

Transport relaxation time of a two-dimensional electron gas due to scattering by surface acoustic waves

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We calculate the dc electrical conductivity σ of a two-dimensional electron gas interacting with surface acoustic waves in a quantum well. It is shown that in the low-temperature regime, the behavior of the conductivity in terms of a characteristic temperature Θ_m follows a fourth-power law and a linear law in Θ_m/T , respectively, for $T \ll \Theta_m$ and $\Theta_m \ll T \ll E_F/k_B$.

It is well established by now that scattering by longitudinal optical (LO) phonons is the important mechanism determining the room temperature mobility of the two-dimensional electron gas (2DEG) in $\text{Ga}_{1-x}\text{Al}_x\text{As-GaAs}$ heterostructures in the absence of a magnetic field.¹ The interaction of the 2DEG with bulk vibrations have been studied by Das Sarma *et al.*² using the fact that in those systems the electron-phonon coupling is very small and so a perturbative treatment can be used. More recently, da Cunha Lima *et al.*³ calculated the relaxation time of the 2DEG interacting with bulk longitudinal acoustical (LA) and LO phonons, using a memory-function—projection-operator formalism.

In some experimental situations, depending on the heterostructure architecture, the scattering by surface acoustic waves near the interface may become also relevant. That should be the case, for instance, in doped heterostructures, where the 2DEG lies close to the interface between GaAs and $\text{Ga}_{1-x}\text{Al}_x\text{As}$.⁴ Moreover, experiments on the effect of this interaction on the electrical conductivity of the 2DEG under strong magnetic field has been reported recently by Wixforth *et al.*⁵

In this work we calculate the transport relaxation time of the 2DEG due to scattering by 2D longitudinal acoustic phonons using the memory-function—projection-operator formalism. This technique becomes very convenient in the case of LA phonons, since it can reproduce exactly, for 3D LA phonons, the classical Boltzmann equation results⁶ and allows several extensions suitable to describe more complex situations.⁷

We start with the Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + U, \quad (1)$$

where $c_{\mathbf{k}}^{\dagger}$ ($c_{\mathbf{k}}$) are the electron creation (annihilation) operators with wave number \mathbf{k} , $a_{\mathbf{q}}^{\dagger}$ ($a_{\mathbf{q}}$) are creation (annihilation) operators for phonons of momentum \mathbf{q} and U is the electron-phonon interaction potential given by

$$U = \sum_{\mathbf{q}} F(\mathbf{q}) \rho(\mathbf{q}) A(\mathbf{q}), \quad (2)$$

$\rho(\mathbf{q})$ is the Fourier transform of the electron density operator and $A(\mathbf{q}) = a_{\mathbf{q}} + a_{-\mathbf{q}}^{\dagger}$. The coupling constant $F(\mathbf{q})$ with longitudinal vibrations, given by⁸

$$F(\mathbf{q}) = (2m_i N \omega_{\mathbf{q}})^{-1/2} q \Xi_d, \quad (3)$$

results from the deformation potential of the 2D system; Ξ_d is the deformation constant due to dilation, m_i is the ionic mass, and N the number of lattice cells.

The dc electrical conductivity for the 2DEG, $\sigma = Ne^2 \tau / m^*$, m^* being the effective electronic mass and N the carrier density, can be obtained via the memory function $\tau^{-1} = \text{Im} M_{xx}(0)$. In the zeroth-order approximation it can be expressed in terms of the force-force correlation function $\Pi_{xx}^R(\omega)$:

$$M_{xx}^{(0)}(0) = - \frac{1}{N_e m^*} \lim_{\omega \rightarrow 0} \Pi_{xx}^R(\omega), \quad (4)$$

where

$$\Pi_{\alpha\beta}^R(\omega) = -i \int_{-\infty}^{+\infty} \Theta(t) \langle [U_{\alpha}(t), U_{\beta}(0)] \rangle e^{i\omega t} dt. \quad (5)$$

$\Theta(t)$ is the Heaviside step function, $\Theta(t) = 1$ if $t > 0$ and $\Theta(t) = 0$ if $t < 0$; U_{α} is the generalized force acting on the center of mass of the 2DEG system, $U_{\alpha} = \partial U(\mathbf{q}) / \partial R_{\alpha}$, α and β being indices which label the x and y coordinates R_{α} .

The Hamiltonian (1) yields

$$U_{\alpha} = i \sum_{\mathbf{q}} q_{\alpha} F(\mathbf{q}) \rho(\mathbf{q}) A(\mathbf{q}). \quad (6)$$

After some algebra, we can express the imaginary part of Π_{xx}^R in terms of the imaginary part of the dielectric constant ϵ_2 of the electron gas in random-phase approximation (RPA):

$$\text{Im}\Pi_{xx}^R(\omega) = -\frac{1}{4\pi} \int dq q^3 |F(q)|^2 \frac{e^{\beta\omega q}(e^{\beta\omega} - 1)}{(e^{\beta\omega q} - 1)(e^{\beta(\omega q + \omega)} - 1)} \frac{\epsilon_2(q, \omega q + \omega)}{V_q} - \text{term } (\omega \rightarrow -\omega). \quad (7)$$

Notice that to work out the above equation we have assumed isotropy in q space. V_q is the Fourier transform of the Coulomb potential. The 2D $\epsilon_2(q, \omega)$ is easily obtained:

$$\epsilon_2(q, \omega) = \frac{m^* V_q}{2\pi \epsilon_q^{1/2}} \left[\int_{E_-}^{\infty} d\epsilon f(\epsilon) (\epsilon - E_-)^{-1/2} - \int_{E_+}^{\infty} d\epsilon f(\epsilon) (\epsilon - E_+)^{-1/2} \right], \quad (8)$$

where $\epsilon_q = q^2/2m^*$, $f(\epsilon)$ is the Fermi-Dirac distribution function and

$$E_{\pm} = \frac{(\epsilon_q \pm \omega)^2}{4\epsilon_q}.$$

In the low-temperature regime, i.e., $k_B T \ll E_F$, E_F being the Fermi energy, one has

$$\frac{\epsilon_2(q, \omega)}{V_q} = \frac{m^*}{\pi \epsilon_q^{1/2}} [(E_F - E_-)^{1/2} \Theta(E_F - E_-) - (E_F - E_+)^{1/2} \Theta(E_F - E_+)] + O(T^2). \quad (9)$$

Introducing this result into Eq. (7) and after performing the limit $\omega \rightarrow 0$ in Eq. (4), we get

$$\tau^{-1} \cong \frac{D^2}{16\pi^2} \frac{(2m^*)^{3/2}}{E_F^{1/2} C^5} \frac{(k_B \Theta_m)^4}{m^* N_e m_i N} \left[\frac{T}{\Theta_m} \right]^4 J_4 \left[\frac{\Theta_m}{T} \right], \quad (10)$$

where we have assumed the Debye approximation, i.e., $\omega = Cq$ up to a maximum $q_m = 2K_F$ that correspond to a maximum momentum transfer to the electron at low temperature. We can associate with q_m a temperature Θ_m which defines a temperature scale in our problem: $\Theta_m = 2K_F C/k_B$. In the case of $\text{Ga}_{1-x}\text{Al}_x\text{As-GaAs}$ heterostructures, $\Theta_m \approx 10$ K. J_n is the Debye integral:⁹

$$J_n(y) = \int_0^y x^n \frac{e^x}{(e^x - 1)^2} dx. \quad (11)$$

The asymptotic values of $J_4(y)$ are

$$J_4(y) = \begin{cases} 4 \frac{\pi^4}{15} & \text{as } y \text{ goes to infinity,} \\ \frac{y^3}{3} & \text{as } y \text{ goes to zero.} \end{cases} \quad (12)$$

Consequently, for $T \ll \Theta_m$ one has

$$\sigma = A (\Theta_m/T)^4, \quad (13)$$

and for $T \gg \Theta_m$

$$\sigma = B (\Theta_m/T), \quad (14)$$

where A and B are constants.

Notice that the temperature dependence of the relaxation time appears as in Eq. (11), as $(T/\Theta_m)^n J_n(\Theta_m/T)$. This gives us different power laws in Θ_m/T for the two temperature regimes $T \ll \Theta_m$ and $T \gg \Theta_m$. It has been shown in Ref. 3 that $n=5$ is obtained for a three-dimensional (bulk) LA-phonon system (3DPS) interacting either with a 2DEG or a 3DEG. In both cases, for $T \ll \Theta_m$ the power law for the conducting obtained there is $(\Theta_m/T)^5$. In the present case, where we study the 2DPS interacting with the 2DEG via LA phonons, we obtain in Eq. (11) $n=4$, resulting, if $T \ll \Theta_m$, in a conductivity depending on $(\Theta_m/T)^4$. Therefore, we conclude that, for LA interaction, the power law is uniquely determined by the dimensionality of the phonon system. Moreover, the conductivity increases faster with the decrease of temperature in the case of 3DPS than in the case of 2DPS, enhancing the importance of the last one, as indeed observed by Wixforth *et al.*⁵

The mobility that we have obtained for $T \ll \Theta_m$ decreases more slowly than that obtained by Price.¹⁰ This is probably due to the screening caused by the electron gas on the interaction potential. In principle this can be included in our formalism and will be the subject of a future analysis.

On the other hand, in the more interesting regime $\Theta_m \ll T \ll E_F/k_B$, where screening effects can be disregarded, we have obtained for both 3DPS and 2DPS the dependence Θ_m/T , as shown in Eq. (14) which is indeed the experimentally observed dependence in the range $10 \text{ K} \leq T \leq 100 \text{ K}$. Hence, one can speculate that in this regime the LA phonon system loses the information of its dimensionality in the interaction with the electron gas.

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