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Voltage fluctuations in small conductors

M. Büttiker

IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

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The mean-squared voltage fluctuations of an ensemble of disordered conductors, measured between probes less than an inelastic scattering length apart, are independent of the separation of the probes. The mean-squared fluctuations of the conductance can exceed e^2/h by many orders of magnitude. To obtain these results, it is essential to take into account that carriers can make large excursions into the voltage leads without being scattered inelastically. Our results complement recent arguments and experimental findings by Benoit, Umbach, Laibowitz, and Webb (unpublished).

In a recent paper, Benoit, Umbach, Laibowitz, and Webb,[1] have presented an intricate argument and experimental results to show that the fluctuations of the voltage drop along a section of a narrow wire in a magnetic field are characterized by a mean-squared deviation.

$$\langle(\Delta V)^2\rangle=(\alpha/2)(e^2/h)^2\mathcal{R}_\phi^2 V_\phi^2 \ , \qquad (1)$$

independent of the separation l_v of the voltage probes as long as l_v is smaller than the phase-breaking length l_ϕ. In Eq. (1) the angle brackets denote an ensemble average and ΔV is the deviation of the voltage away from the ensemble average. We show that α is a constant of order 1. The size of the fluctuations is determined by the average resistance $\mathcal{R}_\phi$ of a section of the conductor with a length equal to the phase-breaking length and V_ϕ is the average voltage drop over a phase-breaking length. In contrast, if the voltage fluctuations are measured over a distance $l_v \gg l_\phi$ each segment of length l_ϕ gives a contribution to ΔV of random sign and magnitude given by Eq. (1) leading to voltage fluctuations which increase linearly[1] with l_v. Voltage fluctuations arise because current flow past impurities gives rise to local field variations,[2,3] and the voltage profile along the conductor is linear only after ensemble averaging. However, I will show that voltage probes, because they are themselves disordered conductors, cannot measure these local potentials with precision, but profoundly influence what is measured. The conductance fluctuations depend on the separation of the voltage probes and are given by

$$\langle(\Delta G)^2\rangle=(\beta/2)(e^2/h)^2(l_\phi/l_v)^4 \ , \qquad (2)$$

where β is a factor of order 1. As the distance l_v between voltage probes is made small, Eq. (1) predicts conductance fluctuations which can exceed e^2/h by many orders of magnitude. This is astonishing in view of recent theoretical claims that conductance fluctuations are *universal*[4-7] and bounded by e^2/h. Experimentally we deal with a conductor which is connected to voltage probes, and conductance is measured in a four-terminal setup.[8,9] In contrast, Refs. 4-7 assume a two-terminal measurement.

To derive Eqs. (1) and (2) we consider the three-terminal conductor[10] shown in Fig. 1. The conductor is connected to three reservoirs with chemical potentials μ_1, μ_2, and μ_3. Consider the case $\mu_1 > \mu_2$. This induces a net current flow from reservoir 1 to reservoir 2. μ_3 is determined by the condition that no net current flows to reservoir 3. Lead 3, thus, is a voltage probe and allows one to investigate the potential differences $\mu_1-\mu_3$ or $\mu_3-\mu_2$. Thus we can study the potential distribution along the conductor by changing the position of lead 3, the voltage probe, by taking μ_1 or μ_2 as a reference potential. The conductor in Fig. 1 scatters only elastically. Inelastic scattering events occur only in the reservoirs. The physical situation is, of course, different; inelastic events occur throughout the conductor. To make contact with a typical experiment, we must make further assumptions: We take the distance between the reservoirs 1 and 2 to be the phase-breaking length $l_\phi=x+y$. Further, since carriers in the conductor can make excursions into the voltage probe over a distance of order l_ϕ before being scattered inelastically, the length z of the voltage probe must also be of the order of an inelastic scattering length. We take $z=l_\phi/2$. A carrier in the conductor due to its wave nature "sees" the whole conductor, including the portion of the voltage probes within a distance l_ϕ of the main conductor.

To investigate the transport properties of the conductor of Fig. 1, we invoke the approach put forward in Refs. 8 and 10. Reference 8 was concerned with the symmetries inherent in a four-probe measurement.[9] Connection of an extra lead to a conductor was invoked in Ref. 10 to introduce incoherence into a conductor in which scattering is only elastic. The approach of Ref. 8 used here differs from that of Refs. 11-15. References 11-15 make a number of assumptions about what constitutes a voltage measurement which are not realized in the present experiments. They assume that the voltage probe is only weakly coupled to the conductor, that phase coherence between the con-

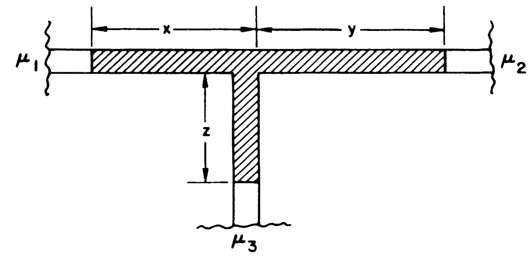

FIG. 1. Three-pole conductor with segments of length x,y,z. Current flow is along the x and y segment and the z segment serves as a voltage probe.

ductor and the voltage probe can be neglected, and that voltage probes measure carrier densities and are insensitive to the direction of motion of the carriers. We treat the voltage probe as part of the conductor and treat both on an equal footing. Quantum transport in the conductor of Fig. 1 is determined by the probabilities T_{ij} for a carrier incident in lead j to traverse the sample into lead i, and the reflection probabilities R_{ii} for a carrier incident in lead i to be reflected into lead i. To obtain a well-defined scattering problem, a piece of perfect conductor is inserted between the reservoir and the conductor. If the leads have a finite cross section A, the number of states at the Fermi energy with positive velocity (from the reservoir toward the conductor) is of the order $N=A/\lambda_F^2$. In the perfect lead, these states can be characterized by their transverse eigenstate and are referred to as quantum channels. The scattering problem is determined by the probabilities $T_{ij,kl}$ for a carrier incident in lead j, in channel l, to traverse the sample into channel k, of lead i, and by the reflection probabilities $R_{ij,kl}$. If each reservoir feeds all channels up to the same chemical potential μ_i, the resistance of the sample depends only on the total probabilities[8] $T_{ij}=\sum_{k,l}T_{ij,kl}$ and $R_{ii}=\sum_{k,l}R_{ii,kl}$. The current in lead i is determined by

$$I_i=(e/h)\left[(N-R_{ii})\mu_i-\sum_{j(\neq i)}T_{ij}\mu_j\right] . \tag{3}$$

For the purpose of this Rapid Communication, we consider the fluctuations of the voltage and conductance from sample to sample of an ensemble of macroscopically identical conductors. Fluctuations from sample to sample arise due to the sample-specific microscopic disorder configuration. In contrast, the experiment of Ref. 1 investigates fluctuations of a given sample as a function of an applied magnetic field. In the absence of a magnetic field, the case of interest here, the transmission probabilities have the symmetry $T_{ij}=T_{ji}$. Equation (3) can now be used to find the chemical potential μ_3 and the resistance of the x and y segments. Current conservation requires $I=I_1=-I_2$. The condition $I_3=0$ determines the chemical potential of the voltage probe,[10]

$$\mu_3=\mu_2+\frac{S_b}{S_b+S_f}(\mu_1-\mu_2) . \tag{4}$$

Here we have used the unitary relation $T_{13}+T_{23}+R_{33}=N$ and introduced the abbreviations $S_b=T_{13}=T_{31}$ and $S_f=T_{23}=T_{32}$. The chemical potential is thus determined by the probability S_b of a carrier incident in the voltage lead to be scattered against the direction of current flow *back* into reservoir 1 and the probability S_f for a carrier incident in lead 3 to be scattered in the direction of current flow *forward* into reservoir 2. The resistance of the x segment (see Fig. 1) is

$$\mathcal{R}_x=(\mu_1-\mu_3)/I=\left[\frac{h}{e^2}\right]\frac{S_f}{T_c(S_f+S_b)+S_fS_b} . \tag{5}$$

Here, $T_c=T_{12}=T_{21}$ is the probability of a carrier incident in lead 1 to traverse the sample coherently, that is without being scattered into lead 3. Similarly we find from Eq. (3) the resistance of the y segment,

$$\mathcal{R}_y=(\mu_3-\mu_2)/I=(h/e^2)\{S_b/[T_c(S_f+S_b)+S_fS_b]\} .$$

The total resistance of the conductor is $\mathcal{R}=(\mu_1-\mu_2)/I=\mathcal{R}_x+\mathcal{R}_y$. The resistance $\mathcal{R}$ describes a continuous transition[10] from completely coherent transmission through the sample ($S_b=S_f=0$) to completely incoherent transmission ($T_c=0$). For completely coherent transmission we find $\mathcal{R}=(h/e^2)T_c^{-1}$. For completely incoherent transmission every carrier has to enter reservoir 3 where its phase and energy are randomized and this yields $\mathcal{R}=(h/e^2)(S_b^{-1}+S_f^{-1})$.

First, we determine the variation of the potential and the resistances in the limit that we can neglect quantum interference effects. In the absence of interference effects, carriers move ballistically and are scattered elastically by randomly distributed impurities in the sample. The trajectory of a single carrier is purely deterministic but looks like that of a particle undergoing random Brownian motion with a diffusion constant $D\sim v_F l_e$, where v_F is the Fermi velocity. In this limit, we can determine the probability of carriers to traverse the sample by solving a classical diffusion problem. The details of such a calculation will be presented elsewhere.[16] Here, we only quote the results. The junction between the conductor and the voltage probe is assumed to be isotropic; that is, a carrier incident from one of the arms of the junction has a probability of $\frac{1}{3}$ to traverse the junction into one of the other arms and is scattered back with a probability $\frac{1}{3}$. For a conductor with a width $\sqrt{A}=\sqrt{N\lambda_F^2}$ much smaller than the length x,y,z of the segments, we find

$$T_c=Nl_ez/q , \tag{6a}$$

$$S_b=Nl_ey/q , \tag{6b}$$

$$S_f=Nl_ex/q . \tag{6c}$$

Through the shape function $q=xy+xz+yz$, the transmission probabilities depend in a nonlocal way on the geometry of the whole conductor. Using these results, and Eq. (4), we find a linear variation of the potential μ_3,

$$\mu_3=\mu_2+\frac{y}{x+y}eV_\phi . \tag{7}$$

Here we have used $(\mu_1-\mu_2)=eV_\phi$ as the voltage drop over a phase-breaking length. Furthermore, the resistance of the x and y segment are given by $\mathcal{R}_x=(h/e^2)(x/Nl_e)$ and $\mathcal{R}_y=(h/e^2)(y/Nl_e)$. The resistances are given by the classical Boltzmann expression. In the absence of interference effects coupling or decoupling a voltage probe to the conductor has no effect.

Next, let us discuss fluctuations away from the classical behavior. Interference effects produce small corrections $\Delta T_c,\Delta S_f,\Delta S_b$ to the classically determined probabilities given above. In a two-port conductor, characterized by a single reflection and transmission coefficient R and T, the quantum corrections ΔR and ΔT are universal[5-7] and satisfy $\langle(\Delta R)^2\rangle=\langle(\Delta T)^2\rangle\cong 1$ independent of the shape of the conductor and the degree of elastic scattering as long as the sample length is long compared to l_e and short compared to the localization length. In a two-port conductor the universality of the fluctuations in the transmission and reflection probabilities implies the universality of the conductance fluctuations. Since $G=(e^2/h)T$ we have

$$\langle(\Delta G)^2\rangle=(e^2/h)^2\langle(\Delta T)^2\rangle=(e^2/h)^2 \ .$$

Below is an argument to show that the fluctuations of the transmission and reflection probabilities of the three-port conductor are also universal and obey

$$\langle(\Delta R_{11})^2\rangle\simeq\langle(\Delta R_{22})^2\rangle\simeq\langle(\Delta R_{33})^2\rangle\simeq 1 \ , \quad (8a)$$

$$\langle(\Delta T_c)^2\rangle\simeq\langle(\Delta S_f)^2\rangle\simeq\langle(\Delta S_b)^2\rangle\simeq \tfrac{1}{2} \ . \quad (8b)$$

Interestingly, in a three-port conductor, the universality of the fluctuations of the reflection and transmission probabilities gives rise to the results of Eqs. (1) and (2) which are not universal.

To arrive at Eqs. (8) we use an argument given in Ref. 15, respectively, a recent refinement of this argument by Lee.[7] As shown in Ref. 15, the fluctuations in the transmission probabilities $T_{ij,kl}$ are of the same order as the transmission probability itself. Reference 15 found $T_{ij,kl}=t+\delta t$, where t is the ensemble average and is positive, and δt is the fluctuation and can be negative or positive but is of the order of t. Similarly, the reflection probabilities obey $R_{ii,kl}=r+\delta r$, where r is the average and δr is the fluctuation. r and δr are of the same order. Whereas the transmission probabilities into different channels might be correlated[4] or only a number of them less than N^2 might effectively be nonzero,[6] such correlations should be absent in the reflection coefficients.[7] We find $\langle R_{ii}\rangle=\sum_{k,l}\langle R_{ii,kl}\rangle\cong N^2 r$ and $\langle(R_{ii}-\langle R_{ii}\rangle)^2\rangle\cong N^2(\delta r)^2$. The average value of the reflection probabilities gives a contribution to the total reflection probability which grows like N^2, whereas the fluctuations, because of their random sign, give a contribution which grows only like N. Since carriers incident from the reservoirs are reflected back into the reservoir with a probability which is almost one, $\langle R_{ii}\rangle=N$, we have $r=\delta r=O(1/N)$. Hence the fluctuations of the reflection probabilities $\Delta R_{ii}\cong N\delta r$ are of order 1 as stated in Eq. (8a). The total probabilities (average plus fluctuations) are related because of current conservation. We have $R_{11}+T_c+S_b=N$ and $R_{22}+T_c+S_f=N$ and hence we obtain

$$1\cong\langle(\Delta R_{11})^2\rangle=\langle(\Delta T_c)^2\rangle+\langle(\Delta S_b)^2\rangle+2\langle\Delta T_c\Delta S_b\rangle$$

and

$$1\cong\langle(\Delta R_{22})^2\rangle=\langle(\Delta T_c)^2\rangle+\langle(\Delta S_f)^2\rangle+2\langle\Delta T_c\Delta S_f\rangle \ .$$

Carriers emanating from different ports experience different portions of the conductor with differing intensity, and ensemble averaging can thus be expected to destroy correlations between the probabilities for transmission into different ports. This is in contrast to the correlations which exist between the probabilities for transmission into the same port into different quantum channels. Correlations between transmission probabilities into the same port arise because the Feynman paths contributing to these transmission probabilities are identical over a large region of the sample.[7] On the other hand, if we consider transmission probabilities of differing ports, say $S_b=T_{13}$ and $S_f=T_{23}$, the Feynman paths traverse the same disordered region only over the length of the z segment and explore the x and y segments with differing intensity. Clearly paths from ports 3 to 1 which cross the junction more than once are less likely the larger the excursions into the y segment are. In the absence of correlations, we have $1=\langle(\Delta T_c)^2\rangle+\langle(\Delta S_f)^2\rangle$ and $1=\langle(\Delta T_c)^2\rangle+\langle(\Delta S_b)^2\rangle$ and hence the magnitude of the fluctuations in the transmission probabilities is given by Eq. (8b).

Let us now explore the consequences of Eq. (8) and discuss the resulting voltage and conductance fluctuations. The voltage measured between reservoirs 1 and 3 is $eV=\mu_1-\mu_3$. For the fluctuations $\Delta V=V-\langle V\rangle$, we obtain, using Eq. (4),

$$\langle(\Delta V)^2\rangle=\frac{S_f^2\langle\Delta S_b^2\rangle+S_b^2\langle\Delta S_f^2\rangle}{(S_f+S_b)^4}(V_\phi)^2 \ . \quad (9)$$

Using Eq. (6) and Eqs. (8) and taking into account that $\mathcal{R}_\phi=(h/e^2)(l_\phi/Nl_e)$, yields for the voltage fluctuations Eq. (1) with

$$\alpha=q^2(x^2+y^2)/(x+y)^4 l_\phi^2 \ . \quad (10)$$

Equation (1), together with Eq. (10), is our key result. The potential fluctuations given by Eqs. (1) and (10) are basically independent of the position $x\equiv l_v$ of the voltage probe in agreement with Benoit *et al.*[1] If the probe is located symmetrically, i.e., for $x=y=z=l_\phi/2$, we find $\alpha=\frac{9}{32}$. If the lead is only an elastic length away from the reservoir 1, we have $x\cong l_e$, $y=l_\phi-l_e\cong l_\phi$, and $z=l_\phi/2$. For this case, we find $\alpha=\frac{1}{4}$. Using $\mathcal{R}_\phi=(h/e^2)(l_\phi/\zeta)$, Eq. (1) becomes

$$\langle(\Delta V)^2\rangle/V_\phi^2=(\alpha/2)(l_\phi/\zeta)^2.$$

Here $\zeta\equiv Nl_e$ is the "localization length."[17] The potential fluctuations are the larger the bigger the ratio l_ϕ/ζ. In a conductor with $l_\phi\cong\zeta$, the potential fluctuations are of the order of the average potential drop $eV_\phi=\mu_1-\mu_2$.

Because we have studied the case where the potentials μ_1 and μ_2 are fixed, the potential fluctuations given by Eq. (10) should approach zero as the voltage probe approaches one of the reservoirs. Equation (10) cannot describe this approach since the transmission probabilities given in Eq. (6) are only valid for segments which are large compared to l_e. That Eq. (10) remains length independent even as x approaches zero is thus an indication that the correlation length of the voltage fluctuations is short and comparable to the elastic scattering length.

Next consider the fluctuations of the conductance $G_x=(\mathcal{R}_x)^{-1}$. We linearize Eq. (5) with respect to the fluctuations of the transmission probabilities and determine the mean-squared average using Eq. (8). We find

$$\langle(\Delta G_x)^2\rangle=\frac{1}{2S_f^4}[S_f^2(S_b+S_f)^2+S_f^2(T_c+S_f)^2+T_c^2S_b^2] \ . \quad (11)$$

Using Eq. (6) yields Eq. (2) with

$$\beta=[x^2(x+y)^2+x^2(y+z)^2+z^2y^2]/(x+y)^4 \ . \quad (12)$$

β is essentially length independent and of order 1. For $x=y=z=l_\phi/2$, we find $\beta=\frac{9}{16}$ and for $x\cong l_e$, $y=l_\phi-l_e\cong l_\phi$, $z=l_\phi/2$, we find $\beta=\frac{1}{4}$. Thus the conductance fluctuations, measured with voltage probes a distance apart which is short compared to an inelastic scattering length, are determined by Eq. (2). Note that even as x approaches l_ϕ the conductance fluctuations do not approach $(e^2/h)^2$ but a value smaller than that. We find $\frac{5}{8}(e^2/h)^2$. Coupling or decoupling a lead to the conductor does effect the size of the conductance fluctuations: The fluctuations of the total conductance $G=(\mathcal{R}_x+\mathcal{R}_y)^{-1}$ are given by $\langle(\Delta G)^2\rangle=\frac{9}{16}(e^2/h)^2$ if the voltage probe is in the center, $x=y$, and approach $\langle(\Delta G)^2\rangle=(e^2/h)^2$ only in the limit that either x or y is small.

Let us now depart from the typical experimental situation discussed above and study the effect of the length dependence of the voltage probe. For this discussion we take a voltage probe made out of a different material with the same elastic properties as the conductor but with a different inelastic scattering length. We consider the two limits, where the carriers "see" a portion of the voltage probe of length $z=l_e$, and $z=\zeta$. Let us focus on the potential fluctuations. In the limit that carriers entering the voltage probe suffer an inelastic event after only an elastic length, we find that $\alpha\cong x^2(l_\phi-x)^2/l_\phi^4$. The voltage fluctuations depend on the lead position. The maximum excursions of the voltage away from the average occur when the probe is in the middle of the conductor. In the "physical" situation described by Eqs. (1) and (2), the forward and backward scattering probabilities are of order Nl_e/l_ϕ independent of the lead position. In the presence of a short lead, however, the probability S_b becomes of order N as x becomes small and is of order Nl_e/l_ϕ only if the probe is near the midpoint between the two reservoirs. The key point is that in this case the relative magnitude of the fluctuations $\Delta S_b/S_b$ is x dependent. That the voltage fluctuations are length independent in the physical situation is thus a consequence of large excursions of carriers into the voltage lead. If the carriers in the voltage lead can travel a distance ζ, that is, as far as a localization length before being scattered inelastically, we obtain $\alpha\cong\zeta^2/l_\phi^2$. If this is inserted into Eq. (1), we find that $\langle\Delta V^2\rangle\cong V_\phi^2$, i.e., the voltage fluctuations are of the order of the average potential drop itself. In this case the fluctuations of S_b and S_f are of order 1: that is, we have fluctuations for which either S_b or S_f is zero. As can be seen by looking at Eq. (4), this causes fluctuations of order $(\mu_1-\mu_2)=eV_\phi$. Finally, we can also consider the case where there is very little elastic scattering in the conductor but the voltage probe remains disordered. For $x=y=l_e$ we have $T_c\sim N$, that is, we have perfect transmission from reservoir 1 to reservoir 2. In this case $\langle(\Delta G_x)^2\rangle$ is determined by the square of a Sharvin contact conductance,[10,18] $(e^2/h)^2N^2$. The voltage fluctuations are given by $\langle(\Delta V)^2\rangle\cong(z^2/\zeta^2)V_\phi^2$. Thus even a perfect conductor exhibits strong voltage fluctuations when the probes are disordered conductors! This case serves as a good illustration of the difference of the approach presented here and in Refs. 11–16. Using the potentiometers invoked in these discussions, one finds for a perfect conductor a potential $\mu_3=(\mu_1+\mu_2)/2$ without any fluctuations at all. In contrast, our approach yields this potential only on the average and yields fluctuations in the measured potential because the quantum channels in the disordered lead couple randomly to those of the perfect conductor. Only on the average is the coupling of the lead symmetric with respect to right- and left-moving carriers in the perfect lead. Admittedly, the cases considered in this paragraph do not correspond to the present experiments, but if leads and conductors can be made out of different materials they can be realized too. The present experiments are characterized by the fact that the current and voltage probes are of the same material and are interchangeable. Taking this into account, I was able to account for the experimentally observed conductance and voltage fluctuations as well as the general symmetry properties of a four-probe resistance measurement.[8]

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