio m^*/m does indeed give rise to the structure of the effective interaction discussed above. Namely, the dimensionless direct interaction $I_{pp'}(q)$ is the physical (dimensional) interaction function $i_{pp'}(q)$ multiplied by the density of states $N_F\propto m^*/m$. Thus for given $i'_{\text{pp}}(q)$ the dimensionless quantity $I'_{pp}(q)$ grows linearly with m^*/m and one may reach the limit where the coefficients $I^{s,a}_{00},I^{s,a}_{01}$ are indeed much greater than unity. The fact that m^*/m may grow large or may even diverge follows from the effective-mass relation if expressed by the dimensional Landau parameter f_1^s:

$$\frac{m^*}{m}=1+\tfrac{1}{3}N_Ff_1^s=\frac{1}{1-\frac{1}{3}N_F^0f_1^s}\ .$$

For $f_1^s\to(\frac{1}{3}N_F^0)^{-1}$, where N_F^0 is the density of states of the noninteracting Fermi gas, the effective mass is actually seen to diverge. This particular behavior of m^* has also been discussed in Ref. 34. It is tempting to connect this divergence to the analogous behavior found in the lattice-gas description,[53] where $m^*/m\propto(1-U/U_c)^{-1}$. The analogy suggests that f_1^s is related to the Hubbard repulsion U, which makes sense in a picture where the backflow around a quasiparticle is envisaged as being created mainly by the hardcore pushing aside particles in its way. It would be interesting to quantify this conjecture.

IV. NUMERICAL EVALUATION OF THE DIRECT INTERACTION MODEL

We now turn to the results of a numerical evaluation of the model discussed in the preceding section. As a principal input we use the local part $v^s(Q)$ of the direct interaction calculated by Krotscheck, shown in Fig. 7. The potential $v^s(Q)$ is available for three densities only, $\rho=0.0142$, 0.0166, and 0.0180 Å^{-3}. The first value of ρ corresponds to the minimum of the ground-state energy at zero external pressure P. The experimental density at $P=0$ is more like $\rho=0.0166$. We will consecutively take the first and the second value of ρ as representing the equilibrium density. The value $\rho=0.0180$ Å^{-3} then corresponds to an applied pressure of 12 and 3 bar, respectively.

A perfect fit of the potential curves $V^s(Q)$ was obtained by the expansion (54b) using $k_{\max}=6$. The equations (47a) and (47b) were solved using an iteration procedure. The resulting first few Landau parameters for $\rho=0.0166$ and taking the value of the ratio density of states N_F to

particle density n to be $(N_F/k_B n)\simeq 0.84\ \mathrm{K}^{-1}$ are given by $F_0^s=14.0$, $F_0^a=-0.533$, $F_1^s=0.303$. As expected, the backflow parameter F_1^s turns out to be much too small. This is in contradiction to the result of Bedell and Quader[34] based on the Babu-Brown equations, who find F_1^s to be proportional to V_1^s. Their result is an artifact of the assumed q independence of F and A in the crossed p-h channel.

In order to explore the quantitative influence of a nonlocal part in the direct interaction we have added a three-parameter model expression for $V^a(Q)$,

$$V^a(Q)=V_0^a+V_1^a Y_{01}(Q)+V_2^a Y_{02}(Q) \tag{64}$$

to the local direct interaction $V^s(Q)$ calculated by Krotscheck.[14,37] The solution was calculated for various forms of single-particle spectrum (22) using the experimental value of m^* at the Fermi surface and several values of the parameter ε_c. The resulting scattering amplitude was employed to calculate the transport relaxation times for viscosity τ_η, spin diffusion τ_D, thermal conductivity τ_κ and the quasiparticle relaxation time on the Fermi surface τ_N^0 from the exact expression in Refs. 54 and 55. As already mentioned, the theoretically determined equilibrium density does not agree with the experimental density at $P=0$. We have therefore tried both possible identifications of the zero pressure state: $\rho=0.0142$ and $0.0166\ \text{Å}^{-3}$.

The results of these calculations are collected in Table I. In the last line, the experimental data[56–61] based on Greywall's effective-mass values[56] are given for reference. In the first line, the Landau parameters resulting from the purely local direct interaction are shown. The next two lines show the result of a calculation, where only the parameter V_1^a has been chosen to be nonzero such that F_1^s is reproduced. This is shown for two values of the width of the wiggle in the energy spectrum: no wiggle ($\varepsilon_c=0$) and wiggle as calculated in Ref. 24 ($\varepsilon_c=0.23\varepsilon_F^0$). It appears as if a slightly larger value of ε_c, $\varepsilon_c\simeq 0.4\varepsilon_F^0$ would reproduce the experimental data more accurately. This is shown in the last two lines of Table I, where the agreement with experiment is good. The pair-interaction parameters G_l for $l=0,1,2$ are also shown in the table. Strongest attraction is always found for p waves, although the actual values of G_l vary quite strongly. We do not attempt to discuss the transition temperature, as the present approach does not allow for a determination of the cutoff energy entering the T_c expression. Ainsworth and Bedell,[36] using a phenomenological parametrization of the direct interaction and a quadratic energy spectrum report somewhat better agreement than we obtain here. Neither Ainsworth and Bedell[36] nor we do as well as did Bedell and Pines[18,19] in their phenomenological calculation of transport properties, the density dependence of the superfluid transition, and the strong-coupling corrections.

In Fig. 8 the direct interaction potentials $V^s(Q)$ and $V^a(Q)$ employed at $P=0$ and $\rho=0.0142\ \text{Å}^{-3}$ are shown. The Landau interaction functions $F_0^{s,a}(Q)$ and $F_1^{s,a}(Q)$ obtained with the direct interaction of Fig. 8 and the value $\varepsilon_F^0/\varepsilon_c=4.3$ are plotted in Fig. 9. It is seen that the spin-symmetric function $F_0^s(Q)$ is close in shape to the input potential $V^s(Q)$, although somewhat larger. The function $F_0^a(Q)$, on the other hand, develops a hump around $Q\approx 1$, where $F_0^a\sim -0.95$, unlike $V^a(Q)$. This may be taken as an indication of a tendency to antiferromagnetic rather than ferromagnetic order. Also, an increase in F_0^a with Q may mean that the spin-density molecular field correction is more important in regions of k space farther away from the Fermi surface, i.e., for excited states, than it is right at the Fermi surface. In this way the rapid drop of the density of states as a function of energy, which is clearly seen in the specific heat at temperatures above ~ 100 mK, may be reconciled with the rather smooth behavior of the spin susceptibility as a function of temperature: The rapid decrease of the effective mass in the momentum range $0.5k_F\lesssim k\lesssim 1.5k_F$ may be compensated to some extent by the enhancement of the Landau interaction function $F_0^a(Q)$. In fact the combination of parameters $[1+\frac{1}{3}F_1^s(Q)]F_0^a(Q)$, where the first factor is meant to describe the Q dependence of the effective mass, is found to stay nearly constant up to $Q\simeq 1.2$. This would also explain why the incoherent neutron-scattering cross section appears to call for a much less momentum-dependent effective mass.[16]

According to Bedell and Pines[18] the functions $F_0^s(Q)$ and $F_0^a(Q)$ should be identified with the density and spin-density polarization potentials of Aldrich and Pines.[16] In fact, comparison shows this to be satisfied reasonably well. The $l=1$ interaction functions $F_1^s(Q)$ and $F_1^a(Q)$ decrease monotonically with Q to zero at $Q=2$. The latter behavior is a consequence of the factor $1\text{-}Q^2/4$ appearing in all the $l=1$ eigenfunctions Y_{lk}. Comparison with the backflow polarization potential $f_v(q)$ of Aldrich and Pines, assuming the relation between $f_v(q)$ and $F_1^s(Q)$ proposed by Bedell and Pines, shows that our $F_1^s(Q)$ drops much faster with Q. Whether this is due (i) to the assumed relationship to be incorrect, (ii) our input direct interaction to be incomplete, or (iii) the determination of $f_v(q)$ by the fit to neutron-scattering data to be uncertain, is not known at present.

V. CONCLUSION

In this paper we have presented a general framework for the calculation of the effective quasiparticle interaction in a Fermi liquid at zero temperature. The theory is based on the assumption of reasonably well-defined quasiparticles in a large portion of momentum space, from zero momentum up to $2p_F$. Such an assumption is to some extent corroborated by microscopic theories using correlated-basis function perturbation theory and by extrapolations of Landau theory to finite temperatures, excitation energies and momenta. We argue that the main part of the renormalizations taking place in a Fermi liquid is due to quasiparticle-quasihole excitation processes. These processes may be singled out and are summed up by two coupled integral equations for the quasiparticle scattering amplitude $A'_{pp}(q)$ and the generalized Landau interaction function $F_{pp'}(q)$ in the direct and crossed p-h channels. At this stage the input into these equations is the quasiparticle-quasihole propagator, which is given in terms of the quasiparticle energy ε_k, and the p-h irreduc-

TABLE I. Landau parameters $F_l^{s,a}$, pair interaction constants G_l and transport times $\tau_\eta, \tau_0, \tau_\kappa, \tau_N$ for various values of pressure P and using different parametrizations of the quasiparticle energy $[\varepsilon_c, (m^*/m)]$ and the local (V_l^s) as well as the nonlocal direct interaction (V_l^a). The available experimental values are listed in the last two lines of the table.

No.	P (bar)	ρ ($\mathring{A}^{-3}$)	$\frac{m^*}{m}$	$\frac{\varepsilon_F^0}{\varepsilon_c}$	V_0^s	V_1^s	V_2^s	V_3^s	V_4^s	V_0^a	V_1^a	V_2^a
1	0	0.0166	2.73	0	17.50	−3.36	−2.48	1.53	−0.59	0	0	0
2	0	0.0166	2.73	0	17.50	−3.36	−2.48	1.53	−0.59	0	1.05	0
3	3	0.0180	3.13	0	16.57	−3.18	−2.35	1.45	−0.56	0	1.30	0
4	0	0.0166	2.73	4.3	17.50	−3.36	−2.48	1.53	−0.59	0	1.00	0
5	3	0.0180	3.13	4.3	16.57	−3.18	−2.35	1.45	−0.56	0	1.25	0
6	0	0.0166	2.73	4.3	17.50	−3.36	−2.48	1.53	−0.59	1.5	0.85	0.19
7	0	0.0142	2.73	4.3	19.42	−3.73	−2.75	1.70	−0.65	−0.9	0.78	0.25
8	0	0.0142	2.73	2.5	19.42	−3.73	−2.75	1.70	−0.65	−0.85	0.85	0.16
9	12	0.0180	3.97	2.5	16.57	−3.18	−2.35	1.45	−0.56	−3.7	1.6	0.2

No.	F_0^s	F_0^a	F_1^s	F_1^a	F_2^s	F_2^a	F_3^s	F_3^a	G_0	G_1	G_2	$\tau_\eta T^2$ (μsec K^2)	$\tau_D T^2$ (μsec K^2)	$\tau_K T^2$ (μsec K^2)	$\tau_N T^2$ (μsec K^2)
1	14	−0.5	0.3	−0.05	−0.03	0.03			1.0	−0.04	0				
2	14	−0.71	5.2	−0.95	1	0.2	0.6	−0.3	0.01	−0.06	0.09	3.1	0.7	0.7	0.6
3	20	−0.73	6.4	−1.0	1.2	0.0	0.2	−0.2	−0.01	−0.25	0.26	2.76	0.57	0.59	0.52
4	14	−0.83	5.2	−1.2	1.9	0.2	0.6	−0.3	0.43	−0.08	0.11	1.33	0.26	0.28	0.28
5	20	−0.85	6.4	−1.2	2.2	0.2	0.7	−0.2	−0.28	0.19	−0.19	1.16	0.23	0.25	0.25
6	9.1	−0.70	5.2	−1.2	0.3	0.6	−0.1	−0.1	0.21	−0.10	−0.08	1.27	0.53	0.52	0.53
7	9.1	−0.69	5.3	−0.7	−0.7	0.5	−0.2	0.4	0.46	−0.28	−0.03	0.96	0.47	0.44	0.56
8	9.1	−0.70	5.2	−0.75	0.0	0.3	0.1	0.1	0.31	−0.19	0.06	1.54	0.51	0.51	0.59
9	35	−0.75	8.9	−0.79	0.3	0.4	0.3	0.0	−0.08	−0.17	0.13	1.11	0.28	0.29	0.33
Expt. $P=0$ bar	9.1	−0.70	5.2	−0.80	−0.5					~ -0.1		1.58	0.38	0.39	
Expt. $P=12$ bar	35	−0.75	8.9	−0.80	0.5					~ -0.2		1.01	0.24	0.24	

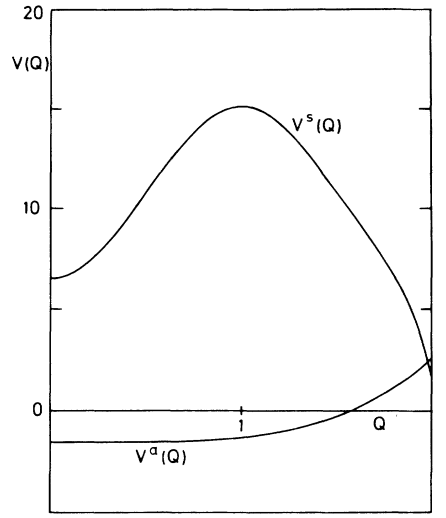


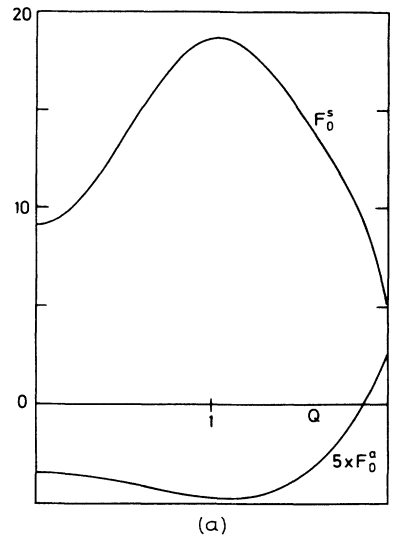
FIG. 8. Direct interaction potentials $V^s(Q)$ and $V^a(Q)$.

ible interaction, the so-called direct interaction $I_{pp'}(q)$. The direct interaction may be considered as a phenomenological quantity, defined on a more microscopic level than A and F, or else it may be calculated in microscopic theory.

Assuming a simple model form for $I_{pp'}(q)$, consisting of a large repulsive weakly q-dependent local potential as suggested by microscopic theory, one finds the remarkable behavior that the Landau parameter F_0^s scales with the interaction potential to large values, whereas F_0^a saturates at a small negative value, just as observed experimentally for liquid ^{3}He. A large value of the backflow parameter F_1^s is not generated unless an exchange potential or a more general nonlocal part is added to $I_{pp'}(q)$. However, the magnitude of this exchange part needed to obtain the experimental value of F_1^s is only 10% of the size of the local potential. We present arguments, based on the particle-particle reducibility properties, showing that such an exchange contribution to $I_{pp'}(q)$ should indeed be expected and that it should scale with $(m/m^*)F_0^s$. As a consequence the approximate scaling relation $F_0^s \propto (m^*/m)^2$ is found, which is well obeyed for ^{3}He at higher pressures where the strong-coupling limit applies. The spin antisymmetric parameter F_1^a is again found to saturate at a small negative value. The universal behavior of the effective interaction obtained in this way is characteristic of an almost localized Fermi system, i.e., a system close to an instability where the effective mass diverges.

We expect the theory presented here to apply to other types of instabilities and phase transitions as well. In particular itinerant magnetism should manifest itself as an instability of the system of equations for $A_{pp'}(q)$ and $F_{pp'}(q)$. In this case the complete set of parameters A_{lk} is likely to diverge in a universal way. In which way this can happen and what forms of direct interaction drive the system towards a magnetic instability remains to be studied.

Finally we have calculated the quasiparticle scattering amplitude for liquid ^{3}He quantitatively by employing the results of a microscopic calculation of the local part of the direct interaction and a simple model for the nonlocal part. The resulting Landau parameters and transport parameters are in good agreement with experiment.



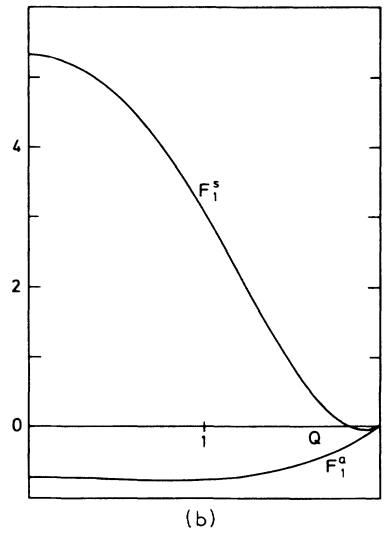


FIG. 9. (a) and (b) Momentum dependent Landau parameters $F_0^s(Q)$, $F_0^a(Q)$ (a) and $F_1^s(Q)$,$F_1^a(Q)$ (b) obtained by using the direct interaction potentials of Fig. 8.

ACKNOWLEDGMENTS

We are grateful to Dr. E. Krotscheck for providing us with numerical results on the direct interaction and for useful discussions. Thanks are due to Dr. D. Vollhardt for stimulating discussions and a careful reading of the manuscript.

*Also at Max-Planck-Institut für Physik und Astrophysik, München, F.R.G. Present address: Department of Physics, University of Florida, Gainesville, FL 32611.

[1]L. D. Landau, Zh. Eksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys.—JETP **3**, 984 (1956)]; **32**, 59 (1957) [**5**, 101 (1957)]; **35**, 97 (1959) [**8**, 70 (1959)].

[2]For an excellent review of Landau theory see G. Baym and C. J. Pethick, in The Physics of Liquid and Solid Helium, Part II, edited by K. H. Bennemann and J. B. Ketterson, (Wiley, New York, 1978), p. 1.

[3]P. Wölfle, Rep. Prog. Phys. **42**, 269 (1979).

[4]R. A. Aziz, V. P. S. Nain, J. S. Carley, W. L. Taylor, and G. T. McConville, J. Chem. Phys. **70**, 4332 (1979).

[5]M. H. Kalos, in *Recent Progress in Many Body Theories,* edited by J. G. Zabolitzky, M. de Llano, M. Fortes, and J. W. Clark (Springer, Berlin, 1981), p. 252.

[6]K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1040 (1958).

[7]T. W. Burckhardt, Ann. Phys. (N.Y.) **47**, 516 (1968).

[8]E. Østgaard, Phys. Rev. **187**, 371 (1969); **180**, 263 (1969); **176**, 351 (1968); **171**, 248 (1968); **170**, 257 (1968).

[9]G. Bertsch, Phys. Rev. **184**, 187 (1969).

[10]S. I. Hernadi and H. R. Glyde, Phys. Rev. B **28**, 141 (1983).

[11]S. Babu and G. E. Brown, Ann. Phys. (N.Y.) **78**, 1 (1973).

[12]O. Sjöberg, Ann. Phys. (N.Y.) **78**, 39 (1973); Nucl. Phys. A **209**, 373 (1973).

[13]E. Feenberg, *Theory of Quantum Fluids* (Academic, New York, 1969).

[14]E. Krotscheck, in *Quantum Fluids and Solids-1983,* Proceedings of the Conference on Quantum Fluids and Solids, AIP Conf. Proc. No. 103, edited by E. D. Adams and G. G. Ihas (AIP, New York, 1983), p. 132; Phys. Rev. A **26**, 3536 (1982).

[15]See the contributions in *The Many-Body Problem: Jastrow Correlations versus Brueckner Theory,* edited by R. Guardiola and J. Ros (Springer, Berlin, 1980) and in *Recent Progress in Many-Body Theories,* edited by J. G. Zabolitzky, M. de Llano, M. Fortes, and J. W. Clark (Springer, Berlin, 1981).

[16]C. H. Aldrich III and D. Pines, J. Low Temp. Phys. **32**, 689 (1978).

[17]C. H. Aldrich III, C. J. Pethick, and D. Pines, Phys. Rev. Lett. **37**, 845 (1976).

[18]K. Bedell and D. Pines, Phys. Rev. Lett. **45**, 39 (1980); Phys. Lett. A **78**, 281 (1980).

[19]K. Bedell, Phys. Rev. B **26**, 3747 (1982).

[20]M. Pfitzner and P. Wölfle, J. Low Temp. Phys. **51**, 535 (1982).

[21]M. Pfitzner and P. Wölfle, in *Quantum Fluids and Solids-1983,* Proceedings of the Conference on Quantum Fluids and Solids, AIP Conf. Proc. No. 103, edited by E. D. Adams and G. G. Ihas (AIP, New York, 1983), p. 148.

[22]K. S. Dy and C. J. Pethick, Phys. Rev. **185**, 373 (1969).

[23]J. Sauls and J. W. Serene, Phys. Rev. B **24**, 183 (1981); Physica **108B**, 1137 (1981).

[24]K. Levin and O. Valls, Phys. Rev. B **20**, 105 (1979); **20**, 120 (1979).

[25]M. Pfitzner, J. Low Temp. Phys. **61**, 141 (1985).

[26]B. L. Friman and E. Krotscheck, Phys. Rev. Lett. **49**, 1705 (1982).

[27]E. Krotscheck and R. A. Smith, Phys. Rev. B **27**, 4222 (1983).

[28]G. E. Brown, C. J. Pethick, and A. Zaringhalam, J. Low Temp. Phys. **48**, 349 (1982).

[29]V. K. Mishra, G. E. Brown, and C. J. Pethick, J. Low Temp. Phys. **52**, 379 (1983).

[30]S. Fantoni, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. **48**, 878 (1982).

[31]M. Pfitzner, J. Low Temp. Phys. **61**, 433 (1985).

[32]T. L. Ainsworth, K. S. Bedell, G. E. Brown, and K. F. Quader, J. Low Temp. Phys. **50**, 319 (1983).

[33]K. S. Bedell and T. L. Ainsworth, Phys. Lett. A **102**, 49 (1984).

[34]K. S. Bedell and K. F. Quader, Phys. Rev. B **32**, 3296 (1985).

[35]K. F. Quader and K. S. Bedell, J. Low Temp. Phys. **58**, 89 (1985).

[36]T. L. Ainsworth and K. S. Bedell (unpublished).

[37]E. Krotscheck (private communication).

[38]M. Pfitzner and P. Wölfle, Phys. Rev. B **33**, 2003 (1986); in *Proceedings of the 17th International Conference on Low Temperature Physics, Karlsruhe, 1984,* edited by U. Eckern *et al.* (Elsevier, Amsterdam, 1984), p. 19.

[39]A. A. Abrikosov, L. P. Gorkov, and I. Y. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon, New York, 1965).

[40]P. Nozières, *Theory of Interacting Fermi Systems* (Benjamin, New York, 1964).

[41]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[42]J. P. Jeukenne, A. Lejenne, and C. Mahaux, Phys. Rep. **25**, 83 (1976).

[43]R. Sartor and C. Mahaux, Phys. Rev. C **21**, 1546 (1979).

[44]J. P. Blaizot and B. L. Friman, Nucl. Phys. A **372**, 69 (1981).

[45]C. Mahaux, in *Recent Progress in Many-Body Theories,* edited by H. Kümmel and M. L. Ristig (Springer, Berlin, 1983).

[46]R. E. Prange and L. P. Kadanoff, Phys. Rev. A **134**, 566 (1964).

[47]R. W. Haymaker and R. Blankenbecler, Phys. Rev. **171**, 1581 (1968).

[48]A. D. Jackson, A. Lande, and R. A. Smith, Phys. Rep. **86**, 55 (1982).

[49]A. Lande and R. A. Smith, in *Recent Progress in Many-Body Theories,* edited by H. Kümmel and M. L. Ristig (Springer, Berlin, 1983).

[50]C. Mahaux and H. Ngô, Phys. Lett. **100B**, 285 (1981).

[51]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1970).

[52]P. W. Anderson and W. F. Brinkman, in *The Physics of Liquid and Solid Helium,* edited by J. B. Ketterson and K. H. Bennemann (Wiley, New York, 1977), Vol. 2.

[53]D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984).

[54]H. H. Jensen, H. Smith, and J. W. Wilkins, Phys. Lett. **27A**, 532 (1968); Phys. Rev. **185**, 323 (1969).

[55]G. A. Brooker and J. Sykes, Phys. Rev. Lett. **21**, 279 (1968); J. Sykes and G. A. Brooker, Ann. Phys. (N.Y.) **56**, 1 (1970).

[56]D. S. Greywall and P. A. Bush, Phys. Rev. Lett. **49**, 146 (1982); D. S. Greywall, Phys. Rev. B **27**, 2747 (1983).

[57]J. C. Wheatley, Rev. Mod. Phys. **47**, 467 (1975).

[58]K. Nara, I. Fujii, K. Kaneko, and A. Ikushima, Physica **108B**, 1203 (1981).

[59]J. M. Parpia, D. J. Sandiford, J. E. Berthold, and J. D. Reppy, Phys. Rev. Lett. **40**, 565 (1979).

[60]D. F. Brewer, D. S. Betts, A. Sachrajda, and W. S. Truscott, Physica **108B**, 1059 (1981).

[61]D. S. Greywall, Phys. Rev. B **29**, 4933 (1984).