# Quantum-interference device without Josephson junctions

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The behavior of a superconducting ring of homogeneous wire of transverse dimensions $d \ll \xi$ and $\lambda$ and uniform cross section connected to two long leads that carry a measuring current is analyzed using the nonlinear Ginzburg-Landau equations. When a magnetic field is present the system behaves like a superconducting quantum-interference device for appropriate values of ring sizes even though it has no Josephson junctions. A practical realization of such a device is within current technical capabilities.

Weak links have played a fundamental role in physics, both in basic research and as a powerful technological tool ever since the discovery of the Josephson effect.[1] The essential feature of a weak link is its ability to produce a sizable change in the phase of the complex order parameter across it upon the passage of a supercurrent. The general expression for the phase difference $\Delta\varphi$ between two points in a superconductor[2] is given by

$$\Delta\varphi=\frac{4\pi}{c}\lambda^2\frac{2\pi}{\phi_0}\int_1^2\frac{\mathbf{J}\cdot d\mathbf{l}}{f^2}+\frac{2\pi}{\phi_0}\int_1^2\mathbf{A}\cdot d\mathbf{l}\ , \tag{1}$$

where $J$ is the current density, $f$ the amplitude of the normalized $(0<f\leq 1)$ order parameter $\Psi(x)=f(x)\exp[i\varphi(x)]$, $\phi_0=hc/2e$ is the fluxoid quantum in Gaussian units, $\lambda$ and $\xi$ are the temperature-dependent penetration depth and coherence length, and $\mathbf{A}$ is the vector potential associated with the magnetic flux density $\mathbf{B}$.

In the bulk of a massive superconductor the contribution due to the first term on the right-hand side of Eq. (1) is negligible. However, at a weak link the value of $f$ can be made very small and this is the origin of its dephasing properties. Constrictions, point contacts, and Josephson tunnel junctions (JJ's) are the most common realizations of weak links.[3]

A conventional superconducting quantum-interference device (SQUID) circuit incorporating two JJ's is shown in Fig. 1(a). Such a circuit has quantum-interference properties which make it possible to construct ultrasensitive magnetometers. It is the aim of this paper to describe a circuit without JJ's, built of homogeneous wire, which behaves like a conventional SQUID for appropriate values of its geometrical parameters. The proposed circuit is depicted in Fig. 1(b). Our system is a ring of radius $R$ made of wire of uniform cross section and transverse dimensions $d \ll \xi$ and $\lambda$.

A long lead $A$-$N$ introduces a transport current $I$ which leaves through $N'$-$B$ after dividing equally (due to symmetry) over branches 1 and 2. In addition, there is a circulating current $I_B$ due to the magnetic field. We shall typically be interested in a range of radii $0<R/\xi\lesssim 2$.

Present day techniques have made it possible to construct circuits of sizes undreamed of a few years ago.[4] Networks with the above given dimensions have been studied by a number of experimentalists[5] recently, in particular with regards to the phase transition diagram, confirming the predictions of theoretical studies.[6] However, since these calculations use the linearized Ginzburg-Landau (GL) equations the results are not applicable to systems carrying a finite current.[7]

We show that if the de Gennes–Alexander[6] approach is extended to include the nonlinear terms of the GL theory, the circuit depicted in Fig. 1(b) has, for a certain range of $R/\xi$ values, quantum-interference properties quite similar to those of the conventional weak link circuit of Fig. 1(a). Since $d \ll \xi$ in the system of Fig. 1(b), the order parameter may be taken constant over the cross section and the GL equations for the order parameter along the wires become effectively one dimensional. Calling $x$ the curvilinear coordinate along the wire [normalized by $\xi(t)$], and eliminating the superfluid velocity $Q$ in terms of the supercurrent density $J$, we obtain from the GL equations:

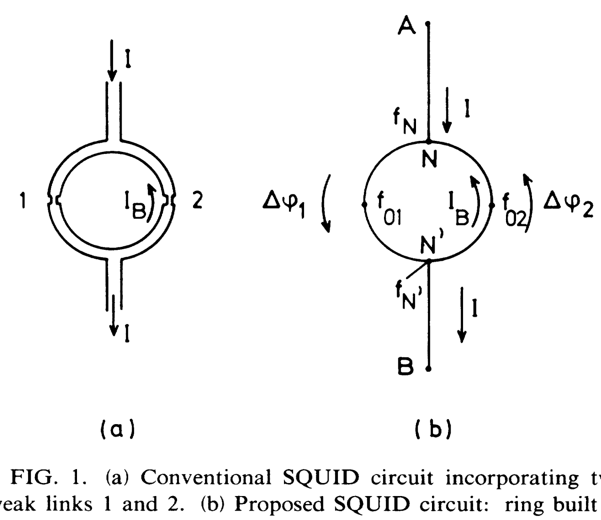


FIG. 1. (a) Conventional SQUID circuit incorporating two weak links 1 and 2. (b) Proposed SQUID circuit: ring built of homogeneous superconducting wire.

$$\frac{d^2f}{dx^2}+(1-f^2-J^2/f^4)f=0 . \quad (2)$$

Complex current conservation imposes the following conditions at the nodes:[6]

$$\sum_n \frac{df}{dx}=0 \text{ and } \sum_n Q=0 . \quad (3)$$

The summations are over all branches joining at a given node. Equation (2) can be solved exactly in terms of Jacobian elliptic functions.[7,8] After some manipulations, one obtains the following general analytical form for the square of the modulus of the order parameter:

$$f^2(x)=f_0^2+t^2(x) , \quad (4)$$

where $f_0$ is the value of $f(x)$ at some extremal point and $t(x)$ is one of the Jacobian elliptic functions which gives rise to the modulation of the order parameter.

We are interested in the general case where both a transport current density $J$ and a circulating current density $J_B$ induced by the magnetic flux are present. For given sets of values of $R/\xi$, $J$, and $J_B$ we have found $f(x)$ and $f_0$ which satisfy Eqs. (2) to (4) along the wire and with $\Psi(x)$ being continuous along the whole circuit, thus finding exact solutions for our problem. Detailed results will be given in a subsequent paper. In order to compare the behavior of our system with that of a conventional SQUID circuit [Fig. 1(a)], we choose the relation between the maximum zero voltage measuring current density $J_m$ and the magnetic flux. Figure 2(a) shows $J_m$ and the corresponding magnetic flux $\phi_m$ for different $R$ values, normalized by $J_c=0.384\,90$ which is the normalized critical-current density of an infinitely long wire. For comparison the equivalent curve for the conventional SQUID, normalized by $2i_c$, is also shown (curve $e$), $i_c$ being the critical current of each weak link. In each case we have normalized the current by the intrinsic critical current of the device.

Also of interest is the relation between the circulating current density $J_B$ and the magnetic flux for fixed measuring current [Fig. 2(b)]. This is obtained by integrating Eq. (1) along the complete circuit which gives

$$2\chi=\Delta\varphi_1+\Delta\varphi_2=(2J_B+J)\,|\,C_1\,|+(2J_B-J)\,|\,C_2\,| , \quad (5)$$

where $\chi=\pi(n-\phi/\phi_0)$ and

$$C_1=\int_{f_{01}}^{f_N} dx/f^2,\;\; C_2=\int_{f_{02}}^{f_N} dx/f^2 .$$

Here the appropriate values for the normalized current densities are $J/2+J_B$ along branch 1 and $J/2-J_B$ along branch 2. The extremal values of $f$ on each branch are $f_{01}$ and $f_{02}$ as shown in Fig. 1(b). Using the results of our computations, we have evaluated Eq. (5) numerically for different sets of values of $R/\xi$, $J$, and $J_B$. It is illuminating to compare our results with those of the conventional SQUID for which the equivalent closed-form expression is[2]

$$I_B/i_c=[\cos^2\chi-(I/2i_c)^2]^{1/2}\tan\chi \quad (6)$$

with the phase difference across the SQUID being

$$\delta=\sin^{-1}[I/(2i_c\cos\chi)] . \quad (7)$$

Figure 2(b) shows $I_B/I_c$ as a function of the flux enclosed by the ring for $I/I_c=0.496\,54$ and radii $R/\xi=0.25$, 0.5, and 1. For comparison $I_B/i_c$ [Eq. (6)] is also shown for $I/2i_c=0.496\,54$ (curve $e$) where $i_c$ is the critical current of the series circuit. It should be noted

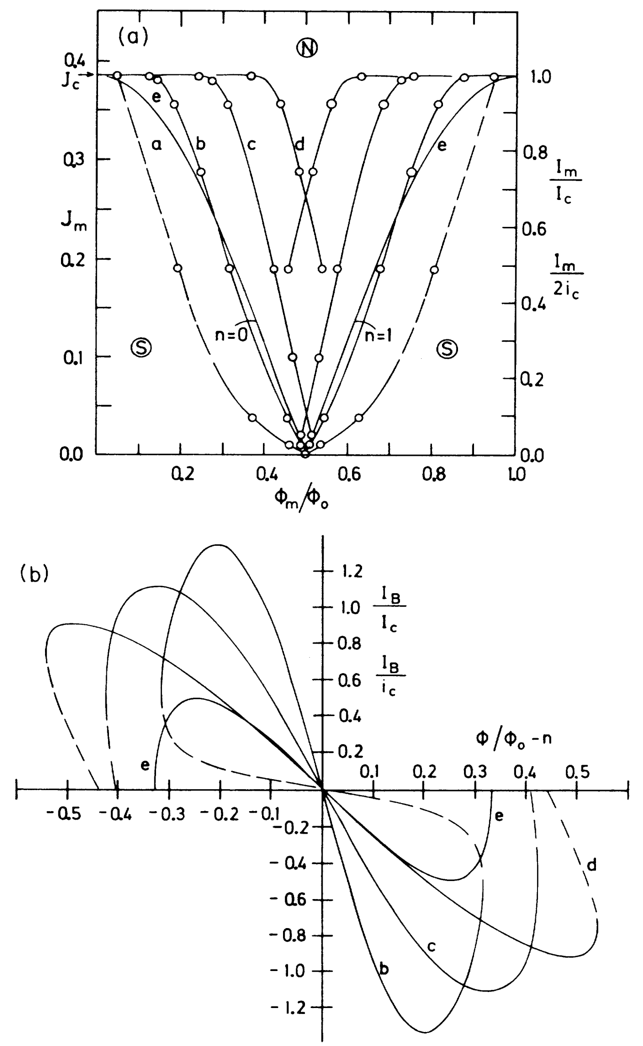


FIG. 2. (a) Maximum measuring current density $J_m$ as a function of the corresponding magnetic flux $\phi_m$ for different values of $R/\xi$. Curves $a$, $b$, $c$, and $d$ are for $R/\xi=0.1$, 0.25, 0.5, and 1. Curve $e$ is the equivalent curve for a SQUID with Josephson junctions [Fig. 1(a)]. In each case the curves are normalized by the intrinsic critical current of the whole device. Some computed values are shown. $S$ and $N$ signify superconducting and normal domains. (b) Circulating current density $J_B$ versus total magnetic flux threaded by the ring circuit. Curves $b$, $c$, and $d$ are for values of $R/\xi=0.25$, 0.5, and 1.0. Curve $e$ shows $I_B$ for the conventional SQUID normalized by $i_c$, the critical current of each Josephson junction [Eq. (6)]. All curves are for the same normalized measuring current $I/I_c=I/2i_c=0.496\,54$. Dashed lines indicate unstable or metastable branches.

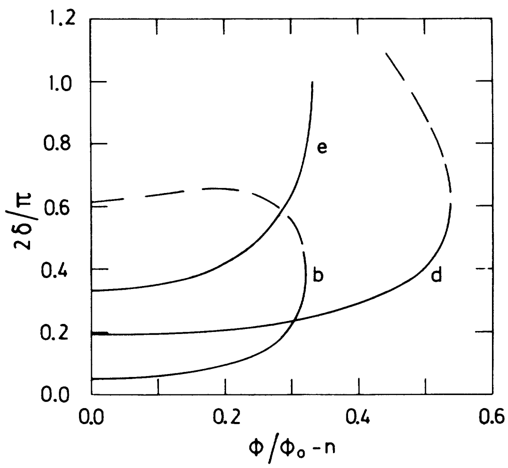


FIG. 3. Plot of the phase difference $\delta$ between nodes $N$ and $N'$ of Fig. 1(b) due to the measuring current $I$. Curves $b$ and $d$ are for $R/\xi=0.25$ and 1.0. Curve $e$ shows $\delta$ for the SQUID with Josephson junctions [Eq. (7)]. All curves are for the same value of the measuring current as in Fig. 2(b). Dashed lines indicate unstable or metastable regions.

that for small values of $R/\xi$ the value of $J_B$ may exceed $J_c$ as a result of the enhancement of superconductivity caused by the two long leads.[9]

Figure 3 shows the angle $\delta$ imposed by the measuring current across the SQUID as a function of $\phi$ for the same values of $J/J_c = I/I_c$ as in Fig. 2(b). This angle is defined by

$$2\delta = \Delta\varphi_1 - \Delta\varphi_2 \ . \tag{8}$$

It can be seen that for the ordinary SQUID $\delta=\pi/2$ at the maximum flux. For the device of Fig. 1(b), $\delta<\pi/2$ at the largest values of the flux as shown in Fig. 2. This is a consequence of the nonsinusoidal character of the current-phase relation obtained from the solutions of the nonlinear GL equations.[8] Figures 2 and 3 are plotted as a function of the total magnetic flux threading the ring. Plots as a function of the applied flux with the self-induction of the ring as a parameter can easily be constructed graphically.[9,10]

As can be seen from Fig. 2(a) the device described here can be used for dc measurements in much the same way as an ordinary SQUID with JJ's. The system we propose has, apart from simplicity, the additional versatility that it can be swept through several characteristic curves due to the variation of $\xi(t)$ with temperature. One other possible application could be measurements of small temperature differences with high accuracy.

The Al circuit used in Ref. 4 for the observation of electron interference effects due to weak localization in normal metals could be used as a realization of our proposed device. Care should be taken, however, in this case because of the known variation of the transition temperature of Al for small sample thicknesses,[11] a fact which might blur the distinction between the two effects.

In conclusion, we have analyzed the properties of a superconducting ring circuit built of homogeneous wire of uniform cross section which for appropriate values of ring sizes behaves like a conventional SQUID and could thus be used as a superconducting quantum-interference device without Josephson junctions.

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