Resistive breakdown of inhomogeneous media

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When an electric field is applied to an inhomogeneous material, the resulting current and power densities may have strong spatial variations. Areas where the dissipated Joule heat is much higher than in neighboring regions are called hot spots. It is natural to assume that the conductivity in these spots is altered as a result of the large Joule heat dissipation . One may ask where hot spots occur in the material, and how they will develop when the material properties change. We use a resistor network model to study the occurrence and evolution of hot spots in inhomogeneous materials, which are two dimensional, two phase, and statistically isotropic. We show that the stability to resistive breakdown of such materials depends upon the concentration of hot spots. When the spots are concentrated, the interaction between the spots makes the evolution of the material rapid. The material is then considered to be unstable to resistive breakdown. The development is in general faster when the spots are made well conducting than when they are made poorly conducting.

I. INTRODUCTION

Composite materials and, more generally, inhomogeneous media are becoming of increasing importance in many fields of physics. Models of such materials are often closely related to problems of fundamental importance, such as percolation[1] and critical phenomena. Much work has been done on transport properties of inhomogeneous media and, in particular, many papers deal with bounds[2–6] to the effective conductivity. Also, many papers are resistor network calculations,[7,8] focusing on critical phenomena near the percolation threshold. Recently, a number of papers dealing with various types of breakdown has appeared. For instance, Niemeyer *et al.*[9] discuss dielectric breakdown, Smalley *et al.*[10] study crack propagation, and Takayasu,[11] de Arcangelis and Redner,[12] and Redner[13] consider resistive breakdown. In this paper we are primarily interested in the resistive properties of inhomogeneous systems and how these properties may change when the medium is subject to a strong electric field. Our method is somewhat similar to that of Takayasu[11] and de Arcangelis and Redner[12] but our breakdown criterion differs from theirs in that it is not the voltage across a single resistor, but rather the dissipated Joule heat in a region around the resistor, which determines its breakdown.

We focus our interest on inhomogeneous two-phase materials, which are two dimensional and statistically isotropic. The constituent conductivities are denoted by σ_1 and σ_2. When a current flows through such a substance, the current density varies sharply due to the irregularities of the phase distribution. Moreover, the power density is also strongly inhomogeneous, especially when the constituent conductivities differ significantly. Regions where the density of the dissipated power is substantially larger than in the surrounding areas are called hot spots.[14] These areas appear as a result of long-range effects of the phase distribution. Assume that the material properties in such regions change due to the large power dissipation. Suppose that the conductivity in the spots either decreases to be σ_2 or increases to be σ_1 ($\sigma_2 \ll \sigma_1$). These changes lead to different distributions of current and power density. We propose a resistor network model to study the evolution of inhomogeneous materials when the conductivity changes in the hot spots.

II. RESISTOR NETWORK SIMULATIONS

Calculations using square networks and two types of resistors, R_1 and R_2, have been made to simulate the evolution. A random, $(N+1)\times(N+1)$ network with concentrations c_2 and c_1 $(=1-c_2)$ of resistors R_2 and R_1 is generated. The network has periodic boundary conditions in the horizontal direction. All nodes in column N are connected through resistors to the corresponding nodes in column 0. $N=80$ turned out to be a large enough size to give consistent results. A voltage is applied across the network in the vertical direction. The node potentials are calculated in an iterative fashion by the Gauss-Seidel procedure. The power distribution is determined by a moving average procedure. The size of the moving average window was chosen to be 5×5 nodes. A larger window suppressed the local variations of the power density too severely, and a smaller window gave an average over just a few resistors. The material properties are altered by changing all resistors connected to nodes belonging to hot spots to R_1 or to R_2 ($R_1 \ll R_2$). This completes the cycle and the potential distribution is recalculated. The results, in the form of a topographical map of the power distribution as well as a network where only resistors R_2 are marked, are recorded after each cycle.

III. ESTIMATION OF POWER DENSITY AND CHARACTERIZATION OF HOT SPOTS

The power value assigned to a particular node is estimated by summing over all resistors which belong to a

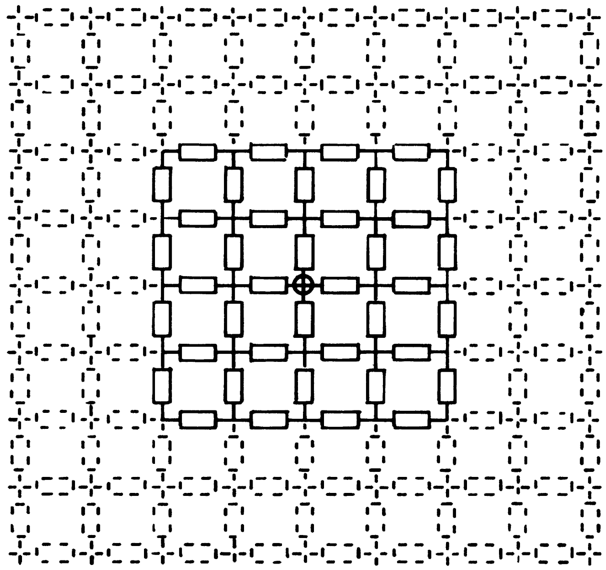


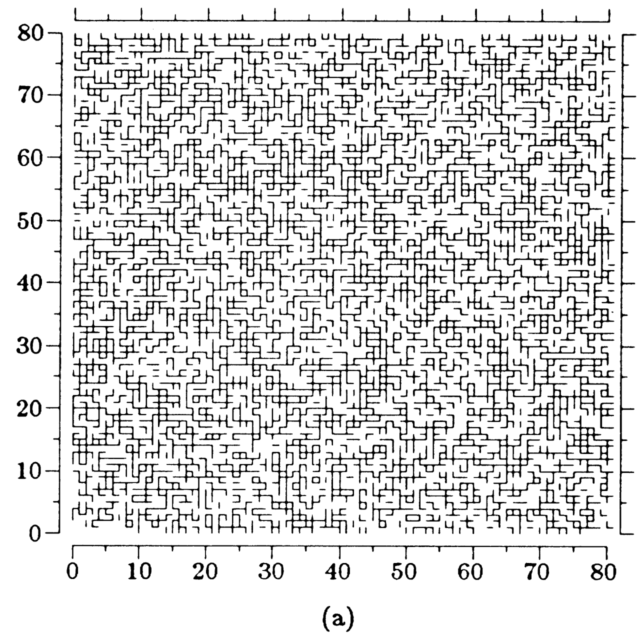
FIG. 1. Moving average window of size 5×5 nodes. The power value assigned to the node marked with a circle is obtained by summing over all resistors denoted by solid lines.

square "window," centered over the node, see Fig. 1. The size of this window should not be too small, since it is not reasonable to interpret few resistors (or a single) to correspond to one phase grain in a real two-phase material. Discretizations with a small number of resistors in each phase grain are too restrictive to give a locally correct representation of the current distribution. Moreover, the smearing effect of the averaging procedure is further justified by noting that heat conduction will make the hot spots somewhat diffuse. However, the size of the window must not be so large as to erase the significant variations in the power density.

Hot spots are regions, where the power density exceeds a certain threshold. It is reasonable to assume that this hot spot threshold depends on material properties, but for practical reasons we have chosen it to be related to the average dissipated power.

Jansson and Grimvall[14] found that there is no conspicuous characteristic of the resistor distribution that leads to the occurrence of hot spots. In order to test the hypothesis, implicit in the paper by Jansson and Grimvall, that there are approximately equal numbers of resistors of each type in hot spots, we studied ten different initial samples, such as that of Fig. 2. We found an average of 55% poorly conducting resistors in hot spots and a standard deviation of 3%. Our simulations further confirm that hot spots are results of strongly nonlocal properties of the resistor distribution, see Fig. 3.

The constriction of the current to a few narrow necks plays an important role in the evolution of hot spots. The type of aging (the conductivity of hot spots changing to become well or poorly conducting) determines whether the growth will be parallel or perpendicular to the direction of the applied electric field.



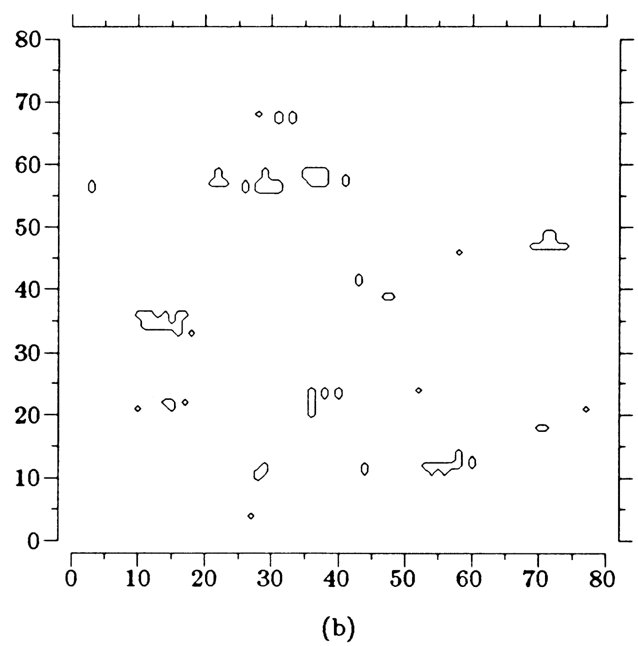


FIG. 2. Initial sample with 81×81 resistors and equal amounts of both types of resistors. $R_1=1$ and $R_2=10$. (a) shows the network, where resistors of type R_2 are marked by solid lines and the node rows and columns are numbered. (b) shows the hot spot distribution when the hot spot threshold equals 1.5 times the average power density.

IV. EVOLUTION BY MAKING HOT SPOTS POORLY CONDUCTING

Random, 81×81 square networks with $c_2=0.5$ and resistors $R_1=1$ and $R_2=10$ were used as starting samples for the simulation. The intermediate results in the form

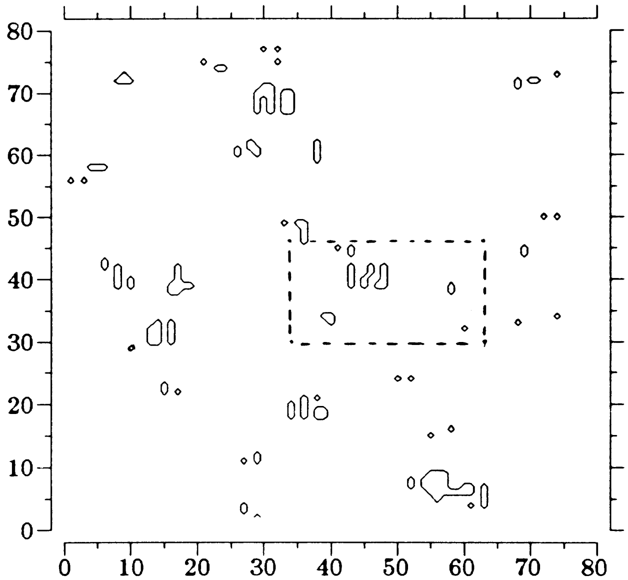


FIG. 3. Hot spot distribution when all hot spots in the sample of Fig. 2, except those in the central enclosed (by dashed lines) region, have been made well conducting. This figure should be compared to Fig. 7(a), which was obtained by making *all* hot spots well conducting. The development of the hot spots is qualitatively the same in both cases. Thus the phase distribution far from the spot is more important than the details of the distribution in the immediate vicinity of the spot, in determining its existence. The hot spot threshold equals 1.5 times the average power density of the sample in Fig. 2.

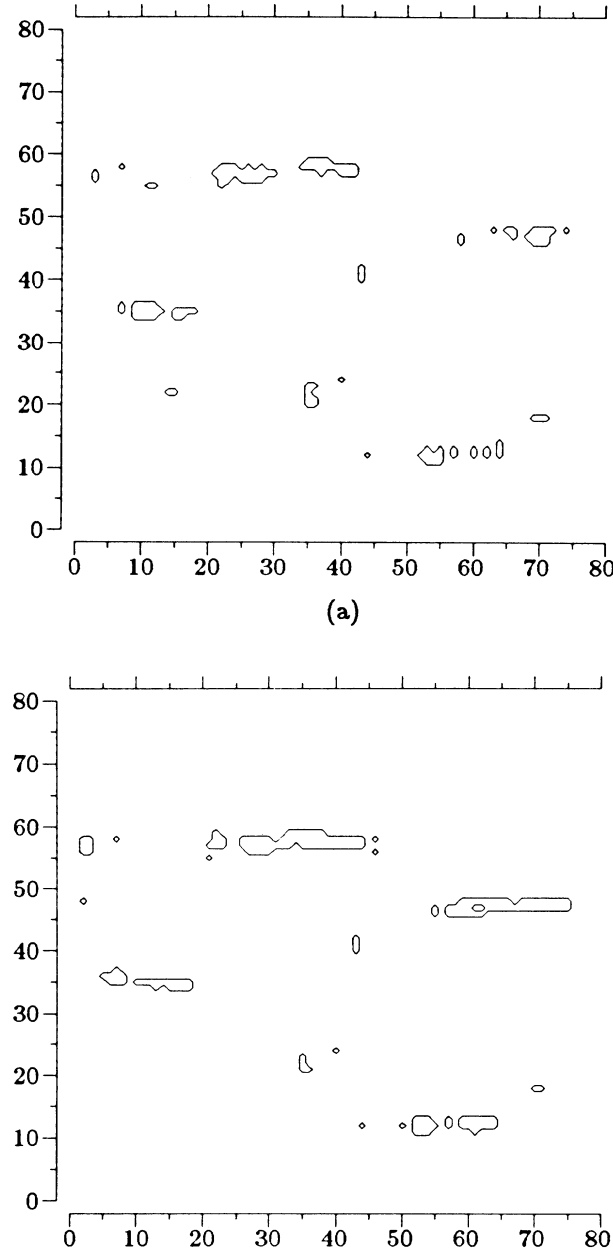


FIG. 4. Hot spot distribution after one cycle (a), and after two cycles (b), respectively, when the sample in Fig. 2 has been changed by making all hot spots poorly conducting. The hot spot threshold equals 1.5 times the average power density of the sample in Fig. 2.

of power plots and maps of the resistor distribution were recorded after each cycle, see Figs. 2 and 4. In an attempt to quantify the evolution, the resistor distribution of the hot spots was determined after each cycle and is shown in Fig. 5.

The growth of hot spots always occurs perpendicularly to the applied electric field. When the material in two hot spots, not far from each other and at approximately the same electrostatic potential, is changed to be poorly conducting, the current will be forced to flow through a narrow neck (an hourglass), see Fig. 6(a). The density of the dissipated power increases in this area and a new hot spot is created. We call this the *hourglass effect.*

When the hot spot threshold is high, the hot spots are dilute, i.e., the diameter of the spots D_{HS} is much smaller than the distance L_{HS} between the spots. The growth of the spots is small or nonexistent. Sometimes the distribution of hot spots changes marginally to begin with and then ceases to evolve. The material is then said to be stable to resistive breakdown. When the threshold is low, D_{HS} is of the same order as L_{HS} and the growth of the spots is rapid, due to a large hourglass effect. The material is then unstable to resistive breakdown.

V. EVOLUTION BY MAKING HOT SPOTS WELL CONDUCTING

Simulations were made with random, 81×81 square networks, with initial concentration $c_2=0.5$ and resistors $R_1=1$ and $R_2=10$. A series of maps in Figs. 2 and 7 show the evolution of such a sample. The resistor distribution of the hot spots is shown in Fig. 5.

The hot spots always grow in the direction of the applied electric field. When the material in two hot spots, not far from each other and aligned with the applied electric field, is changed to be well conducting, the current

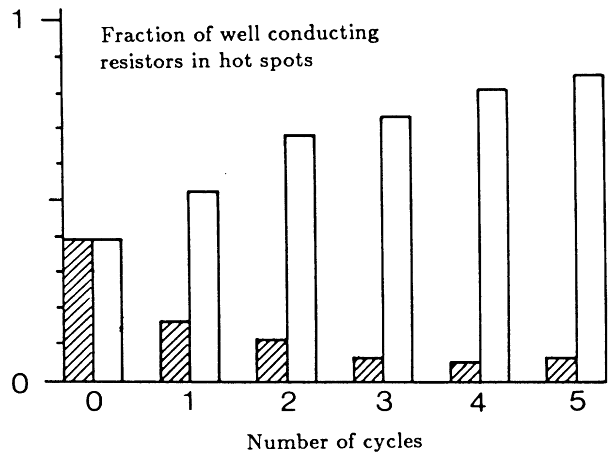


FIG. 5. Fraction of well conducting resistors in hot spots after each cycle, starting from the sample of Fig. 2. The blank columns represent evolution by making hot spots well conducting, and the diagonally striated columns denote development by making hot spots poorly conducting.

penetrates these regions more easily than before. The current density, as well as the Joule heat dissipation increases in the areas closest to the well-conducting material and the hot spots grow parallel to the applied electric field, see Fig. 6(b).

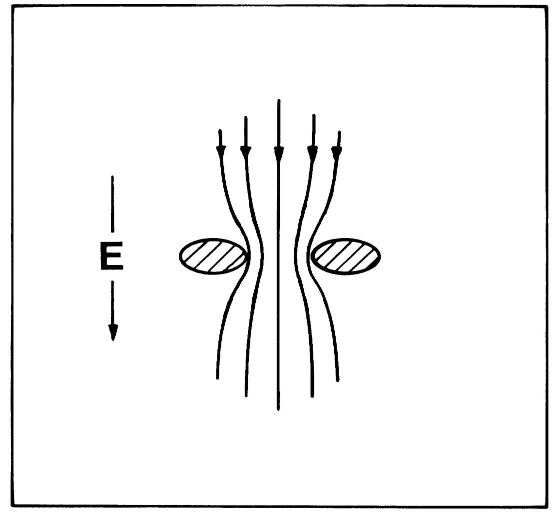
If the hot spot threshold is chosen sufficiently high, the spots are dilute ($D_{\mathrm{HS}} \ll L_{\mathrm{HS}}$) and grow slowly or not at all. For a low threshold, D_{HS} is of the same order as L_{HS} and the evolution is rapid, sometimes "explosive." The material is unstable to resistive breakdown.

VI. INFLUENCE OF BOUNDARY CONDITIONS ON THE EVOLUTION

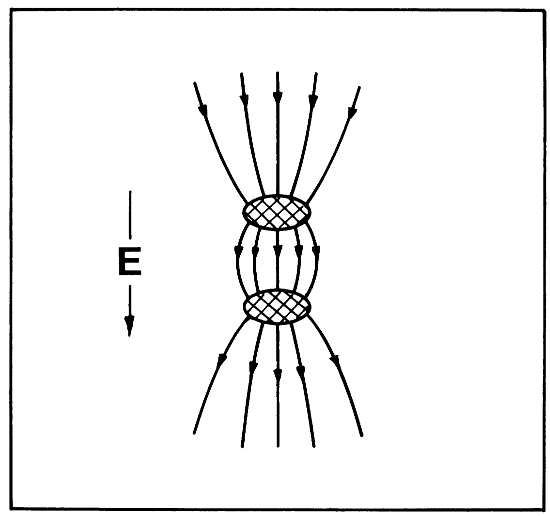
The evolution is found to be more rapid (in some cases even explosive) when hot spots are made well conducting than when they are made poorly conducting. It is a consequence of assuming that the applied electric field **E** is kept constant. This assumption is valid if the sample is part of a larger medium, which has very few hot spots. Then the evolution of the sample will not affect the electric field. In this case it is adequate to write $P = \sigma_e \mathbf{E}^2$, where P is the average power density in the sample and σ_e is the effective conductivity of the sample. When the hot spots become well conducting, σ_e and P increase. The surface fraction of hot spots increases and the evolution accelerates. Correspondingly, when the hot spots are made poorly conducting, σ_e and P decrease. The fraction of hot spots becomes smaller and the process of evolution slows down. However, in some situations it may be more relevant to keep the average current density **J** constant. Then $P = \mathbf{J}^2/\sigma_e$ is appropriate. This gives the result that the evolution, when hot spots change to become poorly conducting, is the most rapid.

VII. CONCLUSIONS AND DISCUSSION

The main concern of this paper has been the occurrence and evolution of so-called hot spots in inhomogeneous



(a)



(b)

FIG. 6. Schematic drawing of the hourglass effect with current lines shown in a small area of a sample when the current is impeded (a) by regions with low conductivity (diagonally striated), and facilitated (b) by regions with high conductivity (cross-hatched). **E** denotes the applied electric field.

materials. These spots are regions with very large power density. Their existence is not determined only by the phase distribution in the immediate vicinity of the hot spot. Instead, it also strongly depends upon the distribution of phases in a large region surrounding the particular spot. Hence, hot spots should be regarded as results of global, rather than local properties of the phase distribution.

We assume that the conductivity in the hot spots increases or decreases and thereby causes the material to evolve. A numerical model has been developed to study this type of evolution in inhomogeneous media. Calcula-

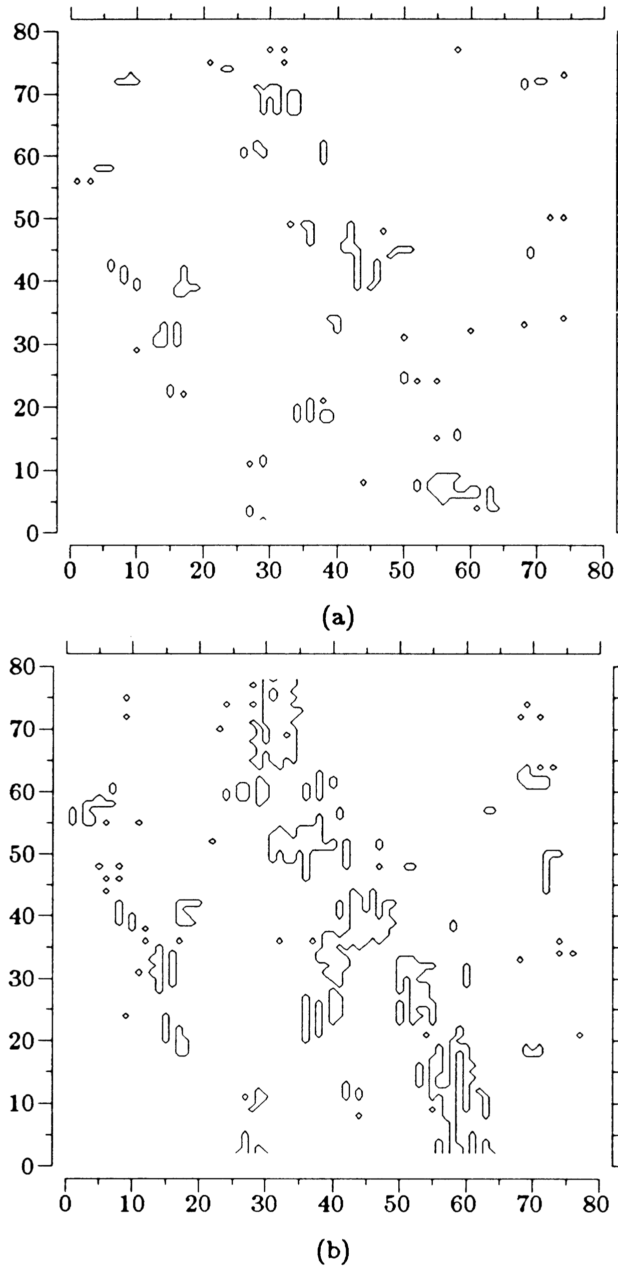


FIG. 7. Hot spot distribution after one cycle (a), and after two cycles (b), respectively, when the sample in Fig. 2 has been changed by making all hot spots well conducting. The hot spot threshold equals 1.5 times the average power density of the sample in Fig. 2.

tions, comparing evolution by making all hot spots well conducting with evolution by making all but the spots in one part of the sample well conducting, confirm that hot spots are results of long-range effects of the phase distribution. It is found that when the hot spots are dilute, the development of the material is slow or nonexistent, whereas when the hot spots are concentrated, the evolution is rapid. Hot spots develop faster when they are made well conducting than when they are made poorly conducting. This is a result of our assumption that the applied electric field is constant.

The speed of evolution of the material is related to the interaction between the hot spots. We have called this interaction the hourglass effect. It is illustrated as follows: Two hot spots, not far from each other and at approximately the same electrostatic potential, are made poorly conducting, see Fig. 6(a). The current is forced to pass through the narrow neck between the spots and the current and power densities increase. This implies that hot spots grow perpendicularly to the applied electric field when the spots are made poorly conducting. Correspondingly, two hot spots not far from each other and aligned with the applied electric field are made well conducting, see Fig. 6(b). The current passes easily through the spots, and tries to penetrate the region in between. We find that hot spots grow parallel to the applied electric field when the spots are made well conducting.

Examples of possible applications are mixtures of conductors and isolators, sintered materials heated by strong current, and breakdown of electrical insulators. By mathematical analogy, results which apply to the electric conductivity are also valid for a number of other transport properties such as thermal conductivity, magnetic permeability, dielectric constant, etc. The analogue of a hot spot in the case of thermal conduction is a region of high-entropy production density, and in the dielectric case a region of high electrostatic energy density. It is not clear what the corresponding mechanism for the evolution would be in the thermal or in the dielectric case, but in the principle the same type of evolution may apply.

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