Interface polaron in a strong magnetic field

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The method recently developed by Larsen is generalized to treat, in the limit of strong magnetic field, the interface polaron in a semi-infinite polar crystal. Both the bulk longitudinal optical phonon and the surface optical phonon and their interactions with the magnetic field are included. Ground-state energy corrections up to the fourth-order perturbation are expressed as functions of the magnetic field and a parameter characterizing the mean distance of the polaron from the interface.

I. INTRODUCTION

There has been a considerable amount of work on the surface or interface polaron in polar crystals in recent years.[1–7] The behavior of such polarons under the influence of an external magnetic field has also been investigated both experimentally[8–11] and theoretically.[12–16]

In the case of bulk samples, various methods have been developed on the basis of polaron theories to treat the magnetic field influence on the polaronic states. In the absence of a magnetic field, Feynman's theory[17] yields accurate approximation to the ground-state energy in both two and three dimensions. Hellwarth and Platzman[18] generalized this theory to the limit of weak field and low temperature. The unitary transformation introduced by Lee, Low, and Pines[19] was extended by Larsen[20] to study the low-lying states of polarons in weak fields. On the other hand, Evrard, Kartheuser, and Devreese[21] introduced a Born-Oppenheimer type of approximation in their treatment of polarons in strong magnetic field. By means of a Green's-function technique, Lepine and Matz[22] have calculated in Fock approximation the ground-state energy at zero temperature for arbitrary electron-phonon coupling and arbitrary magnetic field. Furthermore, Feynman's method has also been generalized by Peeters and Devreese[23] to include arbitrary field at arbitrary temperature. These authors have also calculated on second-order perturbation the polaron energy in two and three dimensions in the presence of an arbitrary magnetic field.[24]

The method described in Ref. 23 has been applied to investigate the two-dimensional polaron in the liquid-helium film[13] and in crystalline solids[14,15] under the influence of an external magnetic field. More recently, a novel perturbation method has been proposed by Larsen[16] who replaces sums over products of matrix elements and energy denominators by much simpler operator algebra. He then calculates the ground-state energy in the strong-field limit up to the sixth-order perturbation and to the fourth order in arbitrary field.

The study of a polaron in an external magnetic field has so far been limited to either three-dimensional bulk crystal or two-dimensional approximation of heterostructure. There does not seem to be sufficient attention to the surface or interface polaron in a semi-infinite polar crystal. In discussions of two-dimensional polaron, it is usually assumed that the electron interacts with only one mode of phonon, bulk longitudinal optical (LO) or surface optical (SO) phonon. Perhaps this is because up to the present time, experiments are only done on $Ga_xAl_{1-x}As$-GaAs-$Ga_xAl_{1-x}As$ quantum wells of width $a \lesssim 50$ Å. It is therefore of interest to investigate the properties of the interface polaron in a magnetic field with both SO and LO phonon included.

In this paper we consider in the strong-field limit the interface polaron in a semi-infinite polar crystal. Larsen's method is generalized to include SO phonon as well as LO phonon. The z-component contribution is calculated by variational method, and expressions for the ground-state energy up to the fourth-order corrections are obtained.

Our result provides a way to calculate the mean distance of the polaron from the nonpolar-polar interface of various dielectric constants. In the limit of large distance, the SO-phonon contribution vanishes and our result reduces to that of Ref. 16, while in the small-distance limit, it approaches approximately to the result of Ref. 15.

II. THE HAMILTONIAN

We consider a semi-infinite polar crystal which occupies the half space $z>0$. The other half of the space is filled by a nonpolar crystal, and the plane $z=0$ is the interface between the two crystals. The electron moves inside the polar crystal but near the interface.

Assuming that a magnetic field $\mathbf{B}=(0,0,B_M)$ exists, the motion of the electron, interacting with both the bulk LO phonon and SO phonon, is described by the total Hamiltonian[5,16]

$$H=\frac{1}{2m}\left[p_x-\frac{\beta^2}{4}y\right]^2+\frac{1}{2m}\left[p_y+\frac{\beta^2}{4}x\right]^2+\frac{p_z^2}{2m}+\frac{e^2(\epsilon_{\infty 1}-\epsilon_{\infty 2})}{4z\epsilon_{\infty 1}(\epsilon_{\infty 1}+\epsilon_{\infty 2})}+\sum_{\mathbf{k}}\hbar\omega_l a_{\mathbf{k}}^\dagger a_{\mathbf{k}}+\sum_{\mathbf{q}}\hbar\omega_s b_{\mathbf{q}}^\dagger b_{\mathbf{q}}$$

$$+\sum_{\mathbf{k}}[V_k^* a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}_{\|}\cdot\rho}\sin(k_z z)+\text{H.c.}]+\sum_{\mathbf{q}}(V_q^* b_{\mathbf{q}}^\dagger e^{-(i\mathbf{q}_{\|}\cdot\rho+qz)}+\text{H.c.}) , \tag{1}$$

where

$$\beta^2=\frac{2e}{c}B_M , \tag{2a}$$

$$V_k^*=\frac{i}{k}(4\pi e^2\hbar\omega_l/\epsilon V)^{1/2} , \tag{2b}$$

$$V_q^*=i\left[\frac{\pi\hbar\omega_s e^2}{\epsilon^* Sq}\right]^{1/2} , \tag{2c}$$

$$\frac{1}{\epsilon}=\frac{1}{\epsilon_{\infty 1}}-\frac{1}{\epsilon_{01}} , \tag{2d}$$

$$\frac{1}{\epsilon_1^*}=\frac{\epsilon_{01}-1}{\epsilon_{01}+1}-\frac{\epsilon_{\infty 1}-1}{\epsilon_{\infty 1}+1} . \tag{2e}$$

The first two terms in (1) represent the electron kinetic energy in the xy plane; the third term is its kinetic energy in z direction. The fourth term is the mirror-image potential energy; the fifth and sixth terms are the longitudinal optical (LO) and surface optical (SO) phonon energy, respectively. The last two terms stand for the interaction energy between electron and the LO and SO phonons. The notation is as follows. $\mathbf{p}=(p_x,p_y,p_z)$ is the electron momentum, m is the band mass of the electron. The volume of the polar crystal is V and the area of the interface is S. $\epsilon_0(\epsilon_\infty)$ is the static (optical) dielectric constant and the subscript 1 (2) stands for polar (nonpolar) crystal. The electron position vector is denoted by $\mathbf{r}=(\rho,z)$. The operator $a_{\mathbf{k}}^\dagger$ creates a LO phonon with wave vector $\mathbf{k}=(\mathbf{k}_{\|},k_z)$, and $b_{\mathbf{q}}^\dagger$ creates a SO phonon with two-dimensional wave vector $\mathbf{q}$. The frequency of LO phonon (SO phonon) is denoted by ω_l (ω_s). If we use ω_T for the frequency of bulk transversal optical (TO) phonon, then we have

$$\omega_s^2=\tfrac{1}{2}(\omega_T^2+\omega_l^2) . \tag{3}$$

We take as our unperturbed Hamiltonian the sum of the first six terms of (1) and consider the last two terms as perturbation. Following Larsen[16] we define the strong-field limit:

$$\lambda_l^2,\lambda_s^2\to\infty , \quad \alpha_l\lambda_l,\alpha_s\lambda_s\to 0 , \tag{4}$$

where

$$\lambda_l^2=\omega_c/\omega_l , \quad \lambda_s^2=\omega_c/\omega_s , \tag{4a}$$

and $\omega_c=\beta^2/2m=eB_M/mc$ is the cyclotron frequency of the electron. It is clear from the definitions (4) that the method works only for small α, or for weak coupling of electron-phonon interactions. To describe the Landau levels, the following harmonic-oscillator operators are introduced:

$$A=\frac{1}{\sqrt{\hbar}\beta}\left[\left[p_x-\frac{\beta^2}{4}y\right]-i\left[p_y+\frac{\beta^2}{4}x\right]\right] , \tag{5a}$$

$$B=A^\dagger-\frac{i\beta}{2\sqrt{\hbar}}(x+iy) , \tag{5b}$$

which satisfy the commutation relations

$$[A,A^\dagger]=[B,B^\dagger]=1 \text{ and } [A,B]=[A,B^\dagger]=0 . \tag{6}$$

The effect of these operators are that $A^\dagger$ lowers the quantum number M of the z angular momentum and raises the Landau quantum number n by one unit, while $B^\dagger$ raises M by one unit without changing n.

In terms of these operators, the total Hamiltonian (1) can be rewritten as

$$H=H_0+H_{e\text{-LO}}+H_{e\text{-SO}} , \tag{7}$$

$$H_0=\frac{\hbar\beta^2}{2m}(A^\dagger A+\tfrac{1}{2})+\frac{p_z^2}{2m}+\frac{e^2(\epsilon_{\infty 1}-\epsilon_{\infty 2})}{4z\epsilon_{\infty 1}(\epsilon_{\infty 1}+\epsilon_{\infty 2})}+\sum_{\mathbf{k}}\hbar\omega_l a_{\mathbf{k}}^\dagger a_{\mathbf{k}}+\sum_{\mathbf{k}}\hbar\omega_s b_{\mathbf{q}}^\dagger b_{\mathbf{q}} , \tag{7a}$$

$$H_{e\text{-LO}}=\sum_{\mathbf{k}}[V_k^* L_k M_k \sin(k_z z)a_{\mathbf{k}}^\dagger+V_k L_k^{-1}M_k^{-1}\sin(k_z z)a_{\mathbf{k}}] , \tag{7b}$$

$$H_{e\text{-SO}}=\sum_{\mathbf{q}}(V_q^* L_q M_q e^{-qz}b_{\mathbf{q}}^\dagger+V_q L_q^{-1}M_q^{-1}e^{-qz}b_{\mathbf{q}}) , \tag{7c}$$

where

$$L_k=\exp\left[\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)A-\frac{\sqrt{\hbar}}{\beta}(k_x-ik_y)A^\dagger\right] , \tag{8a}$$

$$L_k^{-1}=\exp\left[\frac{\sqrt{\hbar}}{\beta}(\mathbf{k}_x-ik_y)A^\dagger-\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)A\right] , \tag{8b}$$

$$M_k=\exp\left[\frac{\sqrt{\hbar}}{\beta}(k_x-ik_y)B-\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)B^\dagger\right] , \tag{8c}$$

$$M_k^{-1}=\exp\left[\frac{\sqrt{\hbar}}{\beta}(k_x+ik_y)B^\dagger-\frac{\sqrt{\hbar}}{\beta}(k_x-ik_y)B\right] , \tag{8d}$$

$$L_q=\exp\left[\frac{\sqrt{\hbar}}{\beta}(q_x+iq_y)A-\frac{\sqrt{\hbar}}{\beta}(q_x-iq_y)A^\dagger\right] , \tag{8e}$$

$$M_q=\exp\left[\frac{\sqrt{\hbar}}{\beta}(q_x-iq_y)B-\frac{\sqrt{\hbar}}{\beta}(q_x+iq_y)B^\dagger\right] . \tag{8f}$$

In the following, we shall treat H_0 as the unperturbed Hamiltonian and $H_{e\text{-LO}}$, $H_{e\text{-SO}}$ as small perturbations.

III. PERTURBATION THEORY

In the limit of strong magnetic field, the unperturbed eigenstates involve only $n=0$ levels of Landau states as has been pointed out in Ref. 16. If the vacuum states of the operators A and B are denoted by $|0\rangle_A$ and $|0\rangle_B$, respectively, then the energy of the system can be calculated by using the effective Hamiltonian

$$H_{\rm eff}={}_A\langle 0|H|0\rangle_A=H^0_{\rm eff}+H' , \tag{9}$$

$$H^0_{\rm eff}=\tfrac{1}{2}\hbar\omega_c+\sum_{\mathbf{k}}\hbar\omega_l a^\dagger_{\mathbf{k}}a_{\mathbf{k}}+\sum_{\mathbf{q}}\hbar\omega_s b^\dagger_{\mathbf{q}}b_{\mathbf{q}} +\frac{p_z^2}{2m}+\frac{e^2(\epsilon_{\infty 1}-\epsilon_{\infty 2})}{4z\epsilon_{\infty 1}(\epsilon_{\infty 1}+\epsilon_{\infty 2})} , \tag{9a}$$

$$H'=\sum_{\mathbf{k}}V_k^* e^{-\hbar k_\parallel^2/2\beta^2}(M_k a^\dagger_{\mathbf{k}}+M_k^{-1}a_{\mathbf{k}})\sin(k_z z) +\sum_{\mathbf{q}}V_q^* e^{-\hbar q^2/2\beta^2}(M_q b^\dagger_{\mathbf{q}}+M_q^{-1}b_{\mathbf{q}}) , \tag{9b}$$

where we have made use of the matrix elements

$$ {}_A\langle 0|L_k|0\rangle_A=e^{-\hbar k_\parallel^2/2\beta^2} , \qquad {}_A\langle 0|L_q|0\rangle_A=e^{-\hbar q^2/2\beta^2} . \tag{10}$$

The unperturbed eigenstates are denoted by

$$|\psi_0\rangle=\phi(z)|M\rangle_B|n_{\mathbf{k}},n_{\mathbf{q}}\rangle , \tag{11}$$

where $\phi(z)$ is the z component of the polaron wave function, $|n_{\mathbf{k}},n_{\mathbf{q}}\rangle$ is the phonon eigenstate in number representation, and $|M\rangle_B=(M!)^{-1/2}(B^\dagger)^M|0\rangle_B$. In Wigner-Brillouin perturbation scheme we expand the perturbation energy in a power series of the small parameters $\alpha_l\lambda_l$ and $\alpha_s\lambda_s$. A straightforward calculation then yields the second-order correction

$$\Delta E^{(2)}=\Delta E_k^{(2)}+\Delta E_q^{(2)} , \tag{12}$$

$$\Delta E_k^{(2)}=(E_p-\hbar\omega_l)^{-1}\frac{\sqrt{\pi}}{2}\alpha_l\lambda_l(\hbar\omega_l)^2\times\left\{1-\left\langle\phi(z)\left|e^{-\beta^2z^2/\hbar}\left[1-\Phi\left[\frac{\beta z}{\sqrt{\hbar}}\right]\right]\right|\phi(z)\right\rangle\right\} , \tag{12a}$$

$$\Delta E_q^{(2)}=(E_p-\hbar\omega_s)^{-1}\frac{\sqrt{\pi}}{2}\alpha_s\lambda_s(\hbar\omega_s)^2\left\langle\phi(z)\left|e^{-\beta^2z^2/\hbar}\left[1-\Phi\left[\frac{\beta z}{\sqrt{\hbar}}\right]\right]\right|\phi(z)\right\rangle , \tag{12b}$$

where the electron—LO-phonon coupling constant α_l is defined by

$$\alpha_l=me^2/\epsilon\hbar^2\kappa_l \tag{13a}$$

with the polaron wave number κ_l given by

$$\frac{\hbar^2\kappa_l^2}{2m}=\hbar\omega_l \tag{13b}$$

and the electron—SO-phonon coupling constant

$$\alpha_s=me^2/\epsilon^*\hbar^2\kappa_s \tag{14a}$$

with

$$\hbar^2\kappa_s^2/2m=\hbar\omega_s . \tag{14b}$$

The function $\Phi(t)$ is the probability integral defined by $\Phi(t)=(2/\sqrt{\pi})\int_0^t e^{-x^2}dx$. Similarly, we find after tedious calculation the fourth-order correction

$$\Delta E^{(4)}=\Delta E_k^{(4)}+\Delta E_q^{(4)} , \tag{15}$$

$$\begin{aligned}\Delta E_k^{(4)}=(E_p-\hbar\omega_l)^{-2}(E_p-2\hbar\omega_l)^{-1}\frac{\pi}{4}(\alpha_l\lambda_l)^2(\hbar\omega_l)^4\langle\phi(z)|&\left\{1-e^{-\beta^2z^2/\hbar}\left[1-\Phi\left[\frac{\beta z}{\sqrt{\hbar}}\right]\right]\right\}^2\\ &+\left[\frac{2}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty e^{-3\hbar k_\parallel^2/2\beta}I_0\left[\frac{\hbar k_\parallel^2}{2\beta^2}\right]dk_\parallel-\frac{2}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty e^{-2k_\parallel z}e^{-3\hbar k_\parallel^2/2\beta^2}I_0\left[\frac{\hbar k_\parallel^2}{2\beta^2}\right]dk_\parallel\right.\\ &-\frac{2}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty e^{-2l_\parallel z}e^{-3\hbar l_\parallel^2/2\beta^2}I_0\left[\frac{\hbar l_\parallel^2}{2\beta^2}\right]dl_\parallel\\ &\left.+\frac{4\hbar}{\beta^2\pi}\int_0^\infty e^{-2k_\parallel z}e^{-\hbar k_\parallel^2/\beta^2}dk_\parallel\int_0^\infty e^{-2l_\parallel z}e^{-\hbar l_\parallel^2/\beta^2}J_0\left[\frac{2\hbar}{\beta^2}k_\parallel l_\parallel\right]dl_\parallel\right]|\phi(z)\rangle ,\end{aligned} \tag{15a}$$

$$\Delta E_q^{(4)}=(E_p-\hbar\omega_s)^{-2}(E_p-2\hbar\omega_s)^{-1}\frac{\pi(\hbar\omega_s)^4}{4}(\alpha_s\lambda_s)^2\langle\phi(z)\,|\,e^{-2\beta^2z^2/\hbar}[1-\Phi(\beta z/\sqrt{\hbar})]^2$$
$$+\frac{4\hbar}{\pi\beta^2}\int_0^\infty e^{-2qz}e^{-\hbar q^2/\beta^2}dq\int_0^\infty e^{-2rz}e^{-\hbar r^2/\beta^2}J_0\left[\frac{2\hbar qr}{\beta^2}\right]dr\,|\,\phi(z)\rangle\ , \tag{15b}$$

where $I_0(t)=J_0(e^{i\pi/2}t)$ is a cylindrical function of imaginary argument, and $J_0(t)$ is the Bessel function of order zero.

IV. GROUND-STATE ENERGY

The perturbation energies calculated above still involve integrals over z which must be evaluated before any further calculation can be done. This is carried out by choosing a trial wave function which is the z-component wave function of an interface polaron[25] when there is no magnetic field. Thus

$$\phi(z)=2\zeta^{3/2}ze^{-\zeta z}\ , \tag{16}$$

where ζ is the variation parameter. We note that being in the z direction the magnetic field has no direct influence on the z component of electron wave function. However, the interaction between the phonon field and electron couples also with the magnetic field. Hence the z-component motion of the electron involves indirect effect of **B**.

Using (16) we find from (12) and (15)

$$\Delta E^{(2)}=\frac{\sqrt{\pi}(\hbar\omega_l)^2\alpha_l\lambda_l}{2(E_p-\hbar\omega_l)}[1-A(\zeta,\beta)]+\frac{\sqrt{\pi}(\hbar\omega_s)^2}{2(E_p-\hbar\omega_s)}\alpha_s\lambda_sA(\zeta,\beta)\ , \tag{17a}$$

$$\Delta E^{(4)}=\frac{\pi(\hbar\omega_l)^4(\alpha_l\lambda_l)^2}{4(E_p-\hbar\omega_l)^2(E_p-2\hbar\omega_l)}\{[1-A(\zeta,\beta)]^2+f_1(\beta)-2f_2(\zeta,\beta)+f_3(\zeta,\beta)\}$$
$$+\frac{\pi(\hbar\omega_s)^4(\alpha_s\lambda_s)^2}{4(E_p-\hbar\omega_s)^2(E_p-2\hbar\omega_s)}[A^2(\zeta,\beta)+f_3(\zeta,\beta)]\ , \tag{17b}$$

where

$$A(\zeta,\beta)=\frac{2\zeta^3}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty\frac{e^{-\hbar t^2/\beta}}{(\zeta+t)^3}dt\ , \tag{18a}$$

$$f_1(\beta)=\frac{2}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty e^{-3\hbar k_{\parallel}^2/2\beta^2}I_0(\hbar k_{\parallel}^2/2\beta^2)dk_{\parallel}\ , \tag{18b}$$

$$f_2(\zeta,\beta)=\frac{2\zeta^3}{\beta}\left[\frac{\hbar}{\pi}\right]^{1/2}\int_0^\infty dt\,I_0(\hbar t^2/2\beta^2)\times e^{-3\hbar t^2/2\beta^2}/(\zeta+t)^3\ , \tag{18c}$$

$$f_3(\zeta,\beta)=\frac{4\hbar}{\beta^2\pi}\zeta^6\int_0^\infty dk_{\parallel}e^{-\hbar k_{\parallel}^2/\beta^2}(\zeta+k_{\parallel})^{-3}\times\int_0^\infty dl_{\parallel}J_0(2\hbar k_{\parallel}l_{\parallel}/\beta^2)\times e^{-\hbar l_{\parallel}^2/\beta^2}(\zeta+l_{\parallel})^{-3}\ . \tag{18d}$$

To obtain the ground-state energy correction to the order $(\alpha_l\lambda_l)^2$ or $(\alpha_s\lambda_s)^2$, we first expand the denominator in (17a), and then combine with (17b). This gives

$$\Delta E=-\frac{\sqrt{\pi}}{2}\hbar\omega_l(\alpha_l\lambda_l)(1-A)-\frac{\sqrt{\pi}}{2}\hbar\omega_s(\alpha_s\lambda_s)A$$
$$+\frac{\pi}{8}\hbar\omega_l(\alpha_l\lambda_l)^2(1-2A+A^2-f_1+2f_2-f_3)$$
$$+\frac{\pi}{8}\hbar\omega_s(\alpha_s\lambda_s)^2(A^2-f_3)$$
$$+\frac{\pi}{4}\hbar(\omega_l+\omega_s)(\alpha_l\lambda_l)(\alpha_s\lambda_s)(A-A^2)\ . \tag{19}$$

Therefore the ground-state energy of the system is

$$E_g(\zeta)=E_0+\Delta E\ , \tag{20}$$

where E_0 is the unperturbed energy. Since the unperturbed eigenstates involve only the lowest Landau levels with $n=0$ in the strong-field limit, we have

$$E_0=\langle\psi_0\,|\,H_0\,|\,\Psi_0\rangle=\tfrac{1}{2}\hbar\omega_c+\frac{\hbar^2}{2m}\zeta^2+\frac{\zeta e^2(\epsilon_{\infty 1}-\epsilon_{\infty 2})}{4\epsilon_{\infty 1}(\epsilon_{\infty 1}+\epsilon_{\infty 2})}\ . \tag{21}$$

The parameter ζ is determined by minimizing E_g. Let the solution to the equation

$$\partial E_g(\zeta)/\partial\zeta=0 \tag{22}$$

be ζ_0. We find finally the ground-state energy of interface polaron in the strong magnetic field limit

$$E_g(\zeta_0)=\tfrac{1}{2}\hbar\omega_c+\frac{\hbar^2}{2m}\zeta_0^2+\frac{\zeta_0 e^2(\epsilon_{\infty 1}-e_{\infty 2})}{4\epsilon_{\infty 1}(\epsilon_{\infty 1}+\epsilon_{\infty 2})}$$

$$+\Delta E_1+\Delta E_2+\Delta E_3+\Delta E_4+\Delta E_5\ , \qquad (23)$$

$$\Delta E_1=-\frac{\sqrt{\pi}}{2}\hbar\omega_l(\alpha_l\lambda_l)[1-A(\zeta_0,\beta)]\ , \qquad (23a)$$

$$\Delta E_2=-\frac{\sqrt{\pi}}{2}\hbar\omega_s(\alpha_s\lambda_s)A(\zeta_0,\beta)\ , \qquad (23b)$$

$$\Delta E_3=\frac{\pi}{8}\hbar\omega_l(\alpha_l\lambda_l)^2\{[1-A(\zeta_0,\beta)]^2-f_1(\beta)$$

$$+2f_2(\zeta_0,\beta)-f_3(\zeta_0,\beta)\}\ , \qquad (23c)$$

$$\Delta E_4=\frac{\pi}{8}\hbar\omega_s(\alpha_s\lambda_s)^2[A^2(\zeta_0,\beta)-f_3(\zeta_0,\beta)]\ , \qquad (23d)$$

$$\Delta E_5=\frac{\pi}{4}\hbar(\omega_l+\omega_s)\alpha_l\lambda_l\alpha_s\lambda_s[A(\zeta_0,\beta)-A^2(\zeta_0,\beta)]\ . \qquad (23e)$$

The first term in (23) is the Landau level energy of the electron in strong magnetic field. The second term is the kinetic energy corresponding to z-component motion of the electron. The third term represents the image potential energy, and the last five terms are the interactions between the electron, magnetic field, and phonon fields. ΔE_1 and ΔE_3 are the coupling energies of the electron with the magnetic field and LO phonon; ΔE_2 and ΔE_4 represent interactions with the magnetic field and SO phonon. Finally, ΔE_5 is the interaction energy of the electron coupled with the magnetic field and both modes of phonon fields.

V. RESULTS AND DISCUSSIONS

Equation (22) can only be solved numerically for the value of ζ_0. Our numerical study reveals that when $\epsilon_{\infty 2}<\epsilon_{\infty 1}$ or when the image potential appears repulsive to an electron in medium 1, ζ_0 determined by (20) is a small number. Since ζ_0 is inversely proportional to the mean distance of the polaron from the interface,[2] it is unlikely to find the electron near the interface for $\epsilon_{\infty 2}<\epsilon_{\infty 1}$. Consequently, the SO phonon has little influence on the polaronic states in this case.

The numerical value of ζ_0 depends mainly upon the materials on both sides, especially the dielectric constants. It also depends on the applied magnetic field. As $\zeta_0\rightarrow\infty$, we can show from (18) and (23) that

$$\Delta E=-\frac{\sqrt{\pi}}{2}\hbar\omega_s\alpha_s\lambda_s+\frac{\pi}{8}\hbar\omega_s(\alpha_s\lambda_s)^2(1-f_3)\ . \qquad (24)$$

Up to the order of $\alpha_s\lambda_s$, this leads to the same result as Eq. (14) of Ref. 15 in the strong-field limit. When $\zeta_0\rightarrow 0$, on the other hand, the SO-mode contribution diminishes and we find

$$\Delta E=-\frac{\sqrt{\pi}}{2}\hbar\omega_l\alpha_l\lambda_l+\frac{\pi}{8}\hbar\omega_l(\alpha_l\lambda_l)^2(1-f_1)\ , \qquad (25)$$

which, as is expected, is identical to Larsen's result.[16] In general Eq. (23) applies to any mean distance of the polaron from the interface.

It is noted that the method is valid only in the limit $\alpha_i\lambda_i\rightarrow 0$. This requires that

$$\alpha_i\ll\frac{1}{\lambda_i}=\left[\frac{\omega_i}{\omega_c}\right]^{1/2}=\left[\frac{mc\omega_i}{\epsilon B_M}\right]^{1/2}\ ,\quad i=l,s\ . \qquad (26)$$

Therefore we must limit our discussion to weak-coupling polarons. As an example, we consider the TiO_2/GaAs interface. The parameters are taken from Ref. 26. These include $\hbar\omega_l=36.7$ meV, $\hbar\omega_s=35.2$ meV $\alpha_l=0.0681$, $\alpha_s=0.113$, $m=0.0657m_e$, $\epsilon_{01}=12.83$, $\epsilon_{\infty 1}=10.9$, and $\epsilon_{\infty 2}=78$. We have computed the energy corrections for magnetic fields in the range $(2-50)\times 10^5$ G, and the results are given in Table I. It is found that, at least in this range, ζ_0 is a very slowly varying function of the field B_M, and decreases with increasing B_M. The energy correction due to SO-phonon contribution amounts only to less than 9% of that due to LO-phonon contribution. This is because for the parameters or the materials we have chosen, the calculated ζ_0 corresponds to a mean distance of about 700 Å. Thus the polaron is formed rather deep in the bulk, and as a consequence the LO phonon predominates in the interaction. When the magnetic field is not so strong, however, the situation may be entirely

TABLE I. Corrections to the ground-state energy of the interface polaron in GaAs. All energies are in units of meV.

B_M (10^5 G)	ζ_0 (cm^{-1})	ΔE_1	ΔE_2	ΔE_3	ΔE_4	ΔE_5
2	2.35 ± 10^5	-2.1108	-0.1991	0.0113	0.000 56	0.0234
4	2.35×10^5	-3.0331	-0.2015	0.0225	0.000 57	0.0340
6	2.30×10^5	-3.7446	-0.1972	0.0338	0.000 55	0.0410
8	2.30×10^5	-4.3424	-0.1983	0.0450	0.000 56	0.0479
10	2.25×10^5	-4.8685	-0.1942	0.0561	0.000 53	0.0526
20	2.2×10^5	-6.9359	-0.1906	0.1128	0.000 51	0.0735
30	2.1×10^5	-8.5248	-0.1822	0.1740	0.000 47	0.0863
40	2.05×10^5	-9.8640	-0.1779	0.2448	0.000 45	0.0976
50	2.0×10^5	-11.0406	-0.1739	0.3271	0.000 43	0.1067

different. Work is being carried out for the case of arbitrary field and arbitrary coupling constant.

Furthermore, it is also seen from this calculation that the fourth-order corrections raise the ground-state energy. Thus the inclusion of SO phonon does not change the conclusion of Ref. 16 that the binding is weakened by the fourth-order correction.

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