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Magnetoplasma modes in thin films in the Faraday configuration

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We have undertaken a theoretical study of magnetoplasma waves in a thin, semiconducting film bounded in general, by dissimilar dielectric or conducting media. The applied magnetic field and the direction of propagation of the wave are parallel to the interfaces (Faraday geometry). The exact dispersion relation has been derived on the basis of local theory. An analytic solution for the propagation constant $q_z(\omega)$ has been found in the nonretarded limit, valid for $\omega/c \ll q_z \ll 1/d$, where d is the thickness of the film. There are two modes, the upper one having a negative group velocity ("backward wave"). These modes—magnetoplasma generalizations of the Fuchs-Kliewer modes—approach the asymptotic frequencies given by $[\epsilon_{xx}(\omega)\epsilon_{zz}(\omega)]^{1/2} = -\epsilon_i$, where $\epsilon_{ij}(\omega)$ is an element of the dielectric tensor of the semiconductor and ϵ_i is the dielectric constant of either one of the bounding media. In the symmetric configuration ($\epsilon_1 = \epsilon_3$), the two asymptotic frequencies coincide. We have also applied to the general dispersion relation a thin-film approximation, $q_z d \ll 1$. This enables us to find analytic solutions for $q_z(\omega)$ in two cases: (1) a very thin semiconducting overlayer on a metallic substrate and (2) a very thin, unsupported, magnetoplasma film. In both cases a splitting in the spectrum occurs in the vicinity of the hybrid cyclotron-plasmon frequency, with the creation of a gap.

I. INTRODUCTION

An electromagnetic wave traveling through a polarizable medium is characteristically modified and coupled with the polarization that it induces in the medium. This coupled mode of excitation is now well known as a polariton. Gradually interest has shifted from the bulk to surface and/or interface polaritons. The latter modes are localized at the crystalline surface and/or interface and decay exponentially away from it. Ever increasing interest in investigations, both experimental and theoretical, of surface or interface polaritons can be attributed to the fact that they have proved to be sensitive probes of surface and/or interface properties.[1,2]

Surface polaritons have been conveniently classified by Burstein *et al.*[3] and by Otto,[4] as summarized by Halevi.[5] In addition to surfaces, the coupling of plasmons and/or phonons across an interface has been also studied.[5] The effect of a magnetostatic field ($\mathbf{B}_0$), which causes various qualitative changes in the electromagnetic (EM) modes, has been investigated in different geometries by Chiu and Quinn,[6] Wallis *et al.*,[7] and Palik *et al.*[8] The applied magnetic field is generally taken to be either perpendicular or parallel to the surface. The most widely investigated configurations belong to the latter case, in which the propagation is either along (Faraday configuration) or across (Voigt configuration) the magnetic field.

Initially, the studies of polaritons at optical frequencies were mostly confined to the determination of the dispersion characteristics of the bulk and surface EM waves. Recently, considerable attention has been directed to the propagation distance[9–12] and the lifetime[13] of polaritons in thin metallic films. The dispersion and lifetime of polaritons that propagate along supported (when the bounding media are unidentical) and unsupported (when the bounding media are identical) thin films have been discussed by Fukui *et al.*[13] Their conclusion was that for an unsupported film there is a mode that has a lifetime which increases as the thickness of the film decreases, whereas for the supported film such a mode does not exist. Subsequently, Sarid[14] investigated the propagation length in a thin metallic film. He found that the two Fuchs-Kliewer modes behave quite differently as the film thickness is decreased: The propagation length of one these modes decreases, while that of the other increases. The latter mode is also associated with an enhancement of the EM fields in the bounding media. Hence, it is of interest to applications in nonlinear optics.[15–22]

In the case of magnetoplasma surface polaritons, semiconductors exhibit a particularly rich diversity of phenomena.[23] Since the free carrier concentration and hence the plasma frequency can be varied over a wide range, there is considerable flexibility in the choice of the spectral range for propagation. The relatively small effective mass of free carriers in many semiconductors, as compared to metals, leads to larger effects of the magnetic field on the surface-plasmon dispersion relation. The investigations of Wallis *et al.*[7] and of Halevi[24] for magnetoplasma polaritons in the Faraday configuration, respectively, for the semiconductor-vacuum and the semiconductor-metal interfaces, reveal some interesting behavior characteristics of the polaritons. For details, the reader is referred to review articles by the cited authors.[5,25]

In this paper we have carried out a theoretical investigation of magnetoplasma polaritons propagating along a lossy, semiconducting thin film bounded by two unidentical dielectric media. We are concerned with the Faraday configuration, i.e., the direction of propagation is parallel to the applied magnetic field. It should be noted that in the absence of $\mathbf{B}_0$ all the three media are isotropic. It is worthwhile mentioning that, although there are numerous publications[25] on magnetoplasma modes, we are not aware of similar studies in a thin-film configuration.

There are several motivations behind the present investigation. (i) We wish to explore the possibility of low-frequency modes or "thin-film helicons." The existence of such modes *for a single interface* has been disputed in spite of experimental reports by Baibakov and Datsko.[26] These experimental observations also found theoretical support for a semiconductor bounded by vacuum, by a metallic screen, and by a ferrite (see references in Ref. 27). However, the validity of these calculations was questioned by Halevi and Quinn,[27] Halevi[28] (for propagation at an angle to the applied field), and Boardman and Irving[29] (for the semiconductor-ferrite interface). Moreover, in an experimental work, Laurinavichyus and Malakauskas[30] found no evidence of a surface-helicon wave. In view of this, it is interesting to inquire if, possibly, a thin-film structure does support low-frequency magnetoplasma modes. (ii) We have previously noted the considerable interest in long-range propagation in thin films related to studies of nonlinear effects and surface roughness. The effect of a magnetic field applied in the direction of propagation is an open question. (iii) The understanding of the behavior of magnetoplasma waves in thin films may lead to device applications.

The present paper is organized as follows. In Sec. II we derive the general dispersion relation for magnetoplasma polaritons in the geometry depicted in Fig. 1. In Sec. III we study the magnetoplasma modes in the nonretarded limit ($c \to \infty$). In Sec. IV we simplify our general dispersion relation by assuming that the film is very thin, and investigate two cases of interest: (a) surface polaritons modified by a magnetized overlayer, and (b) a magnetized film bounded by two identical dielectric media. Some details and mathematical proofs have been relegated to the Appendices.

The limiting cases studied in Secs. III and IV will enable us to plot dispersion relations for the magnetoplasma polaritons in thin films, Figs. 2–5. These results are expected to be reasonably accurate in their respective regions of applicability. However, it should be realized that additional polariton branches (in frequency–wave-vector regions that are not covered by our present restrictions) may arise. This possibility will have to await an exact numerical calculation, to be reported in the future.

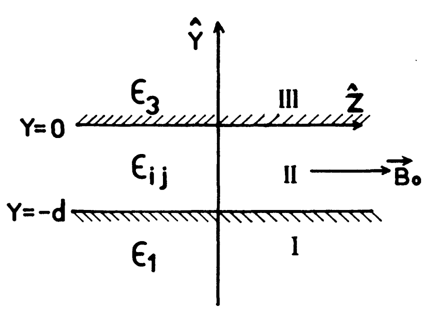

FIG. 1. Schematics of the configuration studied in the present paper. The applied magnetic field $\mathbf{B}_0$ and the direction of propagation of the waves considered are parallel to the two interfaces of the film. We will refer to the cases $\epsilon_1 \neq \epsilon_3$ and $\epsilon_1 = \epsilon_3$ as the asymmetric and symmetric configurations, respectively.

II. GENERAL DISPERSION RELATION

We consider a semiconducting medium (II) of finite thickness characterized by the dielectric tensor $\tilde{\epsilon}$ which is assumed to be independent of the wave vector. Two media, I and III, characterized, respectively, by the dielectric constants, ϵ_1 and ϵ_3, bound the medium II. The magnetostatic field $\mathbf{B}_0$ is assumed to be oriented parallel to the interfaces which are perpendicular to the $\hat{\mathbf{y}}$ axis. The three media constitute the geometry shown in Fig. 1. The direction of B_0 (i.e., $+\hat{\mathbf{z}}$ axis) is the direction of the wave propagation, i.e., we are concerned with the Faraday configuration.

We start with Maxwell's curl field equations. After eliminating the magnetic field variable ($\mathbf{B}$), we obtain the following wave equation for the macroscopic electric field ($\mathbf{E}$):

$$\nabla\times(\nabla\times\mathbf{E})-q_0^2\tilde{\epsilon}\cdot\mathbf{E}=0\ , \tag{1}$$

where q_0 (is equal to ω/c, ω being the angular wave frequency and c the velocity of light in vacuum) is the vacuum wave vector. We assume that the spatial and temporal dependence of the fields is of the form $\sim e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ and $q_x=0$, where q_x is the $\hat{\mathbf{x}}$ component of the wave vector $\mathbf{q}$. In the present configuration (i.e., $\mathbf{B}_0 \| \hat{\mathbf{z}}$) the dielectric tensor ($\tilde{\epsilon}$) is simplified by the symmetry requirements that $\epsilon_{xx}=\epsilon_{yy}$, $\epsilon_{yx}=-\epsilon_{xy}$, and $\epsilon_{xz}=\epsilon_{zx}=\epsilon_{yz}=\epsilon_{zy}=0$. As such, Eq. (1) may be rewritten as follows:

$$\begin{bmatrix} q_0^2\epsilon_{xx}-q_y^2-q_z^2 & q_0^2\epsilon_{xy} & 0 \\ -q_0^2\epsilon_{xy} & q_0^2\epsilon_{xx}-q_z^2 & q_yq_z \\ 0 & q_yq_z & q_0^2\epsilon_{zz}-q_y^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}=0\ . \tag{2}$$

The Cartesian elements of the dielectric tensor ($\tilde{\epsilon}$) are given in Appendix A for a simple model. It is worthwhile to point out that most of the (analytical) results in this paper are independent of our particular model (A1). For instance, we could easily incorporate the frequency dependence of the background dielectric constant which allows for the coupling of magnetoplasmons to the phonons (see Appendix A). Equation (2) is a set of three linear equations satisfied by the electric field in the dispersive, anisotropic semiconducting medium II. The same set of three equations also gives valid solutions of Maxwell's equations in the isotropic media I and III, if we just take $\epsilon_{xy}=0$ and $\epsilon_{xx}=\epsilon_{zz}=\epsilon_i$ ($i\equiv 1,3$ for media I and III, respectively). The nontrivial solution of such a set of three linear equations requires the vanishing of the determinant of the coefficients. This gives two solutions for q_y given by

$$-q_y^2=\beta_\pm^2=\frac{1}{2\epsilon_{xx}}(\,[(\epsilon_{xx}+\epsilon_{zz})k^2-q_0^2\epsilon_{xy}^2] \pm\{\,[(\epsilon_{xx}-\epsilon_{zz})k^2-q_0^2\epsilon_{xy}^2]^2 -4q_z^2q_0^2\epsilon_{xy}^2\epsilon_{zz}\}^{1/2})\ , \tag{3}$$

where

$$k^2=q_z^2-q_0^2\epsilon_{xx} \tag{4}$$

in the semiconducting medium, and

$$-q_y^2=\alpha_i^2=q_z^2-q_0^2\epsilon_i,\quad i=1,3 \tag{5}$$

in the bounding media. In Eqs. (3) and (5), $\beta_\pm(=\pm iq_y)$ refers to the decay constants in medium II and $\alpha_i(=\pm iq_y)$ to those in media I ($i=1$) and III ($i=3$); see Eqs. (7)–(9) below.

We write the spatial and temporal dependence of the fields in the three media in the form (see Fig. 1)

$$\mathbf{E}(r,t)=\mathbf{E}(y)e^{i(q_zz-\omega t)}\ , \tag{6}$$

where $\mathbf{E}(y)$ for the regions I ($y<-d$), II ($-d<y<0$), and III ($y>0$) are expressed as follows:

$$\mathbf{E}^{\rm I}(y)=\mathbf{E}^{\rm II}e^{\alpha_1 y}\ , \tag{7}$$

$$\mathbf{E}^{\rm II}(y)=\mathbf{E}_1e^{-\beta_+y}+\mathbf{E}_2e^{\beta_+y}+\mathbf{E}_3e^{-\beta_-y}+\mathbf{E}_4e^{\beta_-y}\ , \tag{8}$$

and

$$\mathbf{E}^{\rm III}(y)=\mathbf{E}^{\rm III}e^{-\alpha_3 y}\ . \tag{9}$$

Analogous solutions can be written for the magnetic field ($\mathbf{B}$) in the three regions.

In order to determine the dispersion relation for the magnetoplasma modes in the existing configuration, the fields on the two sides of both interfaces ($y=0$ and $y=-d$) have to be matched. The boundary conditions are the continuity of the tangential components of the electric and magnetic fields, that is, the field components E_x, E_z, B_x, and B_z. The use of Faraday's law and of Eq. (2) enables one, after some algebra, to express the E_z, B_x, and B_z components in region II in terms of the E_x components in the same region. Similarly, the B_x and B_z components in regions I and III are expressible, respectively, in terms of E_z and E_x. This greatly reduces the number of unknowns involved; see Appendix B for mathematical details of the conversion of the tangential field components, as described above. The matching of the fields at the two interfaces gives the following relations.

At $y=0$:

$$E_x^{\rm III}=E_{1x}+E_{2x}+E_{3x}+E_{4x}\ , \tag{10}$$

$$\left[\frac{-iq_0^2\epsilon_{xy}}{q_z}\right]E_z^{\rm III}=-\beta_+A_+E_{1x}+\beta_+A_+E_{2x} -\beta_-A_-E_{3x}+\beta_-A_-E_{4x}\ , \tag{11}$$

$$\left[\frac{iq_0^2\epsilon_{xy}\epsilon_3}{q_z\epsilon_{xx}\alpha_3}\right]E_z^{\rm III}=A_+E_{1x}+A_+E_{2x}+A_-E_{3x}+A_-E_{4x}\ , \tag{12}$$

$$-\alpha_3E_x^{\rm III}=-\beta_+E_{1x}+\beta_+E_{2x}-\beta_-E_{3x}+\beta_-E_{4x}\ . \tag{13}$$

At $y=-d$:

$$e^{-\alpha_1d}E_x^{\rm I}=e^{\beta_+d}E_{1x}+e^{-\beta_+d}E_{2x}+e^{\beta_-d}E_{3x}+e^{-\beta_-d}E_{4x}\ , \tag{14}$$

$$\left[\frac{-iq_0^2\epsilon_{xy}}{q_z}\right]e^{-\alpha_1d}E_z^{\rm I}=-\beta_+A_+e^{\beta_+d}E_{1x}+\beta_+A_+e^{-\beta_+d}\times E_{2x}-\beta_-A_-e^{\beta_-d}E_{3x} +\beta_-A_-e^{-\beta_-d}E_{4x}\ , \tag{15}$$

$$\left[\frac{-iq_0^2\epsilon_{xy}\epsilon_1}{q_z\epsilon_{zz}\alpha_1}\right]e^{-\alpha_1d}E_z^{\rm I}=A_+e^{\beta_+d}E_{1x}+A_+e^{-\beta_+d}E_{2x} +A_-e^{\beta_-d}E_{3x}+A_-e^{-\beta_-d}E_{4x}\ , \tag{16}$$

$$\alpha_1e^{-\alpha_1d}E_x^{\rm I}=-\beta_+e^{\beta_+d}E_{1x}+\beta_+e^{-\beta_+d}E_{2x}-\beta_-e^{\beta_-d}E_{3x} +\beta_-e^{-\beta_-d}E_{4x}\ , \tag{17}$$

where $A_\pm$ are defined as follows:

$$A_\pm=\frac{k^2-\beta_\pm^2}{\beta_\pm^2+q_0^2\epsilon_{zz}}\ . \tag{18}$$

Equations (10)–(17) are eight homogeneous equations in terms of eight unknown amplitudes—$E_x^{\rm I}$, $E_z^{\rm I}$, $E_x^{\rm III}$, $E_z^{\rm III}$, E_{1x}, E_{2x}, E_{3x}, and E_{4x}. The condition for the nontrivial solution of such a set of equations (i.e., vanishing of the determinant of the coefficients) gives the required dispersion relation for the magnetoplasma modes. After a considerable amount of algebra we derive the following implicit general disperion relation for $q_z(\omega)$:

$$\begin{aligned}
&A_+^2[(\alpha_1\alpha_3+\beta_-^2)T_-+(\alpha_1+\alpha_3)\beta_-][(\alpha_1\alpha_3\epsilon_{zz}^2+\epsilon_1\epsilon_3\beta_+^2)T_++\epsilon_{zz}(\alpha_3\epsilon_1+\alpha_1\epsilon_3)\beta_+]\\
&\quad+A_-^2[(\alpha_1\alpha_3+\beta_+^2)T_++(\alpha_1+\alpha_3)\beta_+][(\alpha_1\alpha_3\epsilon_{zz}^2+\epsilon_1\epsilon_3\beta_-^2)T_-+\epsilon_{zz}(\alpha_3\epsilon_1+\alpha_1\epsilon_3)\beta_-]\\
&\quad-A_+A_-\{[(\alpha_1\alpha_3\epsilon_{zz}+\epsilon_1\beta_-^2)T_-+(\alpha_1\epsilon_{zz}+\alpha_3\epsilon_1)\beta_-][(\alpha_1\alpha_3\epsilon_{zz}+\epsilon_3\beta_+^2)T_++(\alpha_3\epsilon_{zz}+\alpha_1\epsilon_3)\beta_+]\\
&\quad+[(\alpha_1\alpha_3\epsilon_{zz}+\epsilon_3\beta_-^2)T_-+(\alpha_3\epsilon_{zz}+\alpha_1\epsilon_3)\beta_-][(\alpha_1\alpha_3\epsilon_{zz}+\epsilon_1\beta_+^2)T_++(\alpha_1\epsilon_{zz}+\alpha_3\epsilon_1)\beta_+]\}\\
&\quad+2A_+A_-\alpha_1\alpha_3\beta_+\beta_-(\epsilon_{zz}-\epsilon_1)(\epsilon_{zz}-\epsilon_3)(1-T_+^2)^{1/2}(1-T_-^2)^{1/2}=0\ ,
\end{aligned} \tag{19}$$

where the $T_\pm$ are defined as

$$T_\pm = \tanh(\beta_\pm d) \ . \tag{20}$$

We are interested in the propagating-wave solutions of Eq. (19), namely, modes such that q_z is real when absorption is neglected. Then α_1 and α_3, given by Eq. (5), are either real or pure imaginary quantities. The latter case is of certain interest in waveguide theory ("substrate modes" and "air modes").[31] In the present work we will limit our attention to solutions which decay exponentially away from both interfaces of the film. Such solutions are characterized by both α_1 and α_3 being real and positive quantities. The magnetoplasma modes with real q_z, α_1, and α_3 may be classified according to the nature of β_+ and β_-, given by Eq. (3). Depending on the spectral region the following possibilities may arise.

(i) β_+ and β_- are both real and positive (we may always choose the positive root of $\beta_\pm^2$ if this quantity is positive). This corresponds to ordinary *polariton modes* decaying away from both interfaces inside the film, as well as outside.

(ii) β_+ and β_- are both pure imaginary (one may choose $\mathrm{Im}\beta_\pm > 0$). These are ordinary *waveguide* (WG) *modes* with an oscillatory field dependence inside the film.

(iii) β_+ is real and β_- is pure imaginary, or vice versa. In the case of a surface this would correspond to the superposition of two components: one that decays exponentially in the semiconductor (the surface component) and the other having an oscillatory character with constant amplitude (the bulk component). In the case of a surface, such modes are not well-behaved and are called "pseudosurface waves."[25] On the other hand, in the case of a thin film, both wave components are confined to the interior of the film and, as long as α_1 and α_3 are both real, these are bona fide, propagating waves which are *hybrid* polariton-waveguide *modes.*

(iv) β_+ and β_- are complex conjugates of each other. In the case of a surface both components decay into the semiconductor in an oscillatory fashion; one corresponds to a plane wave leaving the surface, the other to a plane wave approaching it. As far as we are aware these "generalized surface waves"[25] have not been detected experimentally. In case of a thin film, again, such modes are well-behaved. We will refer to them as *complex modes.*

This classification into polariton, waveguide, hybrid, and complex modes is valid only if dissipation is neglected ($\nu=0$). With allowance for absorption ($\nu\neq 0$), q_z, α_1, α_3, β_+, and β_- are, of course, all complex quantities. In this paper we will assume that $\nu=0$, whence ϵ_{xx} and ϵ_{zz} are real and ϵ_{xy} is a pure imaginary quantity. Effects of absorption will be dealt with in a future publication.

It is worthwhile to point out that the dispersion relation, Eq. (19), has been checked by imposing various spacial limits, *viz.*, $d=0$, $d\to\infty$, and $\mathbf{B}_0=0$. It is found that within these special limits our general dispersion relation reproduces exactly the results previously reported for a surface ($\mathbf{B}_0\neq 0$) (Ref. 25) and for a thin film[12,13,31] in the absence of an applied magnetic field ($\mathbf{B}_0=0$).

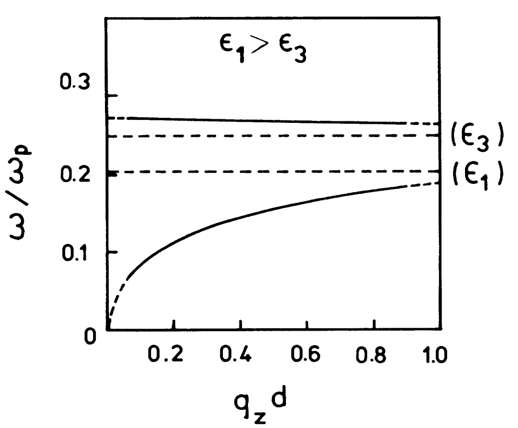

FIG. 2. Normalized frequency ω/ω_p versus normalized propagation constant $q_z d$ in the nonretarded limit for the asymmetric configuration. The two branches (solid lines) are solutions of Eq. (28), which rests on the assumption that $\omega/c \ll q_z \ll 1/d$. The dotted lines indicate that our approximations fail for very small and for very large values of q_z. The asymptotic limits $q_z\to\infty$ are indicated by dashed lines. These lines coincide with the asymptotic solutions for the two decoupled interfaces. The parameters are $\epsilon_1=11.683$, $\epsilon_3=1$, $\epsilon_L=15.7$, $\omega_c/\omega_p=0.1$, and $\nu=0$. This corresponds to an InSb film on a Si substrate in the far infrared.

III. NONRETARDED LIMIT

In the nonretarded (NR or electrostatic) limit we assume that $q_z \gg q_0$; mathematically, this is equivalent to taking $c\to\infty$. Then $\alpha_1=\alpha_3=k=q_z$, $\beta_+=q_z$, and $\beta_-=(\epsilon_{zz}/\epsilon_{xx})^{1/2}q_z$. By definition $\alpha_i>0$; therefore, we limit propagation to $q_z>0$. Consequently, the general dispersion relation, Eq. (19), becomes

$$\left[\epsilon_{zz}^2+\epsilon_1\epsilon_3\frac{\epsilon_{zz}}{\epsilon_{xx}}\right]\tanh[(\epsilon_{zz}/\epsilon_{xx})^{1/2}q_z d] + \epsilon_{zz}(\epsilon_1+\epsilon_3)(\epsilon_{zz}/\epsilon_{xx})^{1/2}=0 \ . \tag{21}$$

This is the dispersion relation for the magnetoplasma polaritons in the NR limit for an arbitrary thickness of the semiconducting film. Interestingly, the off-diagonal element ϵ_{xy} has dropped out of the calculation and is absent in Eq. (21). This should mean that, in the nonretarded limit, the transverse (Hall) field is negligible. Thus there is no dynamical Hall effect and we do not expect helicon-like modes for $q_z \gg q_0$. This however does not preclude long-range propagation under suitable conditions. We will analyze Eq. (21) in two cases.

The case $q_z\to\infty$. First we consider the case $q_z \gg 1/d$; taken together with $q_z\gg q_0$, this implies that $q_z\to\infty$. Because β_- must be positive, $\epsilon_{xx}(\omega)$ and $\epsilon_{zz}(\omega)$ must have the same algebraic sign, and the positive root $(\epsilon_{zz}/\epsilon_{xx})^{1/2}$ must be taken. Then in the limit $q_z\to\infty$ the hyperbolic tangent in Eq. (21) may be replaced by unity. Now, if ϵ_{xx} and ϵ_{zz} are both positive then, for $\epsilon_i>0$, both terms in Eq. (21) are positive and this equation cannot be satisfied. Clearly, the correct behavior is given by

$$\epsilon_{xx}(\omega)<0, \quad \epsilon_{zz}(\omega)<0 \quad (q_z\to\infty) \ . \tag{22}$$

With this understanding, Eq. (21) becomes

$$\epsilon_{xx}\epsilon_{zz}+(\epsilon_{xx}\epsilon_{zz})^{1/2}(\epsilon_1+\epsilon_3)+\epsilon_1\epsilon_3=0 \tag{23}$$

and now the negative root of $(\epsilon_{xx}\epsilon_{zz})^{1/2}$ must be taken. Simple factorization enables one to rewrite Eq. (23) in the form

$$(\epsilon_{xx}\epsilon_{zz})^{1/2}=-\epsilon_i, \quad i=1,3 \ . \tag{24}$$

In the case $\mathbf{B}_0=0$ and hence $\epsilon_{xx}=\epsilon_{zz}=\epsilon_2(\omega)$, say, Eq. (24) reduces to $\epsilon_2(\omega)=-\epsilon_i$, as it should be.[32] Equation (24), for $\epsilon_i=1$ ($i=1$ or 3), gives the solutions for the asymptotic modes at the semiconductor-vacuum interface as specified by Eqs. (28) and (46) in Ref. 25. The relation for $\epsilon_i=1$ was obtained by Pakhamov and Stepanov[33] and by Abdel-Shahid and Pakhomov.[34]

It is then clear that, in the asymptotic limit, the wavefields at the two bounding interfaces of the film are decoupled from each other and the limiting frequencies are given by the solutions for the independent interfaces II/I and II/III, i.e., by Eq. (24). For the magnetoplasma model specified in Appendix A (with $\nu=0$), Eq. (24) may be solved explicitly, namely,

$$\left[\frac{\omega}{\omega_p}\right]^2=\frac{1}{2(\epsilon_L^2-\epsilon_i^2)}\left[\left[(\epsilon_L^2-\epsilon_i^2)\frac{\omega_c^2}{\omega_p^2}+2\epsilon_L\right]\pm\left[(\epsilon_L^2-\epsilon_i^2)^2\frac{\omega_c^4}{\omega_p^4}+4\epsilon_i^2\right]^{1/2}\right] . \tag{25}$$

It can be proved that Eq. (25) is an exact analogue of Eq. (29) in Ref. 25 and of Eq. (46) in Ref. 6.

Considering Eq. (22), the requirement $\epsilon_{zz}(\omega)<0$ implies that $\omega<\omega_p/(\epsilon_L)^{1/2}$, while from $\epsilon_{xx}(\omega)<0$, it follows that, in addition, $\omega>\omega_c$ must hold. Therefore the asymptotic solutions predicted by Eq. (25) must lie in the frequency window

$$\omega_c<\omega<\omega_p/(\epsilon_L)^{1/2} \quad (q_z\to\infty) \ . \tag{26}$$

The upper sign in Eq. (25) gives solutions which lie beyond this window and, therefore, must be discarded. It is the lower sign which gives the valid solutions. Interestingly, Eq. (26) implies that, in the asymptotic limit, only the case $\omega_c<\omega_p/(\epsilon_L)^{1/2}$ yields well-behaved solutions. [In the case $\omega_p/(\epsilon_L)^{1/2}<\omega<\omega_c$ we would have $\epsilon_{xx}(\omega)>0$ and $\epsilon_{zz}(\omega)>0$, and then Eq. (21) could not be satisfied.]

In the special case $\omega_c=0$ the lower sign in Eq. (25) gives the solutions $\omega=\omega_p/(\epsilon_L+\epsilon_i)^{1/2}$, $i=1,3$. These also follow from Eq. (24) if we replace $(\epsilon_{xx}\epsilon_{zz})^{1/2}$ by $\epsilon=\epsilon_L-\omega_p^2/\omega^2$.

The case $q_0\ll q_z\ll 1/d$. Assuming that $|\epsilon_{zz}/\epsilon_{xx}|^{1/2}q_zd\ll 1$, Eq. (21) leads to an explicit solution for q_z:

$$q_z=-\frac{1}{d}\frac{\epsilon_{xx}(\epsilon_1+\epsilon_3)}{\epsilon_{xx}\epsilon_{zz}+\epsilon_1\epsilon_3} \ . \tag{27}$$

Because the retardationless limit requires that $q_0\ll q_z$ this result holds only for very thin films, namely, $q_0d\ll 1$.

In what follows, two different cases regarding the dispersion relation, Eq. (27), have been analyzed.

(i) $\epsilon_1>\epsilon_3\geq 1$. In this case, the substitution of ϵ_{xx} and ϵ_{zz} with the neglect of the collision frequency in Eq. (A1) and some algebra yield a biquadratic equation in the rationalized frequency (ω/ω_p):

$$\left[\frac{\omega}{\omega_p}\right]^4[(\epsilon_L^2+\epsilon_1\epsilon_3)q_zd+\epsilon_L(\epsilon_1+\epsilon_3)]-\left[\frac{\omega}{\omega_p}\right]^2\left\{\left[\epsilon_L\left[2+\epsilon_L\frac{\omega_c^2}{\omega_p^2}\right]+\epsilon_1\epsilon_3\frac{\omega_c^2}{\omega_p^2}\right]q_zd+(\epsilon_1+\epsilon_3)\left[1+\epsilon_L\frac{\omega_c^2}{\omega_p^2}\right]\right\}+\left[1+\epsilon_L\frac{\omega_c^2}{\omega_p^2}\right]q_zd=0 \ . \tag{28}$$

This equation predicts two branches $\omega(q_z)$. For $q_z=0$ we must have either $\epsilon_{xx}(\omega)=0$ or $\epsilon_{zz}(\omega)\to\infty$, as may be seen from Eq. (27). This gives, respectively, for the higher and lower branches

$$\omega=\omega_H \ \text{ and } \omega\to 0 \ \text{ for } q_z=0 \ , \tag{29}$$

where

$$\omega_H=(\omega_c^2+\omega_p^2/\epsilon_L)^{1/2} \ . \tag{30}$$

This is the well-known hybrid cyclotron-plasmon frequency at which $\epsilon_{xx}(\omega)$ vanishes. The asymptotic frequencies, as predicted by Eq. (27), are given by $\epsilon_{xx}(\omega)\epsilon_{zz}(\omega)+\epsilon_1\epsilon_3=0$. This is clearly wrong: We know that the asymptotic frequencies are correctly given by Eq. (24). The discrepancy is hardly surprising since the present approximation is limited to $q_z\ll 1/d$. Similarly, Eq. (29) does not give the true values of $\omega(q_z\to 0)$ because we must satisfy $q_0\ll q_z$. We have calculated the roots of Eq. (28) by choosing the following parameters: $\epsilon_1=11.683$, $\epsilon_L=15.7$, $\epsilon_3=1.0$, and $\omega_c/\omega_p=0.1$. These values of the dielectric constants specify our layered structure (Fig. 1) as made up of a Si-glass substrate (region I), an InSb thin film (region II), and air (region III). The results for the dimensionless frequency (ω/ω_p) versus (q_zd) are shown in Fig. 2. The two curves, represented by the solid lines, are the magnetoplasma analogues of the Fuchs-Kliewer modes in the region $q_0\ll q_z\ll 1/d$. The dashed lines indicate the asymptotic solutions corresponding to Eq. (25). The lower branch starts at the origin; if our calculation were valid for $q_z\gg 1/d$ then it would approach asymptotically the frequency given by $[\epsilon_{xx}(\omega)\epsilon_{zz}(\omega)]^{1/2}=-\epsilon_1$. The upper branch starts at $\omega=\omega_H$ and exhibits a practically constant and negative group velocity ("backward wave"). Again, in an exact calculation this branch would approach asymptotically the solution of $[\epsilon_{xx}(\omega)\epsilon_{zz}(\omega)]^{1/2}=-\epsilon_3$.

Now let us analyze briefly the behavior of the decay constants α_1, α_3, β_+, and β_- corresponding to the two modes in Fig. 2. In the retardationless limit, α_1, α_2, and β_+ all have the value q_z, so they are real and positive. However, $\beta_- = (\epsilon_{zz}/\epsilon_{xx})^{1/2} q_z$ is real (and positive) or pure imaginary depending on whether $\epsilon_{zz}(\omega)/\epsilon_{xx}(\omega)$ is positive or negative. This quantity is positive for $\omega_c < \omega < \omega_p/(\epsilon_L)^{1/2}$; thus, above ω_c the lower branch corresponds to ordinary polariton modes bound to the two interfaces of the film. On the other hand, below ω_c and also for $\omega_p/(\epsilon_L)^{1/2} < \omega < \omega_H$ the decay constant β_- is pure imaginary. Therefore the lowe· part of the lower branch (below ω_c) and the higher part of the higher branch [above $\omega_p/(\epsilon_L)]^{1/2}$ have a mixed polariton—waveguide mode character ("hybrid mode").

(ii) $\epsilon_1 = \epsilon_3 = \epsilon_0$, *say*. In this symmetric configuration Eq. (28) assumes the form

$$\left[\frac{\omega}{\omega_p}\right]^4 [(\epsilon_L^2 + \epsilon_0^2) q_z d + 2\epsilon_L \epsilon_0] - \left[\frac{\omega}{\omega_p}\right]^2 \left\{ \left[\epsilon_L \left[2 + \epsilon_L \frac{\omega_c^2}{\omega_p^2}\right] + \epsilon_0^2 \frac{\omega_c^2}{\omega_p^2}\right] q_z d + 2\epsilon_0 \left[1 + \epsilon_L \frac{\omega_c^2}{\omega_p^2}\right]\right\} + \left[1 + \epsilon_L \frac{\omega_c^2}{\omega_p^2}\right] q_z d = 0 . \quad (31)$$

The roots of Eq. (31) are calculated taking $\epsilon_0 = 1.0$ (unsupported film) and the rest of the parameters are the same as stated in (i). The plotted curves for (ω/ω_p) versus $(q_z d)$ are depicted in Fig. 3. The solid lines reveal the dispersive polariton modes and the dashed (horizontal) line is the asymptotic limit for $\epsilon_i = \epsilon_0$ in Eq. (25). Of course, this line coincides with the line labeled $\epsilon_3 (=1)$ in Fig. 2. The other solution of Eq. (25) with $\epsilon_i = \epsilon_0$ lies outside the frequency region given by Eq. (26) and, as argued before, must be discarded.

The most important difference between the cases $\epsilon_1 \neq \epsilon_3$ and $\epsilon_1 = \epsilon_3$ lies in the fact that, in the latter case, the upper and lower modes have a common asymptotic limit. (The same statement is also true for the Fuchs-Kliewer modes, $\mathbf{B}_0 = 0$.) We have also computed the dispersion curve for $\omega_c/\omega_p = 0.2$ (not shown). The lower branches of both Figs. 2 and 3 are remarkably independent of the value of ω_c. On the other hand, by increasing ω_c the upper branch is considerably shifted to higher frequencies, roughly parallel to itself. The rest of the discussion related to Fig. 2 is still valid. Thus for "small" values of q_z (while observing our assumption that $q_z \gg q_0$) β_+ is real while β_- is imaginary. These waves are interpreted to be "hybrid" polariton—wavequide modes. Neglecting damping and using Eq. (27) our assumptions may be expressed in the form

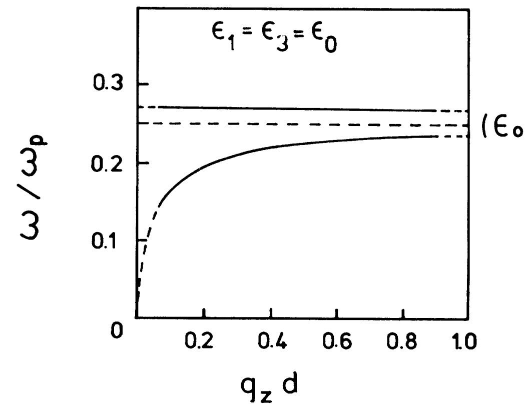

FIG. 3. As in Fig. 2 for the symmetric configuration, $\epsilon_1 = \epsilon_3 = \epsilon_0$ $(=1)$, that is an unsupported InSb film. Both dispersion curves now approach the same asymptotic limit. This is also the asymptotic limit for the corresponding, decoupled surfaces.

$$q_0 d \ll \frac{(\epsilon_1 + \epsilon_3) \mid \epsilon_{xx} \mid}{\mid \epsilon_{xx}\epsilon_{zz} + \epsilon_1 \epsilon_3 \mid} \ll \left[\frac{\epsilon_{xx}}{\epsilon_{zz}}\right]^{1/2} . \quad (32)$$

The first inequality always fails for $\omega \sim 0$ and $\omega \sim \omega_H$. This is to be expected, because for $q_z \sim 0$ the phase velocities for both branches are enormous, while the NR limit requires that $\omega/q_z \ll c$. For other values of the frequency the first inequality is satisfied for sufficiently thin films, $q_0 d \ll \epsilon_i/\epsilon_L$. The second inequality, surprisingly, is satisfied for $\omega = 0$, ω_c, $\omega_p/(\epsilon_L)^{1/2}$, and ω_H.

Therefore, this approximation works well provided that we do not approach too closely one of the asymptotic frequencies (drawn by dashed lines). In brief, for sufficiently thin films we may expect that an exact (numerical) calculation will approximately reproduce those parts of the dispersion curves of Figs. 2 and 3 that are drawn with solid lines.

IV. APPROXIMATE DISPERSION RELATIONS FOR VERY THIN FILMS

We invoke a thin-film approximation (TFA),

$$T_\pm = \tanh(\beta_\pm d) \simeq \beta_\pm d . \quad (33)$$

With this substitution the general dispersion relation, Eq. (19), after a laborious algebra, assumes the form

$$\epsilon_{xx}[\alpha_1^2\epsilon_3 + \alpha_3^2\epsilon_1 + \alpha_1\alpha_3(\epsilon_1+\epsilon_3)] + d\{\alpha_1\alpha_3\epsilon_{xx}[\alpha_1(\epsilon_{zz}+\epsilon_3) + \alpha_3(\epsilon_{zz}+\epsilon_1)] + k^2[\alpha_1\epsilon_3(\epsilon_{xx}+\epsilon_1) + \alpha_3\epsilon_1(\epsilon_{xx}+\epsilon_3)] - (\alpha_3\epsilon_3 + \alpha_3\epsilon_1)q_0^2\epsilon_{xy}^2\} + d^2[\alpha_1^2\alpha_3^2\epsilon_{xx}\epsilon_{zz} + \alpha_1\alpha_3 k^2(\epsilon_{xx}\epsilon_{zz} + \epsilon_1\epsilon_3) + \epsilon_1\epsilon_3 k^4 + q_0^2\epsilon_{xy}^2(q_0^2\epsilon_1\epsilon_3 - \alpha_1\alpha_3\epsilon_{zz})] = 0 . \quad (34)$$

In writing Eq. (34) we have omitted a prefactor (P) defined as

$$P=\left[\frac{(\beta_+^2-\beta_-^2)(k^2+q_0^2\epsilon_{zz})}{(\beta_+^2+q_0^2\epsilon_{zz})(\beta_-^2+q_0^2\epsilon_{zz})}\right]^2 \tag{35}$$

and treated it as nonvanishing in view of the following. The vanishing of the first factor in the numerator of P (i.e., $\beta_+=\beta_-$) was suggested by Rao and Uberoi[35] to lead to so-called "degenerate modes" at a surface. These modes were supposed to account for low-frequency surface helicons as reported by Baibakov and Datsko.[26] However, surface modes characterized by $\beta_+=\beta_-$ do not have physical reality.[27] The vanishing of the second factor (i.e., $k^2+q_0^2\epsilon_{zz}=0$) has no dynamical consequence pertaining to the dispersion of magnetoplasma polaritons in the film configuration. In obtaining Eq. (34), we have retained only the terms up to quadratic in d in the expansion of Eq. (19). We will analyze Eq. (34) in two different cases of interest.

A. Surface polaritons modified by magnetized overlayer

In this case we assume that medium III is air ($\epsilon_3=1.0$) and that medium I is surface-wave active ($\epsilon_1<0$). We use an ansatz[12]

$$q_z^2=q_0^2\frac{\epsilon_1}{1+\epsilon_1}+K_1^2 \tag{36}$$

for a film of small thickness d. In the limit $d\to 0$, K_1^2 must vanish and we are left with a surface-plasmon—polariton dispersion relation. For very small but finite d, we expect that K_1^2 is proportional to some power of d.

Using Eq. (5) and treating K_1^2 as a very small quantity, we calculate α_1 and α_3 (see Appendix C). Substituting in Eq. (34) and retaining only the terms linear in d gives an expression for K_1^2 [see Eq. (C6)]. Then Eq. (36) can be rewritten as follows:

$$q_z\simeq q_0\left[\frac{\epsilon_1}{1+\epsilon_1}\right]^{1/2}+\frac{1}{2}\left[\frac{1+\epsilon_1}{\epsilon_1}\right]^{1/2}\frac{K_1^2}{q_0} \tag{37}$$

$$=q_0\left[\left[\frac{\epsilon_1}{1+\epsilon_1}\right]^{1/2}-\frac{i(q_0d)\epsilon_1^{3/2}}{\epsilon_{xx}(1+\epsilon_1)^2(1-\epsilon_1)}\times[\epsilon_1(1-\epsilon_{xx})-\epsilon_{xx}(1-\epsilon_{zz})]\right]. \tag{38}$$

Thus the propagation constant q_z is linear in the film thickness, to lowest order in d. Equation (38) is a good approximation provided that $q_0d\ll 1$.

First let us look into the case where there is no applied field present, $\mathbf{B}_0=0$. Then $\epsilon_{xx}=\epsilon_{zz}\equiv\epsilon_2(\omega)$ and Eq. (38) reduces to

$$q_z=q_0\frac{|\epsilon_1|^{1/2}}{(|\epsilon_1|-1)^{1/2}}-\frac{q_0^2d\,|\epsilon_1|^{3/2}(|\epsilon_1|+\epsilon_2)}{(|\epsilon_1|-1)^2(|\epsilon_1|+1)}\frac{1-\epsilon_2}{\epsilon_2}\,. \tag{39}$$

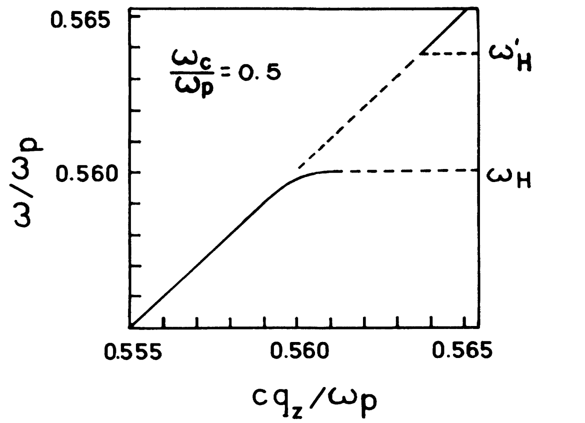

FIG. 4. Normalized frequency ω/ω_p versus normalized propagation constant cq_z/ω_p for surface-plasmon polaritons modified by a semiconducting magnetoplasma overlayer. The dispersion curve shown has been calculated using the thin-film approximation, from Eq. (38) with $\epsilon_3=1$, $\epsilon_3=1-\omega_{p1}^2/\omega^2$, $\omega_p/\omega_{p1}=10^{-3}$, $\omega_c/\omega_p=0.5$, $\epsilon_L=15.7$, $\nu=0$, and $d=2.04$ μm. This corresponds to an InSb film on a Na substrate. At the low frequencies ($\omega\ll\omega_{p1}$) considered, the curve follows closely the light line, except very near to the hybrid cyclotron-plasmon frequency ω_H. The magnetized overlayer creates a gap in the spectrum, of width $\sim\omega_p^2/(2\epsilon_L^2\omega_H)$ just above ω_H.

Since we are interested in the surface-polariton regime we have assumed that ϵ_1 is negative. This formula is an exact analogue of Eq. (3) of López-Rios[36] if ϵ_0, the dielectric constant of medium III in his notation, is equal to 1. This is a justification of our thin-film approximation, Eq. (34). Note that Eq. (39) predicts a splitting of the dispersion curve for $\epsilon_2(\omega)=0$, that is, at the plasma frequency of the thin film (or transition layer). This, in fact, was observed by López-Rios. The splitting is associated with the polariton mode at the interface between two semi-infinite conductors corresponding to media I and II.[5]

Next we put some numbers in Eq. (38), representing an InSb semiconductor film on a Nb metal substrate. The magnetoplasma model for the semiconductor has been specified in Appendix A and the parameters are $\epsilon_L=15.7$, $\omega_c/\omega_p=0.5$, and $d=2.04$ μm. For the metallic substrate, assuming the simple model[37] $\epsilon_1=1-\omega_{p1}^2/\omega^2$, we take $\omega_{p1}/\omega_p=10^3$. Because of this large factor and the requirement that $q_0d\ll 1$, the first term in Eq. (38), corresponding to the bare metallic surface, predominates over most of the spectral range of interest. An exception occurs at the hybrid cyclotron-plasmon frequency ω_H, defined in Eq. (30). At this frequency ϵ_{xx} vanishes and, by Eq. (38), $q_z\to\infty$. While our perturbational approach breaks down, it is clear that a splitting occurs in the dispersion relation of the surface-plasmon polariton. This is shown in Fig. 4 in a narrow range in the vicinity of ω_H. When $\omega\neq\omega_H$ the dispersion curve closely follows the light line $\omega=cq_z$; this is because $\omega\ll\omega_{p1}$. We have disregarded solutions with $q_z<\omega/c$, the reason being that, by Eq. (5), these would give an imaginary decay constant α_3. For ω_{p1} much greater than ω_p and ω_c the expression in the small square brackets of Eq. (38) vanishes at a frequency approximately equal to

$$\omega'_H = \left[\omega_c^2 + \frac{\omega_p^2}{\epsilon_L - 1}\right]^{1/2} . \tag{40}$$

We may conclude that, as a consequence of the thin film, a gap opens up in the spectrum of the surface polariton. For $\epsilon_L \gg 1$ the width of the gap is

$$\omega'_H = \omega_H \frac{\omega_p^2}{2\epsilon_L^2 \omega_H} . \tag{41}$$

In the absence of a static magnetic field $\omega_H = \omega_p / (\epsilon_L)^{1/2}$ and the gap becomes $\omega_p / 2\epsilon_2^{3/2}$.

B. Magnetized film bounded by identical media

Substituting $\epsilon_1 = \epsilon_3 = \epsilon_0$ and hence $\alpha_1 = \alpha_3 = \alpha_0$ in the dispersion relation reduced in TFA, Eq. (34), yields

$$4\epsilon_0\epsilon_{xx}\alpha_0^2 + 2\alpha_0 d[\alpha_0^2\epsilon_{xx}(\epsilon_0+\epsilon_{zz}) + k^2\epsilon_0(\epsilon_0+\epsilon_{xx}) - q_0^2\epsilon_0\epsilon_{xy}^2] + d^2[\alpha_0^4\epsilon_{xx}\epsilon_{zz} + \alpha_0^2 k^2(\epsilon_0^2+\epsilon_{xx}\epsilon_{zz}) + k^4\epsilon_0^2 + q_0^2\epsilon_{xy}^2(q_0^2\epsilon_0^2 - \alpha_0^2\epsilon_{zz})] = 0 . \tag{42}$$

In this case we use the following ansatz for q_z:[38]

$$q_z^2 = q_0^2\epsilon_0 + K_2^2 \tag{43}$$

for small thickness of the film. In the limit $d \to 0$, the film bounded by the identical media becomes a bulk dielectric characterized by the complex dielectric constant ϵ_0. In this limit, the solution for q_z should be $(\epsilon_0)^{1/2} q_0$. For small but finite d, we expect that q_z will differ from $(\epsilon_0)^{1/2} q_0$ by a small amount which is proportional to some power of d. This is the basis of our ansatz, in Eq. (43). Since K_2^2 is a very small correction we can write

$$q_z = q_0\epsilon_0^{1/2} + \frac{1}{2}\frac{K_2^2}{q_0\epsilon_0^{1/2}} . \tag{44}$$

Making use of Eq. (5), with $\epsilon_i = \epsilon_0$ and $\alpha_i = \alpha_0$, we calculate K_2 (see Appendix D). Terms quadratic in d, in Eq. (42), are included in this case because they turn out to contribute to q_z in the same order as the linear terms. Substituting K_2 [Eq. (D4)] in Eq. (44) gives

$$q_z = q_0\left[\epsilon_0^{1/2} + \frac{1}{32}\frac{(q_0 d)^2}{\epsilon_0^{1/2}\epsilon_{xx}^2}((\epsilon_{xx}^2 + \epsilon_{xy}^2 - \epsilon_0^2) \pm \{[(\epsilon_{xx}-\epsilon_0)^2 + \epsilon_{xy}^2]^2 - 4\epsilon_0^2\epsilon_{xy}^2\}^{1/2})^2\right] . \tag{45}$$

Notice that, in the TFA, q_z becomes independent of the element ϵ_{zz}. The necessary requirement for the validity of Eq. (45) is $q_0 d \ll 1$. Because of the higher symmetry, the correction in q_z is proportional to d^2, unlike the linear-d dependence in Eq. (38).

When K_2, Eq. (D4), is subjected to the limit $\mathbf{B}_0 = 0$, it may be written as

$$K_2 = \frac{q_0^2 d}{4\epsilon}(\epsilon - \epsilon_0)[(\epsilon+\epsilon_0) \pm (\epsilon - \epsilon_0)] , \tag{46}$$

where $\epsilon \equiv \epsilon_{xx} = \epsilon_{zz}$. Then K_2, for the lower and upper algebraic signs, respectively, assumes the form

$$K_2^- = -\frac{q_0^2 d}{2}\frac{\epsilon_0}{\epsilon}(\epsilon_0 - \epsilon) \tag{47a}$$

and

$$K_2^+ = -\frac{q_0^2 d}{2}(\epsilon_0 - \epsilon) . \tag{47b}$$

The expression for K_2^-, Eq. (47a), has been derived by Boardman and Halevi[38] for p-polarized polaritons in a very thin film. A similar approach applied to s-polarized modes (for $\mathbf{B}_0 = 0$) leads to Eq. (47b). Now, according to Eq. (D1) for very small d the decay constant α_0 is just K_2, so the right-hand sides of Eqs. (47) must be positive. We may discern three frequency regions, depending on the value of $\epsilon(\omega)$. If $\epsilon(\omega) < 0$ then $K_2^- > 0$; however, $K_2^+ < 0$. This is to say that polariton modes (bound to *both sides* of both interfaces) may have only transverse-magnetic (TM) polarization. Next, if $0 < \epsilon(\omega) < \epsilon_0$ then K_2^+ and K_2^- are both negative and there is a frequency gap in the spectrum of modes. On the other hand, if $\epsilon > \epsilon_0 (> 0)$ then K_2^+, as well as K_2^-, are positive. This corresponds to transverse-electrix (TE) and TM-polarized WG modes. The situation $\epsilon > \epsilon_0$ may be achieved in a semiconductor $(\epsilon = \epsilon_L - \omega_p^2/\omega^2)$ provided that $\omega > \omega_p / (\epsilon_L - \epsilon_0)^{1/2}$. Equations (47a) and (47b), substituted in Eq. (44), give rise to long-range propagation of the *symmetric* TM and TE modes, respectively, in a very thin film ($\mathbf{B}_0 = 0$).

We have computed the dispersion relation (q_z versus ω) using Eq. (45) with $B_0 \neq 0$. The parameters used were $\epsilon_0 = 1$, $\omega_c / \omega_p = 1$ and the rest of them, the same as were cited in the preceding section. Theoretical results in the dimensionless variables are plotted in Fig. 5. For each value of ω, we obtain two values of q_z corresponding to the two algebraic signs in Eq. (45). It is observed that one of the branches effectively coincides with the light line ($\omega = cq_z$). This leads us to infer that the correction term [the second term of Eq. (44)] for this branch is negligibly small. The second branch shows a considerable shift from the light line over almost the whole frequency range.

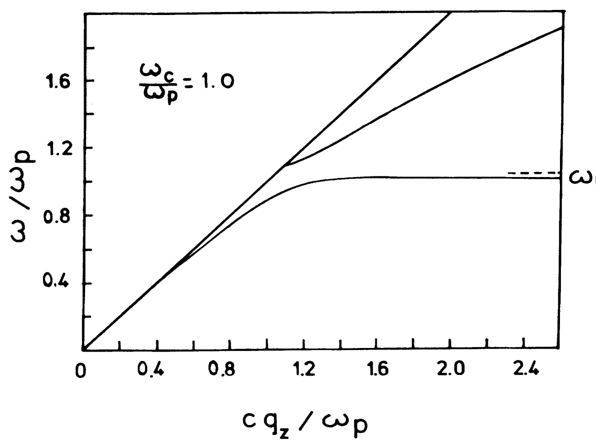

FIG. 5. Normalized frequency ω/ω_p versus normalized propagation constant cq_z/ω_p for a very thin, unsupported semiconducting film. The calculation is based on Eq. (45) with $\epsilon_0=1$, $\epsilon_L=15.7$, $\omega_c/\omega_p=1.0$, $\nu=0$, and $d=2.04$ μm. These are two branches and the strongest deviation from the light line occurs in the vicinity of the hybrid cyclotron-plasmon frequency ω_H.

The second branch exhibits a resonance at $\omega \simeq 1.0314\omega_p$. Indeed, it follows from Eq. (45) that $q_z=\infty$ when ϵ_{xx} vanishes, that is, at the hybrid frequency ω_H, Eq. (30). For $\epsilon_L=15.7$ and $\omega_c/\omega_p=1.0$ this equation gives $\omega_H \simeq 1.0314\omega_p$, the above quoted value. However, the result $\omega=\omega_H$ for the resonance frequency should be viewed with reservation because our TFA breaks down when the second term in Eq. (45) becomes comparable to (or greater than) the first term. Above ω_H, the the region between $1.032\omega_p$ and $1.063\omega_p$, K_2 assumes negative values for the lower sign in Eq. (D4). Such solutions must be discarded because the decay constant $\alpha_0(=K_2)$ should be positive. Thus there is a gap in the second branch between ω_H and a higher frequency at which the correction term in Eq. (45) vanishes.

Figures 5 and 3 both deal with the symmetric configuration, $\epsilon_1=\epsilon_3=1$; however, for different values of ω_c. Figure 5 is valid for $q_z \simeq \omega/c$, near the light line. On the other hand, Fig. 3 is applicable only for $q_z \gg \omega/c$, far away from the light line. It seems that the behavior of the two branches is similar to that of the Fuchs-Kliewer modes (for $\mathbf{B}_0=0$); however, the exact solution in the region $q_z \gtrsim \omega/c$ will have to await the numerical solution of Eq. (19). The nature of the modes (polariton, WG, hybrid, or complex) will be determined from a computation of β_+ and β_-, Eq. (3).

ACKNOWLEDGMENT

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APPENDIX A

The dielectric tensor components are

$$\epsilon_{xx}=\epsilon_{yy}=\epsilon_L+\frac{\omega_p^2(\omega+i\nu)}{\omega[\omega_c^2-(\omega+i\nu)^2]} ,$$
$$\epsilon_{xy}=-\epsilon_{yx}=-i\frac{\omega_p^2\omega_c}{\omega[\omega_c^2-(\omega+i\nu)^2]} , \tag{A1}$$
$$\epsilon_{zz}=\epsilon_L-\frac{\omega_p^2}{\omega(\omega+i\nu)} ,$$

where ϵ_L is the background dielectric constant, ν is the free-carrier collision frequency, and ω_p and ω_c are, respectively, the unscreened plasma frequency and the electron cyclotron frequency, defined as follows:

$$\omega_p^2=\frac{4\pi ne^2}{m_e}$$

and (A2)

$$\omega_c=\frac{e\,|\mathbf{B}_0|}{m_e c} .$$

Here e, m_e, and n are, respectively, the electronic charge, cyclotron mass, and free-carrier concentration in the semiconductor (region II in Fig. 1).

In Eqs. (A1), if we also consider the effect of phonons, which, in a way, embodies the coupling of the magnetoplasma polaritons to the optical phonons, then the background dielectric constant ϵ_L has to be replaced by a frequency-dependent expression,

$$\epsilon_L=\epsilon_\infty\left[\frac{\omega_{\mathrm{LO}}^2-\omega^2-i\Gamma\omega}{\omega_{\mathrm{TO}}^2-\omega^2-i\Gamma\omega}\right] , \tag{A3}$$

where ϵ_∞ is the high-frequency dielectronic constant, Γ is the optical-phonon damping frequency, and ω_{LO} and ω_{TO} are, respectively, the longitudinal and transverse optical phonon frequencies at the zone center of the first Brillouin zone.

APPENDIX B

The tangential field components E_z, B_x, and B_z in terms of E_x in region II are as follows:

$$E_{1,3z}=\frac{-i\beta_{+,-}q_z(k^2-\beta_{+,-}^2)}{q_0^2(\beta_{+,-}^2+q_0^2\epsilon_{zz})\epsilon_{xy}}E_{1,3x} ,$$
$$E_{2,4z}=\frac{i\beta_{+,-}q_z(k^2-\beta_{+,-}^2)}{q_0^2(\beta_{+,-}^2+q_0^2\epsilon_{zz})\epsilon_{xy}}E_{2,4x} , \tag{B1}$$
$$B_{1,2x}=\frac{-q_z(k^2-\beta_+^2)\epsilon_{zz}}{q_0(\beta_+^2+q_0^2\epsilon_{zz})\epsilon_{xy}}E_{1,2x} ,$$
$$B_{3,4x}=\frac{-q_z(k^2-\beta_-^2)\epsilon_{zz}}{q_0(\beta_-^2+q_0^2\epsilon_{zz})\epsilon_{xy}}E_{3,4x} , \tag{B2}$$
$$B_{1,3z}=\frac{-i\beta_{+,-}}{q_0}E_{1,3x} ,$$
$$B_{2,4z}=\frac{i\beta_{+,-}}{q_0}E_{2,4x} . \tag{B3}$$

The tangential (magnetic) field components B_x and B_z in terms of E_z and E_x in regions I and III are as follows:

$$B_x^{\mathrm{I\,(III)}} = +\,(-)\frac{iq_0\epsilon_i}{\alpha_i}E_z^{\mathrm{I\,(III)}} , \tag{B4}$$

$$B_z^{\mathrm{I\,(III)}} = +\,(-)\frac{i\alpha_i}{q_0}E_x^{\mathrm{I\,(III)}} . \tag{B5}$$

The suffix i stands for 1 or 3 specifying the respective quantities in regions I and III.

APPENDIX C

Analytical derivation of K_1 in Sec. IV A

Substituting Eq. (36) in Eq. (5) and treating K_1 as a very small quantity gives

$$\alpha_1 = iq_0\frac{\epsilon_1}{(1+\epsilon_1)^{1/2}} - i\frac{(1+\epsilon_1)^{1/2}}{2q_0\epsilon_1}K_1^2 \tag{C1}$$

and

$$\alpha_3 = -iq_0\frac{1}{(1+\epsilon_1)^{1/2}} + i\frac{(1+\epsilon_1)^{1/2}}{2q_0}K_1^2 . \tag{C2}$$

We have retained opposite signs for α_1 and α_3 in view of the fact that we have made a choice in writing our field solutions, Eqs. (7) and (9), such that $\mathrm{Re}\alpha_1 > 0$ and $\mathrm{Re}\alpha_3 > 0$. The expressions of α_1 and α_3 lead to the following results:

$$\alpha_1\alpha_3 \simeq q_0^2\frac{\epsilon_1}{1+\epsilon_1} - \frac{1}{2}\frac{(1+\epsilon_1^2)}{\epsilon_1}K_1^2 , \tag{C3}$$

$$\alpha_1+\alpha_3 \simeq iq_0\frac{(\epsilon_1-1)}{(1+\epsilon_1)^{1/2}} + \frac{i(1+\epsilon_1)^{1/2}(\epsilon_1-1)}{2q_0\epsilon_1}K_1^2 , \tag{C4}$$

and

$$\alpha_1\epsilon_3+\alpha_3\epsilon_1 \simeq 0 + \frac{i(1+\epsilon_1)^{1/2}(\epsilon_1^2-1)}{2q_0\epsilon_1}K_1^2, \quad \epsilon_3 = 1 . \tag{C5}$$

Substitution of these expressions in Eq. (34), retaining only the terms linear in d, and neglecting the terms proportional to K_1^2 in the coefficient of d, yields

$$K_1^2 = -i\frac{2\epsilon_1^2(q_0^3d)}{\epsilon_{xx}(1+\epsilon_1)^{5/2}(1-\epsilon_1)} \times[\epsilon_1(1-\epsilon_{xx})-\epsilon_{xx}(1-\epsilon_{zz})] . \tag{C6}$$

It should be pointed out that the analytical expressions, throughout this section, have been derived with the choice that $\mathrm{Im}(1+\epsilon_1)^{1/2} < 0$, corresponding to $\epsilon_1 < 0$.

APPENDIX D

Analytical derivation of K_2 in Sec. IV B

Substitution of Eq. (43) in Eqs. (5) and (4) with $\epsilon_i = \epsilon_0$ and $\alpha_i = \alpha_0$ gives us

$$\alpha_0 = K_2 , \tag{D1}$$

$$K^2 = q_0^2(\epsilon_0-\epsilon_{xx}) + K_2^2 . \tag{D2}$$

Substituting the values of α_0 and K in Eq. (42) yields

$$4\epsilon_0\epsilon_{xx}K_2^2 + 2K_2d\{K_2^2\epsilon_{xx}(\epsilon_0+\epsilon_{zz}) + [q_0^2(\epsilon_0-\epsilon_{xx})+K_2^2]\epsilon_0(\epsilon_0+\epsilon_{xx}) - q_0^2\epsilon_0\epsilon_{xy}^2\}$$
$$+ d^2\{K_2^4\epsilon_{xx}\epsilon_{zz} + K_2^2[q_0^2(\epsilon_0-\epsilon_{xx})+K_2^2](\epsilon_0^2+\epsilon_{xx}\epsilon_{zz}) + \epsilon_0^2[q_0^2(\epsilon_0-\epsilon_{xx})+K_2^2]^2 + q_0^2\epsilon_{xy}^2(\epsilon_0^2q_0^2 - K_2^2\epsilon_{zz})\} = 0 . \tag{D3}$$

Neglecting the terms proportional to K_2^2 and K_2^4 in the coefficients of d and d^2 and solving the resulting quadratic equation in K_2 gives

$$K_2 = \frac{q_0^2d}{4\epsilon_{xx}}((\epsilon_{xx}^2+\epsilon_{xy}^2-\epsilon_0^2) \pm \{(\epsilon_{xx}^2+\epsilon_{xy}^2-\epsilon_0^2)^2 - 4\epsilon_0\epsilon_{xx}[(\epsilon_{xx}-\epsilon_0)^2+\epsilon_{xy}^2]\}^{1/2}) . \tag{D4}$$

[1] *Electromagnetic Surface Modes,* edited by A. D. Boardman (Wiley, New York, 1982).
[2] *Surface Polaritons,* edited by D. L. Mills and V. M. Agranovich (North-Holland, Amsterdam, 1982).
[3] E. Burstein, W. P. Chen, Y. J. Chen, and A. Hartstein, J. Vac. Sci. Techol. **11**, 1004 (1974).
[4] A. Otto, Adv. Solid State Phys. **14**, 1 (1974); see also, *Optical Properties of Solids: New Developments,* edited by B. O. Seraphin (North-Holland, Amsterdam, 1975), p. 677.
[5] P. Halevi, in *Electromagnetic Surface Modes,* Ref. 1, p. 249.
[6] K. W. Chiu and J. J. Quinn, Nuovo Cimento B **10**, 1 (1972).
[7] R. F. Wallis, J. H. Brion, E. Burstein, and A. Hartstein, Phys. Rev. B **9**, 3424 (1974).
[8] E. D. Palik, R. Kaplan, R. W. Gammon, H. Kaplan, R. F. Wallis, and J. J. Quinn, Phys. Rev. B **13**, 2497 (1976).
[9] K. Bhasin, D. Bryan, R. W. Alexander, and R. J. Bell, J. Chem. Phys. **64**, 5019 (1976).
[10] D. L. Begley, R. W. Alexander, C. A. Ward, and R. J. Bell, Surf. Sci. **81**, 245 (1979).
[11] R. Miller, D. L. Begley, C. A. Ward, R. W. Alexander, and R. J. Bell, Surf. Sci. **71**, 491 (1978).
[12] R. W. Alexander, Jr., R. J. Bell, and C. A. Ward, in *Electromagnetic Surface Modes,* Ref. 1, p. 201.
[13] M. Fukui, V. C. Y. So, and R. Normandin, Phys. Status Solidi

B **91**, K61 (1979).
[14]D. Sarid, Phys. Rev. Lett. **47**, 1927 (1981).
[15]D. Sarid, R. T. Deck, A. E. Craig, R. K. Hickernell, R. S. Jameson, and J. J. Fasano, Appl. Opt. **21**, 3993 (1982).
[16]D. Sarid, R. T. Deck, and J. J. Fasano, J. Opt. Soc. Am. **72**, 1345 (1982).
[17]R. T. Deck and D. Sarid, J. Opt. Soc. Am. **72**, 1613 (1982).
[18]A. E. Craig, G. A. Olson, and D. Sarid, Opt. Lett. **8**, 380 (1983).
[19]J. Quail, J. G. Rako, and H. J. Simon, Opt. Lett. **8**, 377 (1983).
[20]G. I. Stegeman, J. J. Burke, and D. G. Hall, Opt. Lett. **8**, 383 (1983).
[21]G. I. Stegeman, R. F. Wallis, and A. A. Maradudin, Opt. Lett. **8**, 386 (1983).
[22]G. I. Stegeman and C. T. Seaton, J. Appl. Phys. **58**, R57 (1985), references contained therein.
[23]*Surfaces and Interfaces: Physics & Electronics,* edited by R. S. Bauer (North-Holland, Amsterdam, 1983).
[24]P. Halevi, Phys. Rev. B **23**, 2635 (1981).
[25]R. F. Wallis, in *Electromagnetic Surface Modes,* Ref. 1, p. 575.
[26]V. I. Baikabov and V. N. Datsko, Pis'ma Zh. Eskp. Teor. Fiz. **15**, 195 (1972) [JETP. Lett. **15**, 135 (1972)]; Fiz. Tverd. Tela (Leningrad) **15**, 1616 (1973) [Sov. Phys.—Solid State **15**, 1084 (1973)]; Fiz. Tekhn. Poluprovodn. **12**, 1444 (1978) [Sov. Phys.—Semiconductors **12**, 855 (1978)].
[27]P. Halevi and J. J. Quinn, Solid State Commun. **33**, 467 (1980).
[28]P. Halevi, Solid State Commun. **44**, 1499 (1982).
[29]A. D. Boardman and I. K. Irving, J. Phys. (Paris) Colloq. **45**, C5-275 (1984).
[30]A. Laurinavichyus and P. Malakauskas, Fiz. Tverd. Tela (Leningrad) **23**, 1152 (1981) [Sov. Phys.—Solid State **23**, 669 (1981)].
[31]P. K. Tien, Rev. Mod. Phys. **49**, 361 (1977).
[32]K. L. Kliewer and R. Fuchs, Adv. Chem. Phys. **27**, 355 (1974).
[33]V. I. Pakhomov and K. N. Stepanov, Zh. Tekhn. Fiz. **37**, 1381 (1967) [Sov. Phys.—Tech. Phys. **12**, 1003 (1968)].
[34]N. Z. Abdel-Shahid and V. I. Pakhomov, Plasma Phys. **12**, 55 (1970).
[35]U. J. Rao and C. Uberoi, Phys. Rev. B **18**, 2941 (1978).
[36]T. López-Rios, Optics Commun. **17**, 342 (1976).
[37]The effective mass of an electron in a metal is two orders of magnitude greater than that in a semiconductor like InSb. For this reason the cyclotron frequency of the metal is negligible and the magnetoplasma may be restricted to the semiconducting film.
[38]A. D. Boardman and P. Halevi (unpublished).