Use of scaling plots in phase-transition studies

M. Fähnle, W. -U. Kellner, and H. Kronmüller
Max-Planck-Institut für Metallforschung, Institut für Physik, 7000 Stuttgart 80, Federal Republic of Germany

(Received 17 September 1986; revised manuscript received 21 October 1986)

It is demonstrated that for the phase transition of amorphous ferromagnets the exponent values obtained from optimum data collapsing in a scaling plot depend sensitively on the temperature range used for the fit. Possible implications for the study of spin glasses and random-anisotropy magnets are discussed.

The use of scaling plots is very common for the study of phase transitions in disordered spin systems. For instance, in the case of amorphous ferromagnets the exponent values for β and γ describing the critical behavior of the order parameter $M(T,H=0)$ and the zero-field susceptibility $\chi(T,H=0)$ are chosen in such a way that an optimum collapse of the $M(T,H)$ data to two branches (for T smaller or larger than the critical temperature T_c) is observed for the plot $\ln(M/|t|^{\beta})$ vs $\ln(H/|t|^{\beta+\gamma})$, with $t=(T-T_c)/T_c$. Similarly, for the case of spin glasses and random-anisotropy magnets (RAM) the exponents β and γ [now describing the behavior of the singular susceptibility $\chi_s(T,H=0)$] may be obtained, for example, from the scaling plot $\ln(\chi_s/|t|^{\beta})$ vs $\ln(H^2/|t|^{\beta+\gamma})$. Thereby, in many cases data from a rather extended temperature range are used. For example, for the amorphous ferromagnetic alloys $(Fe_{0.68}Mn_{0.32})_{75}P_{16}B_6Al_3$ and $(Fe_{0.2}Ni_{0.8})_{75}P_{16}B_6Al_3$, Yeshurun, Salamon, Rao, and Chen[1] considered a range of $-0.21 \leqq t \leqq 0.22$ and $-0.2 \leqq t \leqq 0.156$, respectively. For an analysis of the RAM $Tb_{64}Fe_{20}Ga_{16}$, Sellmyer and Nafis[2] considered a range $0.002 \leqq t \leqq 0.13$. Even larger temperature ranges are often considered for spin glasses. Barbara, Malozemoff, and Imry,[3] e.g., observed reasonable data collapse for $0.016 \leqq t \leqq 1.54$ for the spin glass GdAl, and they concluded that this may indicate a large critical regime.

In this note we demonstrate that for the amorphous ferromagnet $Fe_{30}Ni_{50}P_{14}B_6$ the exponent values obtained from optimum data collapsing depend sensitively on the temperature range used for the fit. Figure 1(a) shows the collapse of our data for $-0.005 \leqq t \leqq -0.0002$ and $0.0004 \leqq t \leqq 0.005$, for $\beta=0.34$ and $\gamma=1.34$. These exponents are consistent with those obtained from the modified Arrott plots,[4,5] or from the $\ln M(T,0)$ vs $\ln t$ plot and from $\chi(T,0)$ using the Kouvel-Fisher method.[4,5] Figure 1(b) demonstrates that the collapse is much worse for $\beta=0.37$ and $\gamma=1.7$. However, when considering a larger range, $-0.13 \leqq t \leqq 0.13$ as in Ref. 2, the best collapse is obtained [Fig. 1(c)] for the latter exponent values, whereas $\beta=0.34$, $\gamma=1.34$ now yield a bad collapsing [Fig. 1(d)]. From Fig. 1(c) one might erroneously conclude that $\beta=0.37$ and $\gamma=1.7$, and that the width of the critical regime is large. A comparable data collapsing for the wide t range may also be obtained by slightly different exponent values, for instance, $\gamma=1.75$ and, in principle, we have to

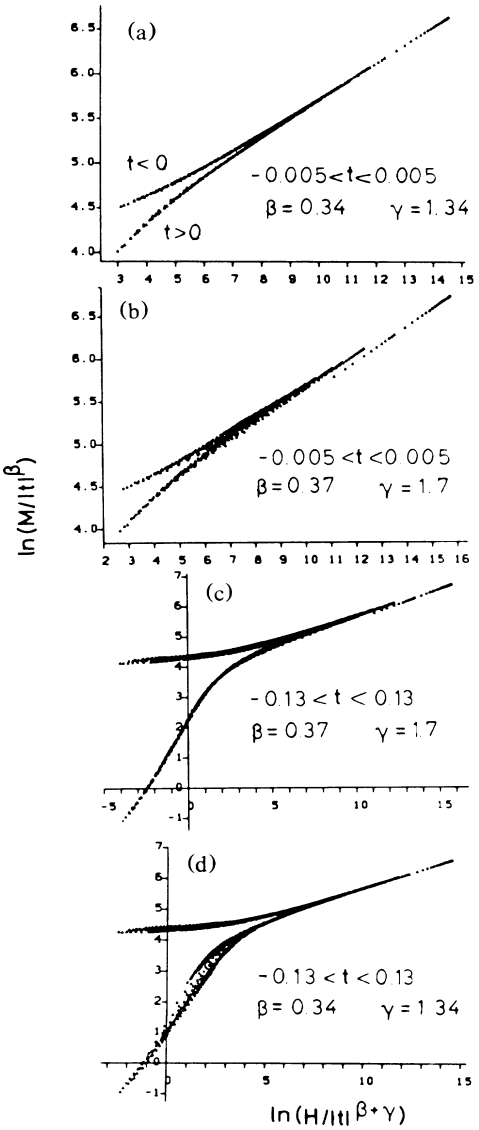

FIG. 1. Scaling plots for $Fe_{30}Ni_{50}P_{14}B_6$ for different exponents and/or different t ranges. Each plot contains 672 (4788) data for the small (wide) t range, from 14 (124) and 10 (47) isotherms below and above T_c, and 28 H values between 0.1 and 9.2 kOe for each isotherm.

determine the optimum values from the best fit found by computer analysis. However, it is obvious to everybody from our figure that the optimum exponent values for the different ranges are drastically different and that the effect is outside the uncertainty in the determination of the optimum exponent values.

Obviously the ln-ln scaling plot is insensitive enough to yield a reasonable good data collapsing even when using data from a t range much larger than the critical regime. However, in this case the exponent values depend on the temperature range used for the fit. To find the correct critical exponents for scaling plots, the following procedure must be performed: Start with the data from a rather large t range and determine the exponent values from optimum data collapse for this range. Then reduce the t range, represent the data on a larger scale, and try to improve the fit by choosing other exponent values. When going to successively smaller t ranges, different optimum exponent values will be found for every step, until the critical regime is entered. The stability of the exponent values against further reduction of the considered t range should be a well-defined criterion for the width of the critical regime. Because the uncertainty in the determination of the optimum exponent values increases with a decreasing number of data, a lot of very accurate data are required to perform this procedure. For example, we consider 4788 data for the wide t range, and the small t range still contains 672 data. For comparison, Yeshurun *et al.*[1] exhibit only 51 data for their rather large t range for the alloy $(Fe_{0.68}Mn_{0.32})_{75}P_{16}B_6Al_3$, and an analysis of this kind most probably is not possible.

One word of caution: Of course, if we just skip the high t data from our plot in Fig. 1(c) (data from the left-hand corner) and present the data on the same scale using the same exponent values, then there is no change in the quality of the scaling plot. When going to the smaller t range one has to present the data every time on a larger scale [Figs. 1(a) and 1(b)] and try to improve the collapse by choosing new exponent values.

The reason for the aforementioned sensitivity of the exponent values on the t range is obvious for the case of amorphous ferromagnets: In a series of recent papers (cf. Refs. 4–8) it has been shown that in these materials the effective exponent values depend very sensitively on temperature. For example, the Kouvel-Fisher[9] exponent $\gamma(T)=(T-T_c)\chi d\chi^{-1}/dT$ starts at the critical value for $T\rightarrow T_c$, increases with increasing t outside the critical regime, runs through a maximum, and decreases only gradually to the mean-field value of 1, in contrast to the monotonically decreasing $\gamma(T)$ of ordered ferromagnets.[6] When performing an optimum ln-ln plot of the data in an extended t range, one therefore does not obtain the correct asymptotic exponents, but something like an average value of $\gamma(T)$ in this range. This has been demonstrated explicitly for another amorphous ferromagnet, $Fe_{32}Ni_{36}Cr_{14}P_{12}B_6$. Whereas an optimum fit of the $\ln\chi$ vs $\ln t$ data for $0.1\leqq t\leqq 0.5$ yields an "average" value[10] of $\gamma=1.71$, the asymptotic analysis according to Kouvel-Fisher[9] reveals the nonmonotonic $\gamma(T)$ with $\gamma(T\rightarrow T_c)=1.38$.[4] Similarly, a scaling analysis of the data for a wide t range yields larger exponent values, as clearly demonstrated by our figure.

Reliable exponent values may only be obtained when considering data from a range equal to or smaller than the critical regime. For amorphous ferromagnets the width of the critical regime is small, comparable to the one of crystalline ferromagnets,[4,8] as long as the composition is far away from the transition to a spin-glass system. There are hints from theory[11] that for spin glasses the critical regime may be larger than for simple ferromagnets, and possibly this is also the case for RAM and amorphous ferromagnets with composition close to the spin-glass transition. We therefore do not argue that the exponent values obtained in the papers previously discussed[1–3] are necessarily wrong because the authors have considered a large t range. On the other hand, there are clues to a nonmonotonic $\gamma(T)$ of the singular susceptibility of spin glasses from the correlated molecular field theory,[12] high-temperature series expansions,[13] and from the Sherrington-Kirckpatrick model.[14] Furthermore, an experimental investigation of the AgMn spin glass[14] yields deviations towards higher apparent values of the exponents when using data from a wide t range, $t_{max}>0.1$, in close analogy to our findings for the case of amorphous ferromagnets. For RAM the correlated molecular field theory[15] predicts a similar behavior of χ_s as for spin glasses. Sellmyer and Nafis[16] investigated χ_s at finite field H for the RAM DyFeB and found decreasing exponent values for T approaching the critical temperature. However, because their determination of γ at a single field value broke down[2] for $t\rightarrow 0$, these results may not be considered as a proof for a possible nonmonotonic $\gamma(T)$ in RAM's. Nevertheless, the possibility of strongly temperature-dependent effective exponents should be taken into account when considering spin glasses and RAM's. As discussed already by Bouchiat,[14] this point of view may help to clarify the diversity existing between the values of critical exponents determined in other spin-glass systems.

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