Large-n-limit model for quantum structural phase transitions with correlated random fields

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The effects of uncorrelated and correlated random fields on the critical properties in the quantum displacive limit in systems undergoing structural phase transitions are investigated within the Hartree approximation. An unusual critical behavior, absent dimensional reduction, and breaking of the scaling exponent relations, occurs as a competition of thermal, quantum, and random field fluctuations when the transition is driven by the temperature. On the contrary, by our approaching the quantum displacive limit in terms of the interaction-strength parameter a picture emerges which is consistent with recent renormalization-group predictions. In particular the dimensional reduction is present also for long-range interactions and correlated random fields due to the absence of perturbative effects as the Griffiths singularities which disappear in the large-n limit.

I. INTRODUCTION

Since the observation of a "central peak"[1] in the dynamic structure factor of $SrTiO_3$ in neutron scattering, considerable effort has been expended in studying the effects of impurities on the critical behavior of systems undergoing structural phase transitions[2-6] at finite temperature. In contrast, the effects of quenched impurities on the critical properties in the displacive limit have received less attention and only partial renormalization-group (RG) information has been obtained quite recently[7-9] for a lattice dynamical model as a member of a wide class of quantum systems.[10] The aim of the present paper is to explore the effects of correlated random fields (describing quenched impurities which couple linearly to the ordering field) on the structural phase transition in the quantum displacive limit[11,12] (QDL) where quantum fluctuations are, in general, extremely important. The starting point is a continuous functional representation of a d-dimensional quantum-lattice dynamic n-vector model[11-13] which provides a good theoretical laboratory for studying several aspects of critical phenomena in displacive structural phase transitions. By using the Hartree approximation, which is exact in the large-n limit, we are able to study systematically the random field (RF) critical properties by approaching the QDL and a great variety of interesting situations appears depending on the nature of the RF correlations. Here we do not include information about the criticality at finite temperature, when quantum fluctuations become irrelevant, because, in this classical regime, the results already obtained for bosonized systems in (Ref. 14) [recently reproduced for the RF classical n-vector model (Ref. 15) in the large-n limit] are true.

The paper is organized as follows. In Sec. II we introduce the n-vector RF quantum model and the self-consistent Hartree equation for the inverse susceptibility is presented. A discussion of the critical properties of the model at QDL is made in Sec. III. Finally, in Sec. IV, we make some concluding remarks.

II. THE MODEL AND THE HARTREE APPROXIMATION

The RF quantum model of interest for us is defined by the following quantum Ginzburg-Landau-Wilson (GLW) functional:[10-13]

$$\mathscr{H}\{\psi,h\}=\frac{1}{2}\int d^dx\int_0^{1/T}d\tau\left[c[\nabla^{\sigma/2}\boldsymbol{\psi}(\mathbf{x},\tau)]^2+r_0\boldsymbol{\psi}^2(\mathbf{x},\tau)+\left[\frac{\partial\boldsymbol{\psi}(\mathbf{x},\tau)}{\partial\tau}\right]^2+\frac{u_0}{2}\boldsymbol{\psi}^4(\mathbf{x},\tau)+2\mathbf{h}(\mathbf{x})\cdot\boldsymbol{\psi}(\mathbf{x},\tau)\right]. \tag{1}$$

Here c, r_0, and u_0 are parameters characteristic of the lattice model (in particular $r_0=-S$, where S is the interaction strength[11-13]),

$$\boldsymbol{\psi}(\mathbf{x},\tau)=\left[\frac{T}{V}\right]^{1/2}\sum_{\substack{q\\0<|\boldsymbol{\kappa}|<\Lambda}}e^{i(\boldsymbol{\kappa}\cdot\mathbf{x}-\omega_l\tau)}\boldsymbol{\psi}(q)$$

is a real n-component vector field, $q\equiv(\boldsymbol{\kappa},\omega_l)$, $\omega_l=2\pi lT$ ($l=0,\pm1,\pm2,\dots$), T is the temperature, V is the volume of the system, and Λ is a wave-vector cutoff related to the presence of a microscopic length scale (the lattice spacing) in the original model. In (1), $[\nabla^{\sigma/2}\boldsymbol{\psi}(\mathbf{x},\tau)]^2$ corresponds to $\kappa^\sigma|\boldsymbol{\psi}(q)|^2$ in the Fourier representation and the value of σ reflects the range of interactions involved in the lattice model: $\sigma=2$ for short-range interactions and $0<\sigma<2$ for long-range interactions which fall as $R^{-(d+\sigma)}$. Finally, the quenched RF $\mathbf{h}(\mathbf{x})$ is governed by a Gaussian distribution with Fourier component averages:

$$[h^j(\boldsymbol{\kappa})]_{\rm av}=0,\quad [h^i(\boldsymbol{\kappa})h^j(\boldsymbol{\kappa}')]_{\rm av}=\delta_{ij}\delta_{\boldsymbol{\kappa},-\boldsymbol{\kappa}'}g(\boldsymbol{\kappa})\ , \tag{2}$$

where[9,10]

$$g(\boldsymbol{\kappa})\approx\Delta_{01}+\Delta_{02}\,|\,\boldsymbol{\kappa}\,|^{\theta}$$

for small $|\,\mathbf{k}\,|$ and arbitrary θ, which corresponds to RF correlations that decay as $|\,\mathbf{x}-\mathbf{y}\,|^{-(d+\theta)}$. Of course, when $\theta\ge 0$ with $\Delta_{0i}\neq 0$ $(i=1,2)$, when $\Delta_{02}=0$ or when $\Delta_{01}=0$, $\Delta_{02}\neq 0$, and $\theta=0$ the short-range correlated RF case will be reproduced.

By using the replica trick,[16] the free energy of the model is given by

$$F=-T\lim_{m\to 0}\left[\frac{1}{m}\int\prod_{\alpha=1}^{m}\mathscr{D}[\psi_\alpha]e^{-\mathscr{H}_{\rm eff}(\{\psi_\alpha\})}\right]\ , \tag{3}$$

where the "effective GLW functional" $\mathscr{H}_{\rm eff}(\{\psi_\alpha\})$ of m replications ($\boldsymbol{\psi}_\alpha$; $\alpha=1,\ldots,m$) of the original field $\boldsymbol{\psi}$ has the following Fourier representation:

$$\mathscr{H}_{\rm eff}(\{\psi_\alpha\})=\tfrac{1}{2}\sum_{\alpha,\beta=1}^{m}\sum_{j=1}^{n}\sum_{\substack{q\\ 0<|\boldsymbol{\kappa}|<\Lambda}}\Gamma^{(0)}_{\alpha\beta}(q)\psi^j_\alpha(-q)\psi^j_\beta(q)+\frac{u_0T}{4V}\sum_{\alpha=1}^{m}\sum_{i,j=1}^{n}\sum_{\substack{\{q_\nu\}\\ 0<|\boldsymbol{\kappa}_\nu|<\Lambda}}\psi^i_\alpha(q_1)\psi^i_\alpha(q_2)\psi^j_\alpha(q_3)\psi^j_\alpha(-q_1-q_2-q_3)\ , \tag{4}$$

where

$$\Gamma^{(0)}_{\alpha\beta}(q)=(r_0+c\kappa^\sigma+\omega_l^2)\delta_{\alpha\beta}-T^{-1}g(\boldsymbol{\kappa})\delta_{\omega_l,0}\ . \tag{5}$$

Equations (3)–(5) allow us to apply to the "effective problem" the usual techniques quite parallel to the pure system, taking $m\to 0$ in the final results.

As shown in Ref. 14, in the large-n limit, for fixed m and arbitrary replica index α, we can split the fourth term of $\mathscr{H}_{\rm eff}(\{\psi_\alpha\})$ in the $(\mathbf{x},\tau)$ representation as (Hartree approximation[17])

$$\boldsymbol{\psi}^4_\alpha(\mathbf{x},\tau)\to 2\langle\boldsymbol{\psi}^2_\alpha\rangle_{\mathscr{H}_{\rm eff}}\boldsymbol{\psi}^2_\alpha(\mathbf{x},\tau)-(\langle\boldsymbol{\psi}^2_\alpha\rangle_{\mathscr{H}_{\rm eff}})^2\ . \tag{6}$$

This makes the effective functional quadratic with $r_0\to r_{\rm eff}=r_0+u_0\langle\boldsymbol{\psi}^2_\alpha\rangle_{\mathscr{H}_{\rm eff}}$. Then, by evaluating the average $\langle\boldsymbol{\psi}^2_\alpha\rangle_{\mathscr{H}_{\rm eff}}$ with the approximate form of $\mathscr{H}_{\rm eff}$:

$$\mathscr{H}_{\rm eff}(\{\psi_\alpha\})\approx\tfrac{1}{2}\sum_{\alpha,\beta=1}^{m}\sum_{j=1}^{n}\sum_{\substack{q\\ 0<|\boldsymbol{\kappa}|<\Lambda}}[(r_{\rm eff}+c\kappa^\sigma+\omega_l^2)\delta_{\alpha\beta}-T^{-1}g(\boldsymbol{\kappa})\delta_{\omega_l,0}]\psi^j_\alpha(-q)\psi^j_\beta(q)-\frac{(r_{\rm eff}-r_0)^2}{4u_0}\frac{mV}{T}\ , \tag{7}$$

we obtain for $r_{\rm eff}$ the self-consistent equation

$$r_{\rm eff}=r_0+nu_0\frac{T}{V}\sum_{\substack{q\\ 0<|\boldsymbol{\kappa}|<\Lambda}}\frac{1}{r_{\rm eff}+c\kappa^\sigma+\omega_l^2}\left[1+\frac{T^{-1}g(\boldsymbol{\kappa})\delta_{\omega_l,0}}{r_{\rm eff}+c\kappa^\sigma+\omega_l^2-mT^{-1}g(\boldsymbol{\kappa})\delta_{\omega_l,0}}\right] \tag{8}$$

with $u_0=O(1/n)$. At this stage we make the limit of Eq. (8) for $m\to 0$ and the result is the self-consistent equation:

$$r=r_0+nu_0\frac{T}{V}\sum_{\substack{q\\ 0<|\boldsymbol{\kappa}|<\Lambda}}\frac{1}{r+c\kappa^\sigma+\omega_l^2}\left[1+\frac{T^{-1}g(\boldsymbol{\kappa})\delta_{\omega_l,0}}{r+c\kappa^\sigma+\omega_l^2}\right] \tag{9}$$

for the parameter $r=\lim_{m\to 0}r_{\rm eff}$ which is just the inverse susceptibility for the RF quantum model under study defined by the inverse value at $q=0$ ($\mathbf{k}=0,\omega_l=0$) of the physical propagator:

$$G(q)=[\langle\psi^j(-q)\psi^j(q)\rangle]_{\rm av}-[\langle\psi^j(-q)\rangle\langle\psi^j(q)\rangle]_{\rm av}\ . \tag{10}$$

This last statement immediately follows from the large-n limit result:

$$G(q)=\lim_{m\to 0}\frac{1}{m}\sum_{\alpha,\beta=1}^{m}\langle\psi^j_\alpha(-q)\psi^j_\beta(q)\rangle_{\mathscr{H}_{\rm eff}}\approx(r+c\kappa^\sigma+\omega_l^2)^{-1} \tag{11}$$

obtained by using the approximate functional (7) via the general replica trick procedure.[14,16]

By making the sum over the Matsubara frequencies in (9) and assuming

$$\frac{1}{V}\sum_{0<|\boldsymbol{\kappa}|<\Lambda}\cdots\ \xrightarrow{V\to\infty}\ K_d\int_0^\Lambda d\kappa\,\kappa^{d-1}\cdots$$

with $K_d=2^{1-d}\pi^{-d/2}/\Gamma(d/2)$, in the low-temperature (which is the temperature region of interest for us) the general self-consistent equation (9) for r reduces to

$$r=r_0+A_0(d,\sigma)nu_0T^{2d/\sigma-1}\Phi(r/T^2)+A_1(d,\sigma)nu_0F_1(r)+A_2(d,\sigma)nw_{01}F_2(r)+A_3(d,\sigma,\theta)nw_{02}F_3(r)\ , \tag{12}$$

where $w_{0i}=u_0\Delta_{0i}$ ($i=1,2$) and

$$A_0(d,\sigma)=\frac{2K_d}{\sigma c^{d/\sigma}},\quad A_1(d,\sigma)=\frac{\Lambda^{d-\sigma/2}K_d}{2\sigma c^{1/2}}\ , \tag{13}$$

$$A_2(d,\sigma)=\frac{\Lambda^{d-2\sigma}K_d}{\sigma c^2},\quad A_3(d,\sigma,\theta)=\frac{\Lambda^{d+\theta-2\sigma}K_d}{\sigma c^2}\ ,$$

$$\Phi(r/T^2)=\int_0^\infty dx\frac{x^{2d/\sigma}-1}{(r/T^2+x^2)^{1/2}}(e^{(r/T^2+x^2)^{1/2}}-1)^{-1}\ ,$$

$$F_1(r)=\int_0^1 dx\frac{x^{d/\sigma-1}}{(\tilde r+x)^{1/2}},\quad F_2(r)=\int_0^1 dx\frac{x^{d/\sigma-1}}{(\tilde r+x)^2}\ , \tag{14}$$

$$F_3(r)=\int_0^1 dx\frac{x^{(d+\theta)/\sigma-1}}{(\tilde r+x)^2}\ ,$$

with $\tilde r=r/c\Lambda^\sigma$.

We are now in a position to investigate the critical properties of our model by approaching the QDL from the disordered phase side. These can be extracted by examining the Eq. (12) near criticality defined by $r=0$. This lies on the possibility to express all the relevant macroscopic quantities in terms of r. Indeed, from (3) and (7), the free-energy density per order parameter component $\mathcal{F}=F/nV$ in the large-n limit is given by

$$\mathcal{F}=-\frac{(r-r_0)^2}{4v_0}+\frac{T}{V}\sum_{0<|\kappa|<\Lambda}\ln\left[2\sinh\left[\frac{(r+c\kappa^\sigma)^{1/2}}{2T}\right]\right]-\frac{1}{2V}\sum_{0<|\kappa|<\Lambda}\frac{g(\kappa)}{r+c\kappa^\sigma}\ , \tag{15}$$

where $v_0=nu_0$. Then, the specific heat at constant volume in the low-temperature limit is expressed as

$$C_V=-T\frac{\partial^2\mathcal{F}}{\partial T^2}\approx\frac{2K_d}{\sigma c^{d/\sigma}}T^{2d/\sigma}\int_0^\infty dx\frac{x^{2d/\sigma-1}e^{(r/T^2+x^2)^{1/2}}}{\left[e^{(r/T^2+x^2)^{1/2}}-1\right]^2}\times\left[\frac{r}{T^2}+x^2-\frac{1}{2T}\frac{\partial r}{\partial T}\right]. \tag{16}$$

Another interesting thermodynamic quantity, which is appropriate when the phase transition is driven by r_0 and constitutes the analogy of the specific heat for a magnetic system, is

$$C=-\frac{\partial^2\mathcal{F}}{\partial r_0^2}=\frac{1}{2v_0}\left[1-\frac{\partial r}{\partial r_0}\right]. \tag{17}$$

Notice that r_0 ($=-S$) does not depend on temperature in contrast to the well known case of the analogous parameter in the GLW functional for classical magnetic systems.

Finally, we note that, from (11) it follows for $G(q)$ the scaling relation:

$$G(\kappa,\omega_l)=\xi^\sigma W(\kappa\xi,\xi^{\sigma/2}\omega_l)\ , \tag{18}$$

where $\xi=c^{1/\sigma}r^{-1/\sigma}$ and $W(x,y)=[c(x^\sigma+1)+y^2]^{-1}$. The relation (18) implies that ξ assumes the role of correlation length for our model.

III. CRITICAL PROPERTIES AT QUANTUM DISPLACIVE LIMIT

A. Critical line and RF quantum displacive limit

As concerning the existence of a critical point defined by $r=0$, we firstly note that $\Phi(r/T^2)$ diverges for $r/T^2\to 0$ when $d/\sigma\le 1$ and $F_i(r)$ ($i=1,2,3$) diverge for $r\to 0$ when $d/\sigma\le\frac{1}{2}$, $d/\sigma\le 2$, and $(d+\theta)/\sigma\le 2$, respectively. This implies that the existence of a phase transition strongly depends on the structure of the RF correlation function $g(\mathbf{k})$ and on appropriate combinations of d,σ,θ.[18]

From Eq. (12) we find that a phase transition occurs:

(i) for $d/\sigma>2$ when $\Delta_{01}\neq 0$, $\Delta_{02}=0$;

(ii) for $d/\sigma>1/2$ and $(d+\theta)/\sigma>2$ when $\Delta_{01}=0$, $\Delta_{02}\neq 0$;

(iii) for $d/\sigma>2$ and $(d+\theta)/\sigma>2$ when $\Delta_{01}\neq 0$, $\Delta_{02}\neq 0$.

In the cases (i)—(iii) for $d/\sigma>1$, a "critical line" $T_c=T_c(r_0)$ exists showing a terminal critical point ($T_c=0$, $r_0=r_{0c}$), where $r_{0c}=r_{0c}(u_0,\{\Delta_{0i}\})$ [$\Longleftrightarrow S_c=S_c(u_0,\{\Delta_{0i}\})$] is the particular choice of r_0 for which T_c vanishes and defines, therefore, the RF QDL. In any case we can write $r_{0c}=r_{0c}^{(P)}+r_{0c}^{(R)}$, where $r_{0c}^{(P)}=-nu_0A_1(d,\sigma)F_1(0)$ is the value of r_0 which defines the QDL in the pure model[11,12] and $r_{0c}^{(R)}$ equal to

$$-nw_{01}A_2(d,\sigma)F_2(0)\ ,$$

$$-nw_{02}A_3(d,\sigma,\theta)F_3(0)\ ,$$

and

$$-n[w_{01}A_2(d,\sigma)F_2(0)+w_{02}A_3(d,\sigma,\theta)F_3(0)]$$

in the cases (i)—(iii), respectively, represents the effects of randomness. Close to the QDL for $r_0\lesssim r_{0c}$, the critical line is, in any case, represented by the power law:

$$T_c(r_0)\approx[nu_0A_0(d,\sigma)\Phi(0)]^{-\sigma/(2d-\sigma)}(r_{0c}-r_0)^{1/\psi}\ , \tag{19}$$

where $\psi=2d/\sigma-1$ is the shift exponent for the present model. Note that it is formally the same as that for the pure model but for $d>\sigma$ and also in the RF case the critical line has an infinite slope at ($T_c=0$, $r_0=r_{0c}$). In the case (ii) for $\frac{1}{2}<d/\sigma<1$ (with $\theta/\sigma>2-d/\sigma$) an isolated critical point ($T_c=0$, $r_0=r_{0c}$) exists, where

$$r_{0c}=r_{0c}^{(P)}-nw_{02}A_3(d,\sigma,\theta)F_3(0)\ .$$

We now investigate the low-temperature critical properties of our RF model by approaching the QDL along the two thermodynamic paths $\mathcal{L}_T\equiv(r_0=r_{0c},T\to T_c=0)$, $\mathcal{L}_S\equiv(T=T_c=0,\ S\to S_c=-r_{0c})$ within the disordered phase region. Correspondingly, for the pure system, experimental[4,19] and theoretical[11,13] results exist so that, predictions about the competition of RF, thermal and quantum fluctuations can be made. Of course, since the critical line, when it exists, has an infinite slope at QDL, the critical behavior along the line $\mathscr{L}_T$ is expected to differ from the one along $\mathscr{L}_S$ which is driven by the variation of $r_0=-S$.

B. Critical properties along the line $\mathscr{L}_T \equiv (r_0=r_{0c},\ T\to 0)$

We are interested to determine the asymptotical behavior for $T\to 0$ of the most relevant macroscopic quantities as the susceptibility $\chi=r^{-1}$, the specific heat C_V and the correlation length ξ. Thus, if we define the critical exponent x_T for a generic quantity X as

$$X \sim T^{-x_T} \quad (T\to 0,\ r_0=r_{0c}) , \tag{20}$$

where $x_T=\gamma_T,\nu_T,\alpha_T,\tilde{\alpha}_T,\ldots$ for $X=\chi,\xi,C_V,\ldots$, respectively, our problem is to calculate the critical exponents (x_T) for different thermodynamic quantities (X). Note that, as in the pure case,[11,12] the exponent for C_V can be defined in two ways since, for $T_c=0$, the additional factor T in $C_V=-T\partial^2\mathscr{F}/\partial T^2$ reduces its singularity by one power in T. Thus, we define $C_V\sim T^{-\tilde{\alpha}_T}$ and $C_V/T\sim T^{-\alpha_T}$, where α_T corresponds to the usual specific-heat exponent. The previous program can be realized simply by a study of the self-consistent equation (12) for $r\to 0$ in combination with the asymptotical behaviors of the functions $\Phi(r/T^2)$ and $F_i(r)$ (see Appendix) and the expressions (16)–(18). The results are conveniently summarized in Table I where R_i, P_i, and B_i denote the regions of the $(d/\sigma,\theta/\sigma)$ plane, shown in Fig. 1, where random (R), pure (P), and borderline (B) behaviors occur, respectively. Here, the white regions correspond to absence of a critical point.

By inspection of Table I and Fig. 1, we firstly note that, when a short-range correlated RF is present (for $\Delta_{01}\neq 0$, $\Delta_{02}=0$; $\Delta_{01}=0$, $\Delta_{02}\neq 0$ and $\theta=0$; $\Delta_{01}\neq 0$, $\Delta_{02}\neq 0$ with $\theta\geq 0$) and the QDL is approached along the line $\mathscr{L}_T$, $d_{\rm CL}=2\sigma$, and $d_{\rm CU}=3\sigma$ assume the role of lower and upper critical dimensionality, respectively, to be compared with $d_{\rm CL}^{(P)}=\sigma/2$ and $d_{\rm CU}^{(P)}=3\sigma/2$ for the corresponding pure system. Thus, the introduction of a RF generates a dimensional shift $d\to d-3\sigma/2$. Here, $d_{\rm CL}$ has the usual meaning but $d_{\rm CU}$ has not to be considered in the usual sense since along $\mathscr{L}_T$ we do not have mean-field behavior for $d>d_{\rm CU}$. It must be considered simply as a borderline dimension below which a RF criticality appears and above which the pure one occurs. When a correlated RF is present we find the following.

(a) If $\Delta_{01}=0$, $\Delta_{02}\neq 0$ with arbitrary $\theta/\sigma\leq 1$, the lower and upper critical dimensionalities are $d_{\rm CL}=2\sigma-\theta$ and $d_{\rm CU}=3\sigma-\theta$, respectively, corresponding to the dimensional shift $d\to d-(3\sigma/2-\theta)$.

(b) If $\Delta_{01}\neq 0$, $\Delta_{02}\neq 0$, we have the same expressions for $d_{\rm CL}$ and $d_{\rm CU}$ as in (a), but with $\theta<0$.

(c) A very strange situation occurs in the $(d/\sigma,\theta/\sigma)$ plane's region $(2-\theta/\sigma<d/\sigma<3-\theta/\sigma,\ 1<\theta/\sigma<3/2)$, where two different RF regimes appear separated, along B_1, by a borderline behavior: the first regime (in R_1) is associated to an isolated critical point, while the second one corresponds to a terminal point of a critical line. In this case, generated by a peculiar competition between the RF and thermal terms in the self-consistent equation (12), and which has not a counterpart along the thermodynamic path $\mathscr{L}_S$, it is difficult to define, without ambiguity, critical dimensionalities to be compared with those of the pure system. Thus, in the following we limit ourselves to

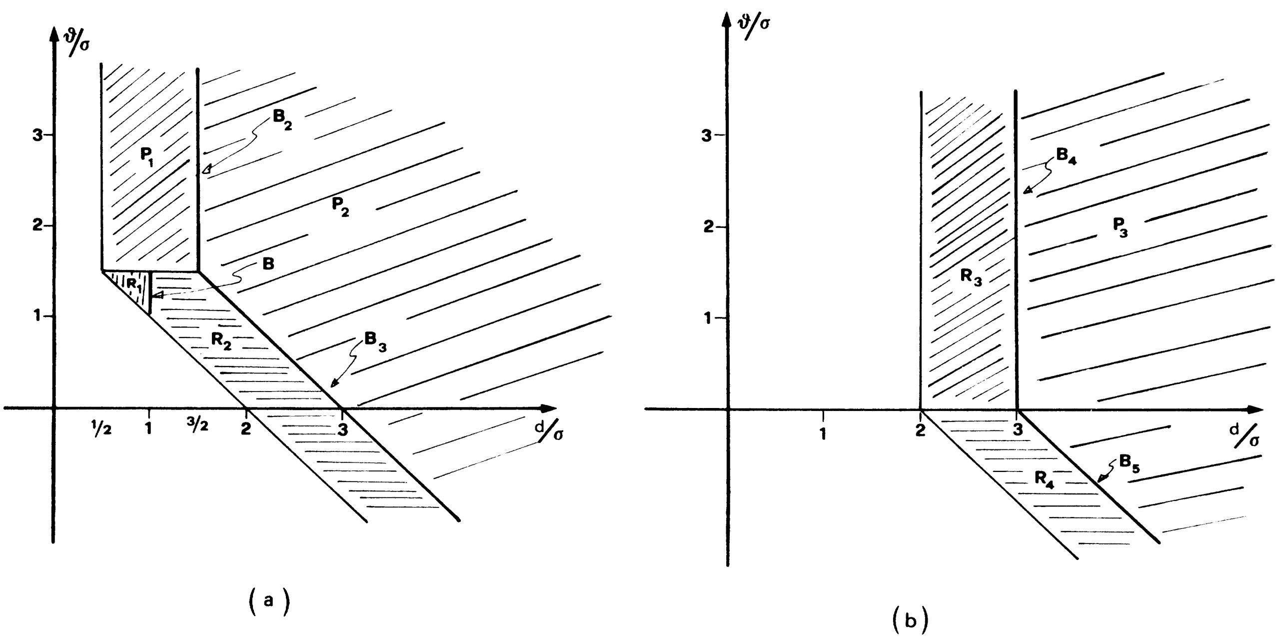

FIG. 1. Regions of the $(d/\sigma,\theta/\sigma)$ plane where different critical regimes occur by approaching the QDL along the line $\mathscr{L}_T$ in the presence of a RF with (a) $\Delta_{01}=0$, $\Delta_{02}\neq 0$; (b) $\Delta_{01}\neq 0$, $\Delta_{02}\neq 0$.

TABLE I. Some critical exponents by approaching the QDL along the line $\mathscr{L}_T \equiv (T \to 0,\ r_0 = r_{0c})$ for different values of the RF correlation function parameters $\{\Delta_{0i}\}$. The asterisks indicate additional logarithmic corrections.

$\{\Delta_{0i}\}$	γ_T	ν_T	α_T	$\tilde{\alpha}_T$	Regions of the $(d/\sigma, \theta/\sigma)$ plane
$\Delta_{01} \neq 0$, $\Delta_{02} = 0$	$\frac{2d-\sigma}{d-2\sigma}$	$\frac{2d/\sigma-1}{d-2\sigma}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	$2 < d/\sigma < 3$
	5^*	$(5/\sigma)^*$	-5	-6	$d/\sigma = 3$
	$\frac{2d-1}{\sigma}$	$\frac{2d-\sigma}{\sigma^2}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	$d/\sigma > 3$
$\Delta_{01} = 0$, $\Delta_{02} \neq 0$	$\frac{\sigma}{\theta-\sigma}$	$\frac{1}{\theta-\sigma}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	R_1
	$\frac{2d-\sigma}{d+\theta-2\sigma}$	$\frac{2d/\sigma-1}{d+\theta-2\sigma}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	R_2
	2	$2/\sigma$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	P_1
	$\frac{2d}{\sigma}-1$	$\frac{2d-\sigma}{\sigma^2}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	P_2
	$\left[\frac{\sigma}{\theta-\sigma}\right]^*$	$\left[\frac{1}{\theta-\sigma}\right]^*$	-1	-2	B_1
	2^*	$(2/\sigma)^*$	-2	-3	B_2
	$\left[\frac{2d}{\sigma}-1\right]^*$	$\left[\frac{2d-\sigma}{\sigma^2}\right]^*$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	B_3
$\Delta_{01} \neq 0$, $\Delta_{02} \neq 0$	$\frac{2d-\sigma}{d-2\sigma}$	$\frac{2d/\sigma-1}{d-2\sigma}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	R_3
	$\frac{2d-\sigma}{d+\theta-2\sigma}$	$\frac{2d/\sigma-1}{d+\theta-2\sigma}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	R_4
	$\frac{2d}{\sigma}-1$	$\frac{2d-\sigma}{\sigma^2}$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	P_3
	5^*	$(5/\sigma)^*$	-5	-6	B_4
	$\left[\frac{2d}{\sigma}-1\right]^*$	$\left[\frac{2d-\sigma}{\sigma^2}\right]^*$	$1-\frac{2d}{\sigma}$	$-\frac{2d}{\sigma}$	B_5

consider situations (a) and (b) only.

As concerning the RF critical exponents for $d_{\rm CL} < d < d_{\rm CU}$ (notice that α_T and $\tilde{\alpha}_T$ have, in any case, the same formal expressions as those in the pure systems), it is immediate to check that they cannot be obtained from the corresponding pure ones[11,12] with the dimensional shift $d \to d - d'$ where $d' = 3\sigma/2$ or $3\sigma/2 - \theta$. Thus, no "dimensional reduction"[20] exists when the QDL is approached along $\mathscr{L}_T$ when a RF is present. On the contrary, as we shall see later, a dimensional reduction occurs when the QDL is approached along $\mathscr{L}_S$, i.e., when the $(T=0)$ transition is driven by the interaction strength $S = -r_0$ of the lattice dynamical model. The absence of dimensional reduction along $\mathscr{L}_T$ is a peculiar manifestation of quantum-thermal fluctuations in the low-temperature limit and not an effect of the Griffiths singularities[21] which have been shown to disappear in the large-n limit.[22]

An additional interesting aspect of our investigation is connected with the effect of the RF fluctuations on the hyperscaling exponent relations which directly involve the spatial dimensionality. It is already known[11,12] that, for pure systems, the usual hyperscaling relations involving the critical exponents $x_T^{(P)}$ appear to fail in the dimensionality range $d_{\rm CL}^{(P)} < d < d_{\rm CU}^{(P)}$, where they are expected to be true for n-vector classical systems. However, for QDL criticality along $\mathscr{L}_T$ it is possible to introduce "modified hyperscaling relations" which are, on the contrary, satisfied. For example, the hyperscaling relation $d\nu = 2 - \alpha$ is modified in $d\nu_T^{(P)} = (2 - \alpha_T^{(P)}) + \zeta_T^{(P)}$, where $\zeta_T^{(P)} = \frac{1}{2}\gamma_T^{(P)} - 2 = -1$, $\gamma_T^{(P)} = 2$, $\nu_T^{(P)} = 2/\sigma$, and $\alpha_T^{(P)} = (\sigma - 2d)/\sigma$. Then, it is easy to show that, when the RF is present, for short-range RF correlations and in the cases (a) and (b) with $d_{\rm CL} < d < d_{\rm CU}$, the above "modified hyperscaling relation" is changed in $(d - d')\nu_T = (2 - \alpha_T) + \zeta_T$, where $\zeta_T = \frac{1}{2}\gamma_T - 2$ and γ_T, ν_T, α_T are the exponents appropriate to the random system. Explicitly we find that $\zeta_T = (7\sigma - 2d)/2(d - 2\sigma)$ or $\zeta_T = [7\sigma - 2(d+\theta)]/2(d + \theta - 2\sigma)$ for uncorrelated and correlated RF, respectively.

The above relations can be also rewritten as

TABLE II. Some critical exponents by approaching the QDL along the line $\mathscr{L}_S \equiv (T=0, r_0 \to r_{0c})$ for different values of the RF correlation function parameters $\{\Delta_{0i}\}$. The asterisks indicate additional logarithmic corrections.

$\{\Delta_{0i}\}$	γ_S	ν_S	α_S	Regions of the $(d/\sigma, \theta/\sigma)$ plane
$\Delta_{01} \neq 0$, $\Delta_{02} = 0$	$\frac{\sigma}{d-2\sigma}$	$\frac{1}{d-2\sigma}$	$\frac{d-3\sigma}{d-2\sigma}$	$2 < d/\sigma < 3$
	1^*	$(1/\sigma)^*$	0^*	$d/\sigma = 3$
	1	$1/\sigma$	0	$d/\sigma > 3$
$\Delta_{01} = 0$, $\Delta_{02} \neq 0$	$\frac{\sigma}{d+\theta-2\sigma}$	$\frac{1}{d+\theta-2\sigma}$	$\frac{d+\theta-3\sigma}{d+\theta-2\sigma}$	R_1
	$\frac{2\sigma}{2d-\sigma}$	$\frac{2}{2d-\sigma}$	$\frac{2d-3\sigma}{2d-\sigma}$	P
	1	$1/\sigma$	0	MF_1
	1^*	$(1/\sigma)^*$	0^*	B_1, B_2
$\Delta_{01} \neq 0$, $\Delta_{02} \neq 0$	$\frac{\sigma}{d-2\sigma}$	$\frac{1}{d-2\sigma}$	$\frac{d-3\sigma}{d-2\sigma}$	R_2
	$\frac{\sigma}{d+\theta-2\sigma}$	$\frac{1}{d+\theta-2\sigma}$	$\frac{d+\theta-3\sigma}{d+\theta-2\sigma}$	R_3
	1	$1/\sigma$	0	MF_2
	1^*	$(1/\sigma)^*$	0^*	B_3, B_4

$(d-\omega^{(P)*})\nu_T^{(P)} = 2-\alpha_T^{(P)}$ and $(d-d'-\omega^*)\nu_T = 2-\alpha_T$, where $\omega^{(P)*} = \zeta_T^{(P)}/\nu_T^{(P)} = -\sigma/2$ and

$$\omega^* = \frac{\zeta_T}{\nu_T} = \frac{\sigma}{2}\,\frac{7\sigma-2d}{2d-\sigma} \quad \text{or} \quad \frac{\sigma}{2}\,\frac{7\sigma-2(d+2\theta)}{2d-\sigma}$$

assume the role of the Fisher "anomalous dimensions of the vacuum"[23] for pure systems and when a (uncorrelated or correlated) RF is switched on, respectively. Due to the possibility to consider different values of d, σ, θ, we believe that the previous aspect of the modified hyperscaling laws for critical properties near the QDL along $\mathscr{L}_T$ may have some relevance from the experimental point of view. A discussion about the scaling relations, which do not contain the dimensionality explicitly, is reserved for the end of Sec. III.

C. Critical properties along the line $\mathscr{L}_S \equiv (T=0,\ S \to S_c)$

We now put $T=0$ in the self-consistent equation (12) so that we can explore the critical properties of the RF model in terms of $r_0 - r_{0c}$ when only quantum and RF fluctuations are involved. By approaching the QDL along $\mathscr{L}_S$ from the disordered phase side, we define the critical exponents γ_S, ν_S, and α_S (for χ, ξ, and $C = -\partial^2 \mathscr{F}/\partial r_0^2$) according to the power law

$$\chi \sim (r_0 - r_{0c})^{-\gamma_S}, \quad \xi \sim (r_0 - r_{0c})^{-\nu_S} \quad (T=0,\ r_0 \to r_{0c}),$$
$$C \sim C_0 + C_1 (r_0 - r_{0c})^{-\alpha_S} \quad (T=0,\ r_0 \to r_{0c}). \tag{21}$$

Then, by using the asymptotical behaviors of the functions $F_i(r)$ $(i=1,2,3)$ near the criticality (see Appendix), a study of the self-consistent equation for the inverse susceptibility yields the results exhibited in Table II for different structures of the RF correlation function.

Here, as in Table I, R_i, P, MF_i, and B_i denote the regions of the $(d/\sigma, \theta/\sigma)$ plane shown in Fig. 2, where random, pure, mean-field, and borderline behaviors occur, respectively. Of course, in the white regions no critical point exists.

The critical properties obtained by approaching the QDL along $\mathscr{L}_S$ drastically differ from those along $\mathscr{L}_T$ and are quite consistent with RG predictions for quantum systems at zero temperature.[7-10] This is due to the fact that $r_0 = -S$ for structural phase transitions (as the chemical potential for Bose systems, the transverse field in spin systems, etc.) is the natural intensive variable involved in a RG treatment of $(T=0)$ quantum criticality and it is the analogous of the corresponding parameter in the GLW functional for classical n-vector model. From Table II and Fig. 2 it immediately follows that, for both uncorrelated and correlated RF, we can speak about a lower and upper critical dimensionality for criticality along $\mathscr{L}_S$ in the usual sense. Specifically one has $d_{CL} = 2\sigma$ and $d_{CU} = 3\sigma$ for short-range correlated RF and $d_{CL} = 2\sigma - \theta$ and $d_{CU} = 3\sigma - \theta$ for long-range correlated ones. Further, for $d_{CL} < d < d_{CU}$, the critical exponents can be obtained from the pure ones $\gamma_S^{(P)} = 2\sigma/(2d-\sigma)$, $\nu_S^{(P)} = 2/(2d-\sigma)$, $\alpha_S^{(P)} = (2d-3\sigma)/(2d-\sigma)$, which occur for $d_{CL}^{(P)} = \sigma/2 < d < d_{CU}^{(P)} = 3\sigma/2$, with the dimensional shift $d \to d - d'$, where $d' = 3\sigma/2$ or $3\sigma/2 - \theta$. Thus, in the present quantum model, when the $T=0$ structural phase transition is driven by $r_0 = -S$, a "dimensional reduction" occurs $[x_S^{(R)}(d) \equiv x_S^{(P)}(\bar{d})$, where $\bar{d} = d - d']$. This form of dimensional crossover is "exact" in the large-n limit since in this case dangerous unperturbative effects[23] as the Griffiths singularities disappear.[22] An immediate consequence is that all the scaling laws, which are true for pure system at $T=0$, preserve their validity in the presence of a RF. Of course also the ($T=0$) hyperscaling exponent relations are simply obtained from the pure ones (in terms of $r_0=-S$ these are valid in absence of RF's in contrast with the situation which appears along $\mathcal{L}_T$ where modified hyperscaling relations must be introduced) with d replaced by $d-d'$. For instance, the ($T=0$) hyperscaling relation $(d+\sigma/2)\nu_S^{(P)}=2-\alpha_S^{(P)}$ for $d_{\rm CL}^{(P)}<d<d_{\rm CU}^{(P)}$ (which for pure quantum systems at $T=0$ is obtained from the classical one for $d\to d+\sigma/2$ as a consequence of the usual quantum-dimensional crossover[24]) is changed in $[(d-d')+\sigma/2]\nu_S=2-\alpha_S$, where ν_S and α_S are RF critical exponents for $d_{\rm CL}<d<d_{\rm CU}$.

We conclude this section by including some considerations about additional critical exponents also involving the "intrinsic critical dynamics" of the quantum model under study. Firstly we observe that, by analytical continuation of the Matsubara propagator (18) just above the real frequency axis ($i\omega_l\to\omega+i\epsilon$, $\epsilon\to0^+$) one obtained the retarded response function $G_R(\kappa,\omega)$ which scales as

$$G_R(\kappa,\omega)=\xi^{2-\eta}W_R(k\xi,\xi^z\omega)\ , \tag{22}$$

where $\eta=2-\sigma$ and $z=\sigma/2$ are the correlation function and dynamical critical exponents. In particular, the critical mode is found to obey the dynamical scaling relation

$$\omega_c(\kappa)=\kappa^z \mathcal{f}\ (\kappa\xi) \tag{23}$$

close to the transition point. Other critical exponents, characteristic of the ordered phase can be derived from the equation of state. It can be obtained in a standard way[17,25] by including in the effective GLW functional (4) a term of the type

$$\mathcal{H}_H=-H\int d^dx\int_0^{1/T}d\tau\sum_{\alpha=1}^{m}\psi_\alpha^1(\mathbf{x},\tau)\ , \tag{24}$$

where H is an external field along the direction $i=1$. Here we omit the details and we only quote the main results for the exponents β_S and δ defined by

$$\begin{aligned} M&\sim|r_0-r_{0c}|^{\beta_S}\ \ (T=0,\ r_0\to r_{oc}^-)\ ,\\ M&\sim|H|^{1/\delta}\ \ (T=0,\ r_0=r_{0c},\ H\to0)\ , \end{aligned} \tag{25}$$

where $M=[\langle\psi^1\rangle]_{\rm av}$ is the order parameter. When the RF is relevant we find $\beta_S=\frac{1}{2}$ in any case and

$$\delta=\frac{d}{d-2\sigma},\quad \delta=\frac{d+\theta}{d+\theta-2\sigma} \tag{26}$$

for short- and long-range random correlations, respectively. Of course the same results can be obtained directly from those in Table II by using the scaling laws:

$$\begin{aligned} \delta&=\frac{2-\alpha_S+\gamma_S}{2-\alpha_S-\gamma_S}\ ,\\ \beta_S&=\frac{\gamma_S}{\delta-1}\ , \end{aligned} \tag{27}$$

or the ($T=0$) hyperscaling relations in the presence of a RF:

$$\begin{aligned} \delta&=\frac{(d-d'+\sigma/2)+2-\eta}{(d-d'+\sigma/2)-2+\eta}\ ,\\ \beta_S&=\tfrac{1}{2}\nu_S\left[\left[d-d'+\frac{\sigma}{2}\right]-2+\eta\right]\ . \end{aligned} \tag{28}$$

Finally, it is immediate to check that, in contrast with the situation along $\mathcal{L}_S$, the usual scaling relations for the QDL critical exponents $\{x_T\}$ break down when the RF is relevant. For instance, apart from the relation $\gamma_T=(2-\eta)\nu_T$ which is trivially verified in any case, we have that the Griffiths inequality $\gamma(\delta+1)\geq(2-\alpha)(\delta-1)$ is fulfilled as strict inequality:

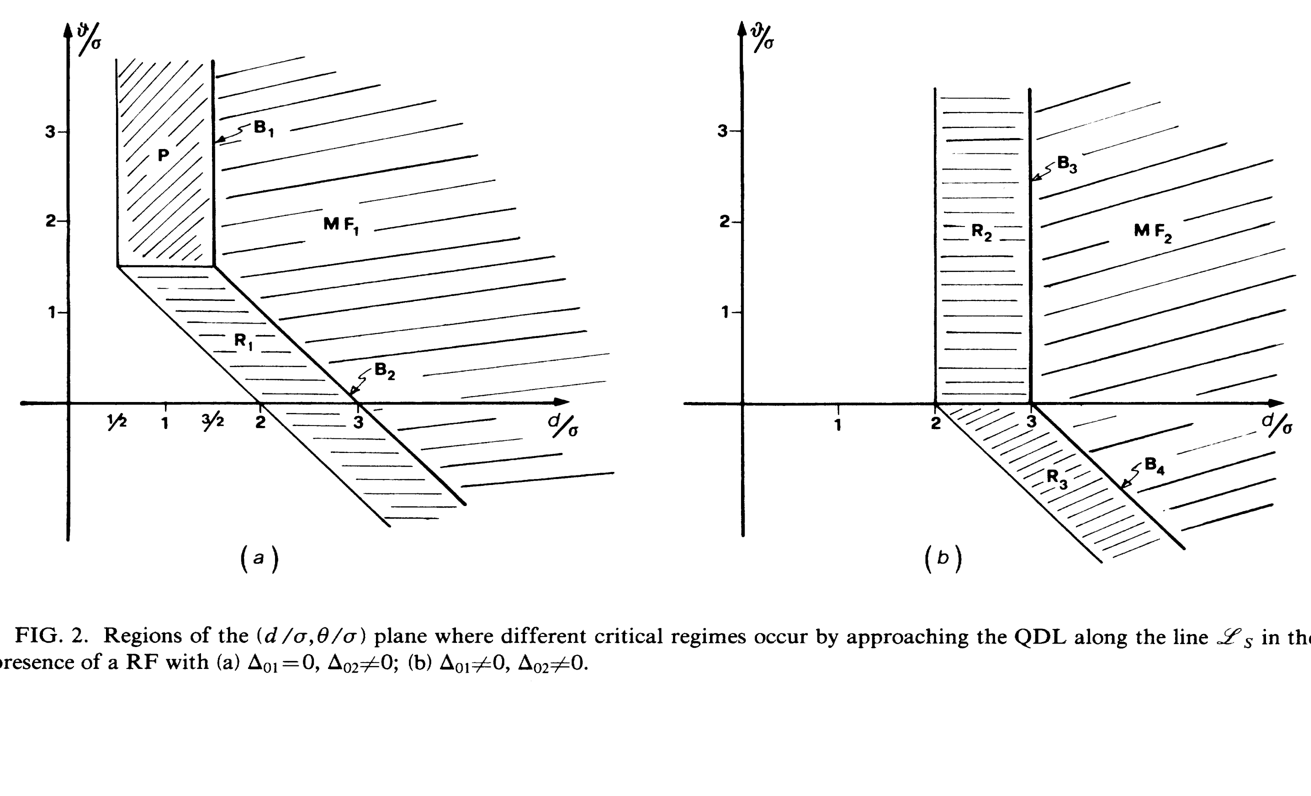

FIG. 2. Regions of the $(d/\sigma,\theta/\sigma)$ plane where different critical regimes occur by approaching the QDL along the line $\mathcal{L}_S$ in the presence of a RF with (a) $\Delta_{01}=0$, $\Delta_{02}\neq0$; (b) $\Delta_{01}\neq0$, $\Delta_{02}\neq0$.

$$\gamma_T(\delta+1) > (2-\alpha_T)(\delta-1) . \quad (29)$$

This is in a sharp contrast with the case of the corresponding pure system[11,12] and appears as a peculiarity of the approach to the QDL along $\mathscr{L}_T$.

IV. CONCLUDING REMARKS

The previous theoretical predictions, which are exact in the large n limit, can give useful informations about real situations in quantum ferroelectrics and other systems which show structural phase transitions when quenched impurities are present. In particular, in our opinion, the analysis of the QDL criticality along the thermodynamic path $\mathscr{L}_T$ assumes a relevant role, from both theoretical and experimental point of view, based on the following,

(i) More sophisticated techniques based on quantum versions of the Wilson RG (Refs. 7 and 10), do not allow to obtain information about QDL critical properties along $\mathscr{L}_T$. The reason of this limitation lies on the fact that in the RG approach the parameter $r_0=-S$ (not depending on temperature) is the natural variable which drives the transition at QDL and not the temperature which assumes the role of external fixed parameter. Thus, only $(T=0)$ predictions in terms of r_0-r_{0c} are available and the exponents $\{x_T\}$ are not accessible via the usual RG treatment.

(ii) The temperature is the intensive parameter which is involved directly in the experiments[3,6] and the situation $(T=0,\ r_0\to r_{0c})$ is beyond the actual experimental possibility, apart from some extrapolations of low-temperature results.[19]

We wish to stress that, while the predictions along $\mathscr{L}_S$ are consistent with those obtained via RG perturbative techniques and the $(T=0)$ dimensional reduction has to be considered exact in the large n-limit (the Griffiths singularities disappear), the results along $\mathscr{L}_T$ change drastically the usual scenario of the RF effects on classical and quantum critical behavior. Unusual QDL critical properties emerge in terms of T which are not governed by the RF fluctuations only, but rather have to be interpreted as a manifestation of the peculiar competition of the three types of fluctuations involved in the problem. The contrary happens for the $(T=0)$ critical behavior along $\mathscr{L}_S$. Indeed, a comparison with the results for a RF Bose system in the classical regime[14] [which are valid also for the present model when the $(T\neq0)$ transition is driven by $r_0=-S$], allows us to assert that, as expected[7,10] the RF fluctuations dominate over thermal and quantum ones along

$$\mathscr{L}_S(T)\equiv[\text{fixed } T \text{ and } r_0\to r_{0c}(T)]$$

and destroy the classical quantum crossover for $T\to0$ which occurs in pure quantum systems.[12,13,24,25]

In conclusion, we believe that the qualitative picture of the QDL problem in structural phase transitions here presented is correct and quite informative and may constitute a useful guide for future experimental and theoretical investigations.

APPENDIX

In this appendix we summarize the asymptotical behaviors of the functions $F_i(r)$ $(i=1,2,3)$ and $\Phi(r/T^2)$ appropriate for a discussion of structural critical properties by approaching the QDL in the presence of a RF. We have

$$F_1(r)\approx F_1(0)+\begin{cases}-A(d,\sigma)r^{d/\sigma-1/2}+O(r), & \frac{1}{2}<d/\sigma<\frac{3}{2}\\ (\frac{1}{2}c\Lambda^\sigma)r\ln r+O(r), & d/\sigma=\frac{3}{2}\\ -\dfrac{\sigma}{c\Lambda^\sigma(2d-3\sigma)}r+O(r^{\min\{d/\sigma-1/2;2\}}), & d/\sigma>\frac{3}{2}\end{cases} \quad (\text{A1})$$

where $F_1(0)=2\sigma/(2d-\sigma)$ and

$$A(d,\sigma)=-\frac{\pi^{1/2}\Gamma(d/\sigma)}{\Gamma(d/\sigma+\frac{1}{2})}\sin[\pi(d/\sigma+\tfrac{1}{2})]>0\ ,$$

since $\sin[\pi(d/\sigma+\frac{1}{2})]<0$ for $\frac{1}{2}<d/\sigma<\frac{3}{2}$. Further, if we put $F_2(r)=G_{d/\sigma}(r)$ and $F_3(r)=G_{(d+\theta)/\sigma}(r)$, one has

$$G_\alpha(r)\approx G_\alpha(0)+\begin{cases}(1-\alpha)\dfrac{\pi/(c\Lambda^\sigma)^{\alpha-2}}{\sin(\pi\alpha)}r^{\alpha-2}+O(r), & 2<\alpha<3\\ 2/c\Lambda^\sigma)r\ln r+O(r), & \alpha=3\\ -\dfrac{2/(c\Lambda^\sigma)}{\alpha-3}+O(r^{\min\{\alpha-2;2\}}), & \alpha>3\end{cases} \quad (\text{A2})$$

where $\alpha=d/\sigma,\ (d+\theta)/\sigma$, $G_\alpha(0)=1/(\alpha-2)$, and $\sin(\pi\alpha)>0$ for $2<\alpha<3$. Finally,

$$\Phi\left[\frac{r}{T^2}\right] \approx \Phi(0) + \begin{cases} \dfrac{\pi}{2\sin(\pi d/\sigma)}(r/T^2)^{d/\sigma-1} + O(r/T^2), & 1 < d/\sigma < \frac{3}{2} \\ -\dfrac{\pi}{2}(r/T^2)^{1/2} + O[(r/T^2)\ln(r/T^2)], & d/\sigma = \frac{3}{2} \\ \dfrac{\pi}{2\sin(\pi d/\sigma)}(r/T^2)^{d/\sigma-1} + O(r/T^2), & \frac{3}{2} < d/\sigma < 2 \\ \frac{1}{2}(r/T^2)\ln(r/T^2) + O(r/T^2), & d/\sigma = 2 \\ -\dfrac{\Gamma\left[\dfrac{2d}{\sigma}\right]\Gamma(d/\sigma - 3/2)\zeta\left[\dfrac{2d}{\sigma} - 3\right]}{8\Gamma(d/\sigma + \frac{1}{2})}(r/T^2) + O[(r/T^2)^{\min\{d/\sigma-1;2\}}], & d/\sigma > 2 \end{cases} \tag{A3}$$

where $\Phi(0) = \Gamma[(2d/\sigma) - 1]\zeta[(2d/\sigma) - 1]$ for $d/\sigma > 1$, $\zeta(s) = \sum_{\kappa=1}^{\infty}\kappa^{-s}$ is the ζ function and $\sin(\pi d/\sigma) < 0$ for $1 < d/\sigma < 2$.

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