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Intersubband optical absorption in a quantum well with an applied electric field

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We present new results for the electric field dependence of the intersubband optical absorption within the conduction band of a quantum well. We show that for increasing electric field the absorption peak corresponding to the transition of states $1 \rightarrow 2$ is shifted higher in energy and the peak amplitude is increased. These features are different from those of the exciton absorption. It is also found that the transition $1 \rightarrow 3$, forbidden when $F=0$, is possible when F is nonzero.

Quantum confinement of carriers in a semiconductor quantum well leads to the formation of discrete energy levels and the drastic change of optical-absorption spectra.[1] The interband absorptions near the band gap have been extensively studied, and it has been shown that their absorption and luminescence spectra are dominated by excitonic effects.[2,3] Some more recent studies have concentrated on the electric field dependence of energy levels[4–7] and band-edge optical absorption including the exciton effect.[8–10] Very recently, experimental studies of the intersubband absorption within the conduction band of a GaAs quantum well without an applied electric field have been reported.[11] A very large dipole strength and a narrow bandwidth were observed. In this paper, we present theoretical calculations for the electric field dependence of the optical absorption between the discrete subbands within the conduction band of a quantum well based on the infinite-potential-barrier model. One of the reasons for increased interest in this area is the possibility of practical device application. For example, in 1970, Kazarinov and Suris[12] proposed a new type of infrared laser amplifier using the intersubband transition and resonant tunneling. A far-infrared photodetector with high wavelength selectivity based on the intersubband absorption and the sequential resonant tunneling has also been suggested.[13]

The Hamiltonian of the system (a single quantum well) subject to a uniform electric field perpendicular to the quantum well (the z direction) in the presence of optical radiation (Fig. 1) is written as

$$H=H_0+H'_{\rm op} , \quad (1)$$

where H_0 is the unperturbed Hamiltonian for an electron in the quantum well in the presence of perpendicular electric field, and the interaction Hamiltonian $H'_{\rm op}$ is given by[14]

$$H'_{\rm op}=-\frac{e}{m_0}\mathbf{A}\cdot\mathbf{p}=-\frac{e}{2m_0}A_0[e^{i\mathbf{q}\cdot\mathbf{r}-i\omega t}+\text{c.c.}]\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{p} , \quad (2)$$

where $\mathbf{A}$ is the vector potential, $\hat{\boldsymbol{\varepsilon}}$ is the polarization vector, $\mathbf{q}$ is the wave vector for incoming optical radiation, e is the magnitude of the charge of the electron, m_0 is the free-space electron mass, and $\mathbf{p}$ is the momentum vector of the electron in the crystal. The first term in (2) gives the absorption $H'^{\rm abs}_{\rm op}$ and the second term gives the emission of photon.

Then, for a given interaction potential $H'_{\rm op}$, the transition rate from the initial state ψ_i to the final state ψ_f for absorption is given by[14]

$$W_{fi}=\frac{2\pi}{\hbar}|\langle\psi_f|H'^{\rm abs}_{\rm op}|\psi_i\rangle|^2\delta(E_f-E_i-\hbar\omega) , \quad (3)$$

where E_i and E_f are the energies of the electron in the initial state and the final state, respectively, and ω is the angular frequency of the incident photon. If we neglect the interaction between the electrons in the well, the wave functions for the initial state ψ_i and the final state ψ_f after absorption can be written as[15]

$$\psi_i=u_c(\mathbf{r})\xi_i(\mathbf{r})$$
$$=A^{-1/2}u_c(\mathbf{r})e^{i\mathbf{k}_t\cdot\mathbf{r}_t}\phi_i(z),\ |z|<\frac{L}{2} , \quad (4a)$$

$$\psi_f=u_{c'}(\mathbf{r})\xi_f(\mathbf{r})$$
$$=A^{-1/2}u_{c'}(\mathbf{r})e^{i\mathbf{k}'_t\cdot\mathbf{r}_t}\phi_f(z),\ |z|<\frac{L}{2} , \quad (4b)$$

where A is the area of the well, L is the width of the well, $\mathbf{k}_t$, $\mathbf{k}'_t$ are the wave vectors of the electron in the x-y plane for the initial and the final states, respectively, $\mathbf{r}_t$ is the po-

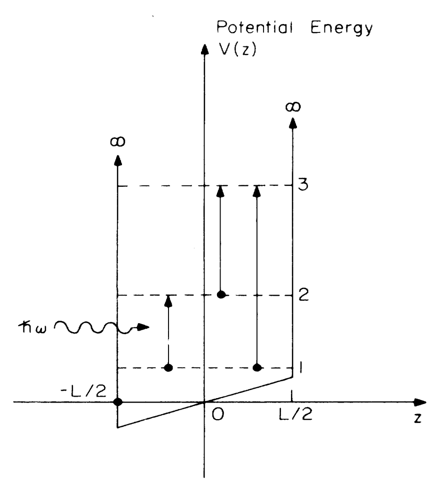

FIG. 1. Potential-energy profile for an infinite quantum well with width L subject to an external electric field F in the presence of incoming radiation with angular frequency $\hbar\omega$.

sition vector in the x-y plane, and u_c and $u_{c'}$ are the cell periodic functions near the conduction-band extremum. The envelope functions ϕ_i and ϕ_f satisfy the following Schrödinger's equation in the effective-mass approximation:[5,7]

$$-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2}\phi(z)+|e|Fz\phi(z)=E\phi(z),\quad |z|\le\frac{L}{2}\ , \qquad (5)$$

and are given by the linear combination of two independent Airy functions $\mathrm{Ai}(\eta)$ and $\mathrm{Bi}(\eta)$, where η is defined by

$$\eta=-\left[\frac{2m^*}{(e\hbar F)^2}\right]^{1/3}(E-|e|Fz)\ . \qquad (6)$$

In Eqs. (5) and (6), m^* and F denote the effective mass of an electron and the electric field, respectively.

For intersubband transitions, the matrix element $\langle\psi_f|H_{\mathrm{op}}^{\prime\mathrm{abs}}|\psi_i\rangle$ can be approximated by[16]

$$\begin{aligned}\langle\psi_f|H_{\mathrm{op}}^{\prime\mathrm{abs}}|\psi_i\rangle\approx\langle\xi_f|H_{\mathrm{op}}^{\prime\mathrm{abs}}|\xi_i\rangle&=-\frac{eA_0}{2m_0}\langle\xi_f|e^{i\mathbf{q}\cdot\mathbf{r}}\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{p}|\xi_i\rangle\\&\approx-\frac{eA_0}{2m_0}\hat{\boldsymbol{\varepsilon}}\cdot\langle\xi_f|\mathbf{p}|\xi_i\rangle\\&=-\frac{eA_0}{2i\hbar}(E_i-E_f)\hat{\boldsymbol{\varepsilon}}\cdot\langle\xi_f|\mathbf{r}|\xi_i\rangle\ ,\end{aligned} \qquad (7)$$

where the cell periodic function part has been taken care of as in Ref. 16, and we have used the dipole approximation. Since

$$\int_{-L/2}^{L/2}\phi_{f^*}(z)\phi_i(z)dz=\delta_{fi}\ ,$$

we find that the major contribution to the optical matrix element is the z component of the $\mathbf{r}$ vector, and so the absorption is strongly polarization dependent. The absorption constant α in the well is defined as[17] $\hbar\omega$ times the number of transitions per unit volume per unit time divided by the incident power per unit area

$$\alpha=\frac{1}{V}\sum_i\sum_f\sum_{\mathbf{k}_t}\sum_{\mathbf{k}_t'}\left[\hbar\omega W_{fi}\Big/\frac{n_r\omega^2A_0^2}{2\mu c}\right]\ , \qquad (8)$$

where the summations over i and f are for the quantized initial and final energies, respectively, for the z components of the momenta. If we calculate the total transition rate and take into account the line broadening,[14] we obtain

$$\alpha=\sum_i\sum_f\frac{\mu c m^* k_B T e^2}{\pi\hbar^2 m_0^2 L n_r\omega}(\cos^2\theta)|M_{fi}|^2\ln\left[1+\exp\left[\frac{E_F-E_i^{(z)}}{k_BT}\right]\Big/1+\exp\left[\frac{E_F-E_f^{(z)}}{k_BT}\right]\right]\frac{(\Gamma/2)}{(\hbar\omega-E_{fi})^2+(\Gamma/2)^2}\ , \qquad (9)$$

with the matrix element

$$M_{fi}=\frac{m_0(E_i^{(z)}-E_f^{(z)})}{i\hbar}\int_{-L/2}^{L/2}\phi_f^*(z)z\phi_i(z)dz\ , \qquad (10)$$

where $E_{fi}=E_f^{(z)}-E_i^{(z)}$, and $E_i^{(z)}$ and $E_f^{(z)}$ denote the quantized energy levels for the initial state and final state, respectively, μ is the permeability, c is the speed of light in free space, k_B is Boltzmann's constant, T is the temperature, θ is the angle between the polarization vector and the normal to the quantum well, n_r is the refractive index, E_F is the Fermi energy which depends on the density of electrons in the well, and Γ is the linewidth.

The oscillator strength f is given by[11]

$$\begin{aligned}f&=\frac{2m_0(E_f^{(z)}-E_i^{(z)})}{\hbar^2}\langle z\rangle^2\\&=\frac{2|M_{fi}|^2}{m_0(E_f^{(z)}-E_i^{(z)})}\ .\end{aligned} \qquad (11)$$

In the zero-field limit, $f=14.45$ for the $1\to2$ transition, which is independent of the width of the well for the infinite-potential-well model. The experimental result[11] of f for a well width of 65 Å (or effective $L=101.27$ Å) is 12.2 and slightly depends on the well width.

We calculated α for the first three states with field dependence numerically for $T=300$ K. In Fig. 2, we plot the absorption coefficient α for the incident photon with polarization perpendicular to the well ($\theta=0$), taking into account the first three states as a function of the energy of the photon with $F=0$ (dashed line) and $F=250$ kV/cm (solid line) for an effective well width 101.27 Å, which gives the same ground-state energy for $F=0$ with the true well width of 65 Å and the barrier height $\Delta E_c=245$ meV.[11] We use $E_F=6.49$ meV which corresponds to about 1.6×10^{17} cm^{-3} electrons and $\Gamma=10$ meV from the experimental results[11] and is assumed to be independent for the variation of F. One can easily see that the transi-

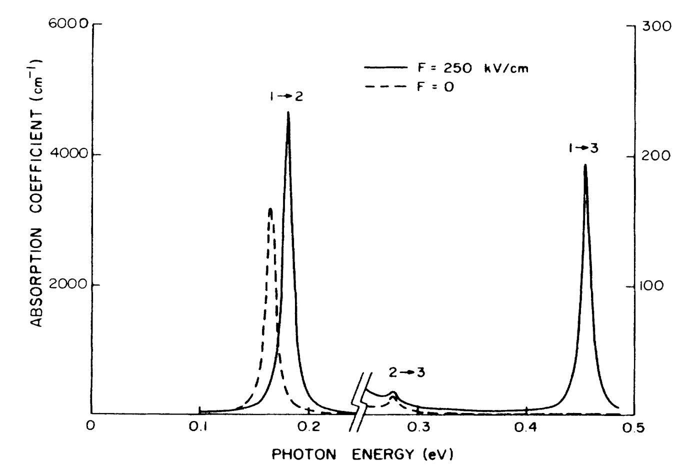

FIG. 2. Comparison of the intersubband absorption coefficient α for an infinite well width $L=101.27$ Å for the first three states with electron density 1.6×10^{17}/cm^3 electrons for the zero electric field (dashed line) and for the electric field of 250 kV/cm (solid line).

tion $1 \rightarrow 2$ is dominant for both electric fields $F=0$ and 250 kV/cm. For $F=250$ kV/cm the absorption peak is shifted by 16 meV from 165 to 181 meV, and the peak amplitude is increased from 3153 to 4619 cm^{-1}. There are two distinct features for the case of the intersubband absorption compared with the exciton absorption.[9]

(i) The absorption peak for intersubband optical absorption is increased in energy with increasing electric field over a wide range of the electric field, because for increasing electric fields the energy of the ground state decreases rapidly, while those of the higher subband states increase slightly then decrease slowly as cited in Ref. 6. On the other hand, for the exciton absorption, both the ground states of the electrons and the holes decrease. Thus the absorption peak is decreased in energy with increasing electric field.

(ii) The absorption peak for intersubband optical absorption is increased in magnitude with increasing electric field because the electrons are shifted to the same side of the well for both the initial and the final states with increasing electric field, and the energy difference E_2-E_1 also increases for the reason mentioned in (i). As a result, the absolute value of the overlap integral M_{fi} for $1 \rightarrow 2$ transition increases. For the exciton absorption, increasing electric field causes further separation of electrons and holes in the well as well as the decrease of the energy difference between the electron and the hole ground states, thus, the decrease of the absolute value of the overlap integral.[4]

It is also remarkable that the forbidden transition $1 \rightarrow 3$ for $F=0$ becomes possible when F is nonzero because the parity which prohibits the transition $1 \rightarrow 3$ no longer exists when F is nonzero. In Fig. 3 we plot $(M_{fi}/M_{fi}^{(0)})^2$ as a function of F for the $1 \rightarrow 2$ transition, where $M_{fi}^{(0)}$ is the value of M_{fi} for the $1 \rightarrow 2$ transition with $F=0$. One can easily see, as expected, that the ratio increases slightly from 1 as F increases. In our calculation, we assume Γ is constant; however, to account for the effect of the electric field on the absorption completely, further analysis of the electric field dependence of the line broadening is desired.

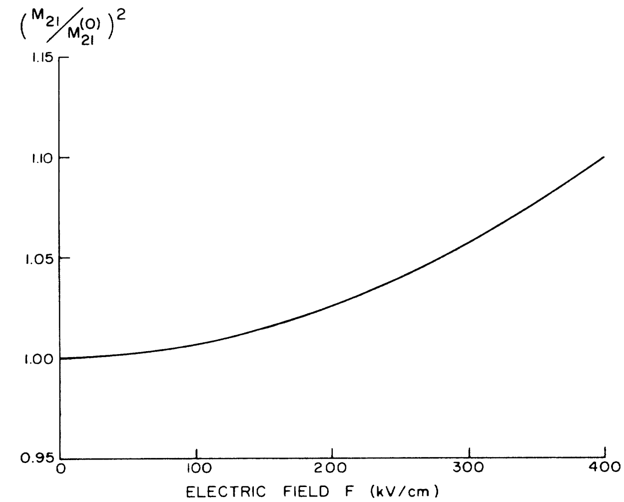

FIG. 3. The normalized overlap integral $|M_{21}/M_{21}^{(0)}|^2$, where $M_{21}^{(0)}$ is for the zero electric field, is plotted vs electric field F.

In conclusion, we have calculated the electric field dependence of the intersubband absorption within a conduction band of a quantum well. It is found that the absorption peak is shifted in energy and is also increased in magnitude with increasing electric field. The forbidden transition $1 \rightarrow 3$ when $F=0$ becomes allowable for the nonzero electric field.

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