Temperature dependence of the critical Josephson current in superconductor–insulator–normal-metal proximity junctions near T_c

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The temperature dependence of the Josephson critical current $I_c(T)$ of a superconductor–insulator–normal-metal proximity junction near its T_c is derived from Ginzburg-Landau theory and the de Gennes boundary conditions. The theoretical result is in good agreement with experimental measurements of the proximity-induced Josephson effect obtained from Ta/Mo and Nb/Ta point-contact junctions.

I. INTRODUCTION

Measurements of the temperature dependence of the Josephson critical current have been used by several authors[1–5] to study the proximity effect in superconductor–normal-metal (*S*-*N*) sandwiches. The junctions involved in these studies were (superconductor–insulator–normal-metal–superconductor (*S*-*I*-*N*-*S*) proximity Josephson junctions. Most of the theoretical explanations for these experiments were based on the Ginzburg-Landau (GL) theory. The $I_c(T)$ was found to be proportional to $(T_c-T)^{3/2}$ near the T_c of the junction. This $(T_c-T)^{3/2}$ behavior near T_c was confirmed by the early experiments.[1–4]

Recently, the Josephson effect between a superconductor and a normal metal (or another superconductor at the temperatures above its transition temperature T_{cn}) through a very thin insulating barrier (*S*-*I*-*N* junction) was observed experimentally on the Nb/UBe_{13}, Ta/UBe_{13}, and Ta/Mo point-contact junctions and interpreted theoretically using the GL theory.[6,7] This "proximity-induced Josephson effect" had been observed earlier,[8–10] but was not successfully explained. In the present paper we use the linearized GL equation to derive the temperature dependence of the critical current $I_c(T)$ of the *S*-*I*-*N* junction and compare it with experimental results on Ta/Mo and Nb/Ta point-contact junctions. We find generally good agreement.

II. THEORY

A model of the *S*-*I*-*N* junction structure is shown in Fig. 1. Here, d_s and d_n are the thickness of the superconductor and the normal metal, respectively. Both d_s and d_n are assumed to be much larger than their coherence length, i.e., $d_{s,n}/\xi_{s,n} \gg 1$. The thickness of the barrier, d_I, is small compared to the electron mean free path $l_{s\,(n)}$, so that we can treat the tunneling process as diffusion. The temperature considered is close to the T_c of the junction, and both superconductor and normal metal are supposed to be in the dirty limits, so that the use of the Ginzburg-Landau equation is appropriate and de Gennes boundary conditions (3) and (4) below can be applied.[11,12]

In the *S* region, the order parameter $\Delta_s(x)$, obtained by solving the linearized GL equation, is given by[3]

$$\Delta_s(x)=\begin{cases}\Delta_{\mathrm{BCS}}(t)\sin\left[\dfrac{\pi(b-x)}{2\xi_{\mathrm{GL}}}\right], & -\xi_{\mathrm{GL}}+b<x<0 \quad \text{(1a)}\\ \Delta_{\mathrm{BCS}}(t), \quad x<-\xi_{\mathrm{GL}}+b\ . & \text{(1b)}\end{cases}$$

Here, $t=T/T_c$ is the reduced temperature and b is the so-called "extrapolation length." The induced order parameter in the *N* region is given by[3,13]

$$\Delta_n(x)=\Delta_n(x=0^+)\left[\frac{\cosh[k_n(d_n-x)]}{\cosh(k_n d_n)}\right], \quad x>0 \tag{2}$$

where

$$k_n=\xi_n^{-1}[(1-T_{cn}/T)^{4/\pi^2}]^{1/2}\equiv\xi_{nF}^{-1}\ .$$

The de Gennes boundary conditions are[11,12]

$$\frac{\xi_s^2}{V_s}\frac{d\Delta_s}{dx}=\frac{\xi_n^2}{V_n}\frac{d\Delta_n}{dx} \quad \text{at } x=0\ , \tag{3}$$

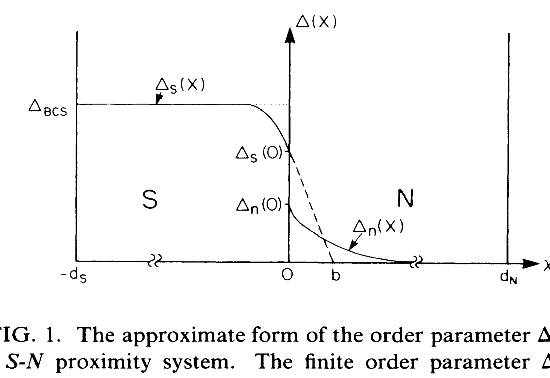

FIG. 1. The approximate form of the order parameter $\Delta(x)$ of the *S*-*N* proximity system. The finite order parameter Δ_n extends into the *N* side by a distance about ξ_n. The suppression of the Δ_s occurs only in the region about $\xi_{\mathrm{GL}}-b$ from the *S*-*N* interface.

$$\frac{\Delta_s}{N_s V_s} = \frac{\Delta_n}{N_n V_n} \quad \text{at } x=0 \,, \tag{4}$$

where

$$\xi^2_{n\,(s)} = \frac{\hbar D_{n\,(s)}}{2\pi T}\,, \quad \xi^2_{\rm GL} = \frac{3\pi\hbar D_s}{8k_b(T_{cs}-T)}\,,$$

and

$$D_{n\,(s)} = \tfrac{1}{3} v_{Fn\,(s)} l_{n\,(s)}\,,$$

where $\xi_{n\,(s)}$ is the coherence length of the normal metal (superconductor), and $\xi_{\rm GL}$ is the Ginzburg-Landau coherence length of the superconductor. T_{cs} is the superconducting transition temperature of the isolated superconductor. $v_{Fn\,(s)}$ and $N_{n\,(s)}$ are the Fermi velocity and the electronic density of states on the Fermi surface, respectively, and $V_{n\,(s)}$ are the pairing interactions in the N (S) region.

Combining Eqs. (1) and (3)–(5), we have

$$\Delta_n(0^+) = \frac{N_n V_n}{N_s V_s} \Delta_{\rm BCS}(t) \sin\left[\frac{\pi b}{2\xi_{\rm GL}}\right]\,, \tag{5}$$

$$k_n \Delta_n(0^+) \tanh(k_n d_n) = \frac{\pi}{2\xi_{\rm GL}} \Delta_{\rm BCS}(t) \cos\left[\frac{\pi}{2\xi_{\rm GL}}\right] \frac{V_n \xi_s^2}{V_s \xi_n^2}\,. \tag{6}$$

Using Eqs. (5) and (6), we can eliminate the extrapolation length b. The final results for Δ_n and Δ_s at the interface are

$$\Delta_n(0^+) = \Delta_{\rm BCS}(t) \left[1 + \left[\frac{2\xi_n \xi_{\rm GL} N_{nF}}{\pi \xi_s^2 N_s}\right]^2\right]^{-1/2} \frac{N_n V_n}{N_s V_s}\,, \quad -\xi_{\rm GL} < x < 0 \tag{7}$$

$$\Delta_s(0^-) = \Delta_{\rm BCS}(t) \left[1 + \left[\frac{2\xi_n \xi_{\rm GL} N_{nF}}{\pi \xi_s^2 N_s}\right]^2\right]^{-1/2}\,, \quad x > 0\,. \tag{8}$$

To obtain the above results, the approximation $\tanh(k_n d_n) = 1$ has been made. At the vicinity of the transition temperature T_c of the junction, $\Delta_{\rm BCS} \propto (T_c - T)^{1/2}$, $\xi_{\rm GL} \propto (T_{cs} - T)^{-1/2}$, and the dc Josephson critical current $I_c \propto \Delta_n(0^+)\Delta_s(0^-)$.[14,15] Thus, we obtain the dependence of the dc critical current in the vicinity of the junction T_c:

$$I_c \propto \Delta_n(0^+)\Delta_s(0^-) = \Delta^2_{\rm BCS}(t) \left[1 + \left[\frac{2\xi_n \xi_{\rm GL} N_{nF}}{\pi \xi_s^2 N_s}\right]^2\right]\,, \tag{9}$$

i.e.,

$$I_c(t) \propto (T_c - T) \left[1 + \left[\frac{\alpha T F^2}{T_{cs} - T}\right]\right]^{-1}\,, \tag{10}$$

where

$$\alpha = \frac{3N_n^2 v_{Fn} l_n}{N_s^2 v_{Fs} l_s}\,;$$

α is determined by the material properties of the N and S, and essentially is proportional to l_n / l_s.

The temperature dependence differs from those of the S-I-N-S and S-N-S junctions. In the latter case, $I_c \propto (T_c - T)^{3/2}$ (Ref. 4) and $I_c \propto (T_c - T)^2$ (Ref. 12) are reported for S-I-N-S and S-N-S junctions, respectively.

In the earlier paper,[6] we used the Beasley model to show that the Josephson effect can occur in a S-I-N proximity junction. In the Beasley model, the order parameter Δ_s in the S side was assumed spatially independent (see Fig. 1) to simplify the procedure. This spatially constant order parameter has the same temperature dependence as that of an isolated superconductor. This assumption is applicable as long as only the existence of the phase-dependent part of the free energy is concerned. However, when the temperature dependence of the critical current is concerned, the assumption of spatially constant Δ_s cannot be used anymore. In this case, the influence of the N side on the S side will give the order parameter in the S side Δ_s a different temperature dependence from that of an isolated superconductor. This influence must be taken into account to give the correct result.

III. EXPERIMENT

The $I_c(T)$ were measured on Ta/Mo and Nb/Ta point-contact junctions. The Ta and Nb points used in the experiments were obtained by mechanically polishing 1-mm high-purity Ta and Nb wires. The Mo used for the Ta/Mo contacts was of a high-purity (99.999%) Mo ingot; the Ta used for the Nb/Ta contacts was a high-purity MRC Marz-grade 50-μm-thick cold-rolled Ta foil. The flat surface of the sample (Mo ingot or Ta foil) was first lightly mechanically polished to remove oxide and then mounted adjacent to a hole in the wide wall of a length of a K-band waveguide, as schematically illustrated in the earlier work of Noer, Chen, and Wolf.[14] The Ta (Nb) point was inserted through the waveguide and was brought into contact with Mo (Ta) surface to form the point-contact junction. Adjustment of the contact could be carried out using a worm gear or screw drive operable via a rotary shaft extending through a vacuum seal outside the ^{4}He Dewar. The whole point-contact assembly was contained in a brass vacuum exchange-gas can.

Before placing the assembly in the vacuum can, both sample and tip were exposed in air for about 1 h to allow a thin oxide layer to grow on the surfaces. The tunneling contacts were made at 4.2 K or below. During the measurements, the temperature was controlled by passing a small current (<1 mA) through a heater attached to the waveguide near the sample or by pumping on the ^{4}He bath. The temperature was varied very slowly, about 20 mK/min and was measured by a calibrated Lake Shore Cryogenics Model 200A-100 Ge-resistance thermometer glued to the gear box in the vicinity of the sample. I-V characteristics of the contacts were taken from the standard four-probe technique. I_c was determined from the I-V curves.

The contact region of the Ta foil, after low-temperature measurement, was studied by use of a scanning Auger microprobe and revealed no Nb residue on the Ta surface.

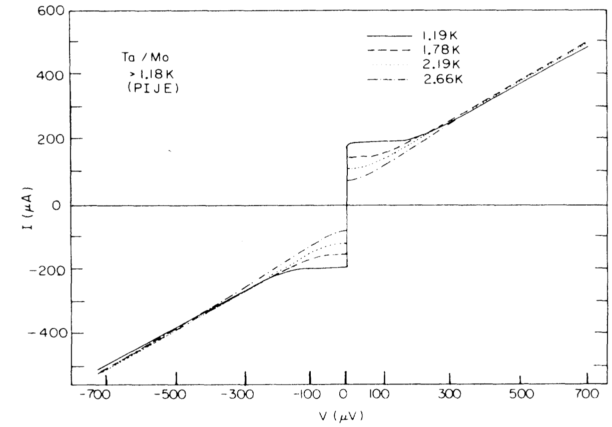

FIG. 2. I-V characteristic of a typical Ta/Mo point contact at various T. Solid line, $T=1.19$ K; dashed line, $T=1.78$ K; dotted line, $T=2.19$ K; dashed-dotted line, $T=2.66$ K. The critical current $I_c(T)$ is taken from these I-V curves. The spreading resistance of the $V=0$ branch is about 0.5 mΩ, which is too small to see in this figure.

These I-V curves of the proximity-induced Josephson effect are the same as those of the ordinary Josephson effect, except for the finite value of dV/dI at $V=0$, which is believed to be from the spreading resistance.[6] The actual values of dV/dI at $V=0$ of the junction shown in Fig. 3 are less than 2 mΩ and, therefore, one cannot recognize them from this small figure.

The experimentally measured $I_c(T)$ of Ta/Mo and Nb/Ta point-contact junctions are compared with the theoretical values calculated from Eq. (10) (see Fig. 3). In all cases, $T_{c\mathrm{Tb}}=9.20$ K, $T_{c\mathrm{Ta}}=4.47$ K, and $T_{c\mathrm{Mo}}=0.92$ K. The electron mean free paths l_s and l_n have not been measured, so we treat α as a fitting parameter. Another parameter in fitting experimental and theoretical curves is the current normalization. From Fig. 3 one can see that the agreement between experiment and theory is generally very good. However, there are still small deviations. Except for small errors in the measurement, the fact that the materials we used are not strictly in the dirty limit and

Thus, the observed Josephson effect must have occurred between the Nb tip and the Ta foil. We have observed good microwave-induced rf steps in these Ta/Mo and Nb/Ta contacts. This is described elsewhere.[15] In general, these steps are similar to those reported earlier.[7] The typical I-V characteristics of a Ta/Mo point contact at temperatures from 1.19 to 2.66 K are shown in Fig. 2.

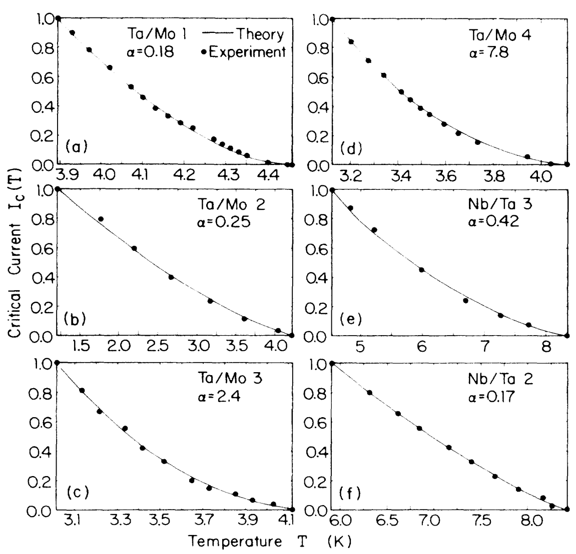

FIG. 3. The temperature dependence of the critical proximity-induced Josephson current of Ta/Mo and Nb/Ta point-contact junctions near their T_c. The dots are the experimental data and the solid lines are values calculated from Eq. (10). The values of α were obtained by making least-squares fits to the experimental data. The critical currents $I_c(T)$ in panels (a)–(f) are normalized as follows: (a), 207.5 μA; (b), 190.0 μA; (c), 130.6 μA; (d), 8.8 μA; (e) 385.0 μA; (f), 87.5 μA.

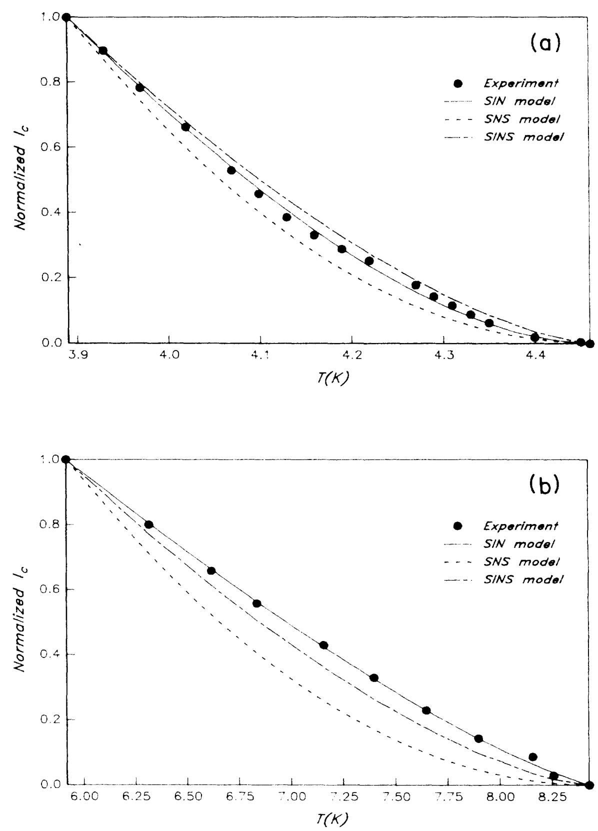

FIG. 4. The $I_c(T)$ curves calculated from the S-I-N model [Eq. (10)], the S-N-S model $[I_c(T)\propto(T_c-T)^2]$, and the S-I-N-S model $[I_c(T)\propto(T_c-T)^{3/2}]$ are compared to the experimental results of the (a) Ta/Mo and (b) Nb/Ta point contacts previously shown in Figs. 3(a) and 3(f), respectively. It is clear from this figure that the best fit to the experimental data is obtained from the S-I-N model.

that GL theory is not strictly applicable when T is far from the T_c are thought to be mainly responsible for the discrepancies. The experimental data are also compared with the *S-N-S* (superconductor–normal-metal–superconductor) junction's $(T_c - T)^2$ and the *S-I-N-S* junction's $(T_c - T)^{3/2}$ behavior. We find that the *S-I-N* model agrees with the experimental data much better than do the other two models (see Fig. 4).

One may notice that the values of α obtained from fitting are diverse. For example, the values of α of Ta/Mo contacts made from the same Mo sample (but with different surface oxidation time and in the four experimental runs) vary from 0.18 to 7.8, a factor of about 43. This scatter occurs also in the other *S-N* system. We tentatively attribute this large diversity in α to variations in $l_{n\,(s)}$, the electron mean free path, due to the local conditions at the interface. In our case the value of $l_{n\,(s)}$ is different from its bulk value, but is related to external conditions, such as the pressure of the contact, the thickness of the insulating layer, and the transmission coefficients of the barrier and of the contact. All these conditions can affect the value of $l_{n\,(s)}$ near the interface and, hence, lead to diverse values of α. One of the disadvantages of using a point contact is that the junction's parameters are relatively hard to control and the value of $l_{n\,(s)}$ near the interface almost cannot be measured experimentally.

In conclusion, the temperature dependence of the Josephson critical current of the *S-I-N* proximity junction has been obtained by solving the linearized GL equation under de Gennes boundary conditions. The result is qualitatively different from those for *S-I-N-S* and *S-N-S* junctions. The best agreement, between the theories and our experimental data, was obtained by using the *S-I-N* model. A disadvantage in the use of the point-contact junctions is that the junction parameters are relatively difficult to control. More detailed checks on the predictions of our *S-I-N* model should be possible using thin-film fabrication techniques.

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[1] J. P. Romagnan, A. Gilabert, J. C. Noiray, and E. Guyon, Solid State Commun. **14**, 83 (1974).

[2] A. Gilabert, C. Van Haessendonck, L. Van den Dries, and Y. Bruynseraede, Solid State Commun. **31**, 109 (1979).

[3] N. L. Rowell and H. J. T. Smith, Can. J. Phys. **54**, 223 (1976).

[4] S. Greenspoon and H. J. T. Smith, Can. J. Phys. **49**, 1350 (1971).

[5] J. A. Blackburn, H. J. T. Smith, and N. L. Rowell, Phys. Rev. B **11**, 1053 (1975).

[6] Siyuan Han, K. W. Ng, E. L. Wolf, H. F. Braun, Lee Tanner, Z. Fisk, J. L. Smith, and M. R. Beasley, Phys. Rev. B **32**, 7567 (1985).

[7] Siyuan Han, K. W. Ng, E. L. Wolf, A. J. Millis, Z. Fisk, and J. Smith, Phys. Rev. Lett. **57**, 238 (1986).

[8] U. Kaiser-Dieckhoff, SQUID **76**, 165 (1976).

[9] I. Taguchi and H. Yoshioka, J. Phys. Soc. Jpn. **27**, 1074 (1969).

[10] H. Happ and U. Kaiser-Dieckhoff, Phys. Lett. **29A**, 161 (1969).

[11] P. G. de Gennes, Rev. Mod. Phys. **36**, 225 (1964).

[12] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1969).

[13] N. R. Werthamer, Phys. Rev. **132**, 2440 (1963).

[14] R. J. Noer, T. P. Chen, and E. L. Wolf, Phys. Rev. B **31**, 647 (1985).

[15] Siyuan Han, Ph.D. dissertation, Iowa State University, 1986.