Ferromagnets with weak random anisotropy

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(Received 2 October 1986)

We consider a continuous-symmetry ferromagnet for $d=3$ with random anisotropy which is weak compared to exchange ($H_r \ll H_{ex}$), in the presence of an external field H. At low fields the system is macroscopically disordered (the Imry-Ma or correlated spin-glass regime), and the external field may be treated as a perturbation for $H \ll H_r^4/H_{ex}^3$. For large fields the system is basically aligned, and the random anisotropy may be treated as a perturbation, thus leading to a unique state with a large but wandering magnetic moment (ferromagnet with wandering axis, or FWA). We show that the system retains its alignment only for $H \gg H_r^2/H_{ex}$. Thus we deduce that there is an intermediate field regime which is well aligned but not accessible to perturbation theory. In addition, we consider a number of questions pertaining to the macroscopic collective modes of this system. We point out that the prediction of a longitudinal resonance depends upon a perturbation approach which may not be valid. Also, we observe that, even in $H=0$, samples small compared to the correlation length will have a spontaneous moment, therefore defeating the disordering effect of the random anisotropy. For well-aligned systems with random anisotropy (with the alignment produced either by an external field or by the fine particle effect, or both), the predicted longitudinal resonance, assumed to be valid, can be obtained solely in terms of the transverse resonance shift, the magnetization, and the differential susceptibility. For the FWA we explicitly calculate the anisotropy constants for uniform rotations, and we apply them to electron spin resonance.

I. INTRODUCTION

In a recent paper, Chudnovsky *et al.*[1] considered the behavior of a ferromagnet with random orientational anisotropy, concentrating on the weak-anisotropy limit, and including the effects of an external magnetic field H. They considered the macroscopic energy density

$$\epsilon=\tfrac{1}{2}\alpha(\nabla_i M_\mu)(\nabla_i M_\mu)-\tfrac{1}{2}\beta_r(\mathbf{M}\cdot\hat{\mathbf{n}})^2-\mathbf{M}\cdot\mathbf{H}\ , \tag{1}$$

where the local magnetization $\mathbf{M}$ is assumed to be of fixed length M_0 (determined by the temperature and short-range exchange constants), the constant α is proportional to Ja^2 (where J is a microscopic exchange constant and a is an interatomic separation), and the constant β_r is proportional to a microscopic anisotropy D_r. (Randomness enters because one permits the anisotropy axis $\hat{\mathbf{n}}$ to point in arbitrary directions and to change significantly over a spatial scale R_a.) This model is a continuum version of the discrete model due to Harris, Plischke, and Zimmermann.[2] It has three characteristic fields: the exchange field

$$H_{ex}\equiv\alpha M_0/R_a^2\ , \tag{2}$$

the random anisotropy field

$$H_r\equiv\beta_r M_0\ , \tag{3}$$

where $H_r \ll H_{ex}$, and the external field $H \ll H_{ex}$. Equivalently, there are two small dimensionless parameters

$$h\equiv H/H_{ex},\quad h_r\equiv H_r/H_{ex}\ . \tag{4}$$

Reference 1 summarizes the well-known result that, for three-component spins in three spatial dimensions, if $h=0$ the system is in a globally disordered state with no net magnetization, although there is local ferromagnetic order with a characteristic correlation length[3,4]

$$R_F\sim R_a h_r^{-2}\ . \tag{5}$$

For this field regime the system has been given the name correlated spin glass (CSG).[1] It has a large magnetic susceptibility[4,5]

$$\chi\sim(M_0/H_{ex})h_r^{-4}\ , \tag{6}$$

where M_0 is the local magnetization. Moreover, in this regime one expects a very large ground-state degeneracy. From (6), the field is a weak perturbation for $h \ll h_r^4$.

Naively, for $h_r^4 \ll h \ll 1$ one would expect the system to be nearly aligned, with a ground state that is essentially unique. Using perturbation theory, its properties when aligned (the ferromagnet with wandering axis, or FWA) were discussed in some detail in Ref. 1. In particular, the magnetization M deviates from the saturated value M_0 by an amount

$$\delta M\sim M_0(h_r^4/h)^{1/2}\ , \tag{7}$$

and there is a transverse correlation length

$$R_F^{\perp}\equiv R_a h^{-1/2}\ . \tag{8}$$

In the present work, we show that the FWA description of Ref. 1 is valid only for the regime

$$h_r^2 \ll h \ll 1 \quad (FWA \text{ regime}) . \tag{9}$$

This is done both by elementary and by more analytical considerations of the effects of the fluctuations in the exchange and anisotropy energies. In the elementary approach (Sec. II) we show that, for $d=3$, there is need for an ultraviolet cutoff, so the fluctuations are larger than one would expect from the simplest of arguments; thus, for the Zeeman energy to dominate over the fluctuations in the exchange and anisotropy energies, it is found that Eq. (9) must hold. In the analytical approach (Sec. III), we show that the fluctuations in the direction of the magnetization cause an increase in the transverse correlation length, and perturbation theory is only consistent if Eq. (9) holds. In addition, we indicate how to proceed into the regime

$$h_r^4 \ll h \ll h_r^2 \quad (\text{modified FWA regime}) ,$$

where the system develops the possibility of more than one local energy minimum as the field is decreased.

In Sec. IV we consider a number of issues relating to the macroscopic collective modes of this system. We point out that the prediction of a longitudinal resonance depends upon a perturbation approach which may not be valid. Also, we observe that even in $H=0$, samples small compared to the correlation length will have a spontaneous moment, therefore defeating the disordering effect of the random anisotropy. For well-aligned systems with random anisotropy (with the alignment produced either by an external field or by the aforementioned fine-particle effect, or both) we show how the position of the predicted longitudinal resonance can be obtained solely in terms of the transverse resonance shift, the magnetization, and the differential susceptibility. In addition, for the large-field FWA we explicitly calculate the anisotropy constants for uniform rotations, and we apply them to electron spin resonance.

Section V provides a summary, and indicates the extent to which experiments support the prediction of an intermediate field regime as described in Secs. II and III.

II. A SIMPLE ARGUMENT AND ITS BREAKDOWN

A. The argument

According to (6), in a field on the order of h_r^4 the system becomes nearly aligned. Nevertheless, the system is not completely ordered, since the random anisotropy causes the local magnetization axis to "wander" slightly as one moves about the system. For this reason, we described the system in this regime as a ferromagnet with wandering axis (FWA).[1] The angle of deviation from alignment θ is correlated over a (field-dependent) correlation length $R_F^{\perp}$. To see this, we first present a simplified qualitative argument, which has the virtue that it correctly obtains both $R_F^{\perp}$ and θ; on the other hand, it does not give correctly the smallest H for which the theory is valid.

As usual, one minimizes the energy, Eq. (1), which is written as a sum of each of its three contributions, expanded about the angle θ made by the magnetization with respect to the field, and with the characteristic scale over which the magnetization varies taken to be R. To the terms explicitly given in (1) we also add in the exchange energy due to local order, and the orientation-independent part of the anisotropy energy:

$$\epsilon \approx \left[-\left[\frac{R_a^2}{2a^2} \right](M_0 H_{\text{ex}}) + \left[\frac{\alpha}{2R^2} \right](M_0^2\theta^2) \right] - \left[\tfrac{1}{6}(\beta_r M_0^2) + \tfrac{1}{2}(\beta_r M_0^2 \theta)(R_a/R)^{3/2} \right] - \left[(M_0 H)(1-\theta^2/2) \right] . \tag{10}$$

The second and sixth terms give the increase in the exchange and Zeeman energies due to the noncollinearity; these increases have no terms linear in θ because $\theta=0$ will minimize each of these energies. The fourth term gives the decrease in the random anisotropy energy arising from the adjustment of the local magnetization to the spatial fluctuations of the anisotropy axis orientations; it is linear in θ because the anisotropy energy is not minimized at $\theta=0$.

Since all three terms in θ are coupled by the minimization conditions, one would expect them to be of the same order of magnitude. The θ^2 terms in the exchange and Zeeman energies lead to the characteristic length $R_F^{\perp}$ given in (8). $R_F^{\perp}$ has the meaning of a ferromagnetic correlation length because the FWA, although aligned along the field, preserves a nearly rigid ferromagnetic order only over a length on the order of $R_F^{\perp}$. Note that $R_F^{\perp} \approx R_F$ for $h \approx h_r^4$, but $R_F^{\perp} < R_F$ for larger fields h. When $R_F^{\perp}$ becomes comparable to R_a, one has $h \approx 1$; for larger h the system maintains its stability about $\theta \approx 0$ dominantly due to the presence of the field alone.

Note that (8) is independent of the nature of the perturbation to the system, and thus would be relevant to the case where the perturbation is due to random exchange, as can occur in reentrant ferromagnetic spin-glass systems.[6]

The full minimization of (10) leads to (8) and the characteristic tipping angle

$$\theta \sim (R_a/R_F^{\perp})^{3/2}(H_r/H) \sim H_r/(H_{\text{ex}}^3 H)^{1/4} \sim (h_r^4/h)^{1/4} . \tag{11}$$

From θ one obtains the magnetization deviation via

$$\delta M \approx (M_0/2)\theta^2 \sim M_0 H_r^2/(H_{\text{ex}}^3 H)^{1/2} \sim M_0 (h_r^4/h)^{1/2} \tag{12}$$

in the approach to saturation. If one now requires that the leading-order term in the Zeeman energy (i.e., $-M_0 H$) be of the same order or greater than the nonuniform part of the exchange or anisotropy energy, then one finds the condition that $(H_r^2 H^{1/2}/H_{\text{ex}}^{3/2}) < H$, which implies that $h > h_r^4$. In fact, as will be discussed below, both the nonuniform exchange and the random anisotropy energies [the second and fourth terms in (10)] are actually larger by a factor of $h^{-1/2}$. This changes the above condition to $(H_r^2/H_{ex})<H$, which implies that the system is well aligned only if $h>h_r^2$.

Note that if one employs (8) and (11) in (10), one finds that in the FWA regime the largest energy density (in units of M_0H_{ex}) is the uniform part of the anisotropy energy [of $O(1)$], followed by the aligned part of the Zeeman energy [of $O(h)$], the random average of the anisotropy energy [of $O(h_r)$], the nonuniform part of the exchange energy and the fluctuating part of the anisotropy energy [of $O(h_r^2)$], and finally the nonaligned part of the Zeeman energy [of $O(h_r^2h^{1/2})$].

B. Breakdown of the argument

The reason the above argument breaks down is that, in two or more spatial dimensions, there are so many short-wavelength degrees of freedom that one must employ a cutoff, and for $d=3$ this involves an enhancement by $R_F^{\perp}/R_a \sim h^{-1/2}$ in the gradient and anisotropy energies. (For $d=2$ there is a log h enhancement, and for $d=1$ there is no enhancement.) To see this, we note that, in the FWA regime, Imry-Ma k-space arguments[3] indicate that the gradient energy is proportional to

$$\langle(\nabla M_\alpha)^2\rangle \equiv \langle(\nabla M_\perp)^2\rangle \sim \int \frac{d^3k k^2 |\mathbf{H}_{\perp k}^{an}|^2}{(k^2+ch)^2}, \quad (13)$$

where we have taken $\mathbf{M}_{\perp k}=\chi_\perp \mathbf{H}_{\perp k}^{an}$, with $\mathbf{H}_{\perp k}^{an}$ the perpendicular component (with respect to the direction of the external field) of the random anisotropy field, and the transverse susceptibility $\chi_\perp \sim (k^2+ch)^{-1}$. [For $h=0$, (13) would give the $k=0$ divergence of Imry and Ma.[3]] In the FWA regime, with $|\mathbf{H}_{\perp k}^{an}|^2$ flat in k-space up to $k_{max}\sim a^{-1}$ (where it goes to zero), (13) yields $\langle(\nabla M_\alpha)^2\rangle$ proportional to $H_r^2 k_{max}$ rather than $H_r^2 k_{char} \sim H_r^2(ch)^{1/2} \sim H_r^2/R_F^{\perp}$. We will determine $\langle(\nabla M_\alpha)^2\rangle$ in detail in Sec. III, using real-space (rather than Fourier-space) considerations.

Now consider the anisotropy energy, which is proportional to

$$\begin{aligned}\langle(\mathbf{M}\cdot\hat{\mathbf{n}})^2\rangle &= \langle M_{\parallel}^2 \hat{n}_{\parallel}^2\rangle + 2\langle M_{\parallel}\hat{n}_{\parallel}(\mathbf{M}_\perp\cdot\hat{\mathbf{n}}_\perp)\rangle \\ &\quad + \langle(\mathbf{M}_\perp\cdot\hat{\mathbf{n}}_\perp)^2\rangle \\ &\approx (\tfrac{1}{3})M_0^2 + 2M_0\langle(\mathbf{M}_\perp\cdot\hat{\mathbf{n}}_\perp)\hat{n}_{\parallel}\rangle, \quad \hat{n}_{\parallel}\equiv\hat{\mathbf{n}}\cdot\hat{\mathbf{H}}. \end{aligned} \quad (14)$$

This establishes that $\mathbf{H}_\perp^{an} \sim \hat{\mathbf{n}}_\perp \hat{n}_{\parallel}$. Hence

$$\langle(\mathbf{M}_\perp\cdot\hat{\mathbf{n}}_\perp)\hat{n}_{\parallel}\rangle \sim \int \frac{d^3k\, |\mathbf{H}_{\perp k}|^2}{(k^2+ch)} \sim H_r^2 k_{max}, \quad (15)$$

so that both the nonuniform exchange and the fluctuating part of the anisotropy are enhanced by the same factor $k_{max}R_F^{\perp} \sim h^{-1/2}$.

III. ANALYTICAL RESULTS

We now turn to a more precise treatment. Minimization of (1) for $n=3$, subject to the constraint that $|\mathbf{M}|=M_0$, gives

$$0=\mathbf{M}\times\frac{\delta\epsilon}{\delta\mathbf{M}}=\mathbf{M}\times[-\mathbf{H}-\beta_r\hat{\mathbf{n}}(\mathbf{M}\cdot\hat{\mathbf{n}})-\alpha\nabla^2\mathbf{M}], \quad (16)$$

$$\hat{\mathbf{M}}\cdot\nabla^2\mathbf{M}=-\left[\frac{1}{M_0}\right](\nabla M_\alpha)^2, \quad (17)$$

which are true for all H. Since H_r need not be small in comparison with H, we must treat these equations with some care. Equation (16) involves the part of the bracketed term which is perpendicular to the local direction of $\mathbf{M}$, rather than perpendicular to the applied field $\mathbf{H}$. Nevertheless, in the FWA regime, where $\mathbf{M}_\perp \equiv \mathbf{M}-\hat{\mathbf{H}}(\mathbf{M}\cdot\hat{\mathbf{H}})$ is much smaller than M_0, we may make the approximation (correct to second order $M_\perp$) that $\mathbf{H}_p \approx -(H/M_0)\mathbf{M}_\perp$, where the subscript p denotes the part which is perpendicular to the local direction of $\mathbf{M}$. Then, with (8) to define $R_F^{\perp}$, (16) becomes

$$\nabla^2\mathbf{M}_p-(R_F^{\perp})^{-2}\mathbf{M}_\perp \approx -\frac{h_r}{R_a^2}\hat{\mathbf{n}}_p(\mathbf{M}\cdot\hat{\mathbf{n}}), \quad \hat{\mathbf{n}}_p\equiv\hat{\mathbf{n}}-(\hat{\mathbf{n}}\cdot\hat{\mathbf{M}})\hat{\mathbf{M}}. \quad (18)$$

Combining (17) and (18) yields

$$[\nabla^2-(R_F^{\perp})^{-2}]\mathbf{M} \approx -(R_F^{\perp})^{-2}\mathbf{M}_{\parallel} - \left[\frac{\mathbf{M}}{M_0^2}\right](\nabla M_\alpha)^2 - \frac{h_r}{R_a^2}\hat{\mathbf{n}}_p(\mathbf{M}\cdot\hat{\mathbf{n}}). \quad (19)$$

The part of this which is perpendicular to $\mathbf{H}$ satisfies

$$[\nabla^2-(R_F^{\perp})^{-2}]\mathbf{M}_\perp \approx -\frac{h_r}{R_a^2}(\hat{\mathbf{n}}_p)_\perp(\mathbf{M}\cdot\hat{\mathbf{n}}) - \left[\frac{\mathbf{M}_\perp}{M_0^2}\right](\nabla M_\alpha)^2, \quad (20)$$

where, to first order in $\mathbf{M}_\perp$,

$$\begin{aligned}(\hat{\mathbf{n}}_p)_\perp(\mathbf{M}\cdot\hat{\mathbf{n}}) &\approx [\hat{\mathbf{n}}_\perp-(\mathbf{M}_\perp/M_0)(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})](M_0\hat{\mathbf{n}}\cdot\hat{\mathbf{H}}+\mathbf{M}_\perp\cdot\hat{\mathbf{n}}_\perp) \\ &\approx M_0\hat{\mathbf{n}}_\perp(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})-\mathbf{M}_\perp(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2+\hat{\mathbf{n}}_\perp(\hat{\mathbf{n}}_\perp\cdot\mathbf{M}_\perp). \end{aligned} \quad (21)$$

In this equation, the terms proportional to $\mathbf{M}_\perp$ have zero angular average for $\mathbf{M}_\perp$ uncorrelated to random $\hat{\mathbf{n}}$. Thus they cannot contribute to (20) to order h_r. Moreover, from (13) we know that the average value of the term in $(\nabla M_\alpha)^2$ is of order h_r^2. For simplicity, therefore, let us temporarily drop terms proportional to $\mathbf{M}_\perp$ on the right-hand side of (21), so that (20) becomes

$$[\nabla^2-(R_F^{\perp})^{-2}]\mathbf{M}_\perp \approx -\frac{h_rM_0}{R_a^2}\hat{\mathbf{n}}_\perp\hat{n}_{\parallel}, \quad \hat{n}_{\parallel}\equiv\hat{\mathbf{n}}\cdot\hat{\mathbf{H}}. \quad (22)$$

This establishes that $R_F^{\perp}$ given by (8) still holds in the FWA regime, despite the fact that the simple argument given in Sec. II A and leading to (8) is no longer valid. The solution to (22) is given by

$$\mathbf{M}_\perp(x)=\frac{M_0h_r}{4\pi R_a^2}\int d^3x'\frac{\exp[-|x-x'|/R_F^{\perp}]}{|x-x'|}\hat{\mathbf{n}}'_\perp\hat{n}'_{\parallel}. \quad (23)$$

If the anisotropy decorrelates over a distance R_a, then it is reasonable to take[1]

$$\langle \hat{\mathbf{n}}'_\perp \cdot \hat{\mathbf{n}}''_\perp \hat{n}'_{\parallel} \hat{n}''_{\parallel} \rangle = \langle (1-\hat{n}^2_{\parallel})\hat{n}^2_{\parallel} \rangle_{\hat{\mathbf{n}}} \exp(-|x'-x''|/R_a) = \tfrac{2}{15}\exp[-|x'-x''|/R_a] . \quad (24)$$

Use of (24) in (23) then leads to[1]

$$\langle |\mathbf{M}_\perp(x)|^2 \rangle \approx \left[\frac{M_0 h_r}{4\pi R_a^2}\right]^2 \left[\frac{16\pi R_a^3}{15}\right] \times \int d^3x' \frac{\exp(-2|x-x'|/R_F^\perp)}{|x-x'|^2} = \tfrac{2}{15} M_0^2 \left[\frac{h_r^4}{h}\right]^{1/2} \quad (25)$$

and

$$(\delta M/M_0) \equiv (M_0 - M)/M_0 \approx \frac{1}{2M_0^2} \langle |\mathbf{M}_\perp(x)|^2 \rangle = \frac{1}{15}\left[\frac{h_r^4}{h}\right]^{1/2} , \quad (26)$$

thus substantiating (12).

To obtain the corrections to (22), we substitute (21) into (20), which yields

$$[\nabla^2 - (R_F^\perp)^{-2}]\mathbf{M}_\perp \approx -\frac{h_r}{R_a^2}[M_0 \hat{\mathbf{n}}_\perp - \mathbf{M}_\perp(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2 + \hat{\mathbf{n}}_\perp \hat{\mathbf{n}}_\perp \cdot \mathbf{M}_\perp] - \left[\frac{\mathbf{M}_\perp}{M_0^2}\right](\nabla M_\alpha)^2 . \quad (27)$$

On the right-hand side there is a pure source term (independent of $\mathbf{M}$) and three terms which depend on $\mathbf{M}$. Two of these terms have coefficients which average to zero if there is no correlation with $\mathbf{M}$, but the term in $(\nabla M_\alpha)^2$ has a nonzero average. For this reason we wish to replace it by its average value [of $O(h_r^2)$], plus a term that fluctuates. The fluctuations are expected to be of $O(h_r^4)$, and therefore they will be neglected. The result is that (27) can be approximated by

$$\left[[\nabla^2 - (R_F^\perp)^{-2} + M_0^{-2}\langle (\nabla M_\alpha)^2 \rangle]\mathbf{1} - \frac{h_r}{R_a^2}[(\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2\mathbf{1} - \hat{\mathbf{n}}_\perp\hat{\mathbf{n}}_\perp] \right] \cdot \mathbf{M}_\perp = -\frac{h_r}{R_a^2}[M_0 \hat{\mathbf{n}}_\perp] , \quad (28)$$

where $\mathbf{1}$ is the unit tensor. This is a tensor equation for the magnetization: the additional random terms have the effect of making the response anisotropic, in the sense that a transverse anisotropy field can induce a magnetic response in the second transverse direction. In addition, they prevent the response from becoming singular, if the diagonal part is accidentally zero. However, the most obvious of the additional terms is the one proportional to $\langle (\nabla M_\alpha)^2 \rangle$, which has the effect of increasing the correlation length. This means that the system finds it more difficult to adjust to the random anisotropy.

In principle, (28) can be solved by an iteration scheme treating the off-diagonal terms as a perturbation. Neglecting the off-diagonal terms, one would find an equation like (22), but with a longer correlation length given by

$$(R_F^{\perp\prime})^{-2} = R_a^{-2}h - M_0^{-2}\langle (\nabla M_\alpha)^2 \rangle . \quad (29)$$

One could then require that the term $\langle (\nabla M_\alpha)^2 \rangle$ be computed in a self-consistent fashion. In principle, the self-consistency condition could lead to multiple solutions; however, we will not pursue this question further. For $R_F^\perp \gg R_a$, we can easily compute $\langle (\nabla M_\alpha)^2 \rangle$ from (23), for then the gradient term dominantly comes from acting on the denominator of (23). As a consequence, $\langle (\nabla M_\alpha)^2 \rangle$ is similar to $\langle |\mathbf{M}_\perp|^2 \rangle$ of (25), except that one has $|x-x'|^4$ in the denominator, thus requiring a cutoff at $x_{\min} \sim R_a$ to make the integral convergent. The net result is that

$$M_0^{-2}\langle (\nabla M_\alpha)^2 \rangle \approx \frac{4}{15}\frac{h_r^2}{R_a^2}, \quad R_F^\perp \gg R_a \quad (30)$$

so that (29) becomes

$$(R_R^{\perp\prime})^{-2} \approx R_a^{-2}(h - \tfrac{4}{15}h_r^2) . \quad (31)$$

If $\delta M/M_0$ is recomputed, as in (26), one finds that h is replaced by h minus a term proportional to h_r^2. Thus, we again find that the simplest form of analysis can be expected to hold only for $h \gtrsim h_r^2$, since otherwise the magnetization deviation becomes too large to be consistent with our assumption that the system is basically aligned. Inclusion of the additional random terms can be expected to further increase the tendency of the system to wander, but we surmise that they should make themselves felt no more strongly than the $(\nabla M_\alpha)^2$ term.

IV. MACROSCOPIC MODES, ANISOTROPY CONSTANTS, AND ESR

In an earlier work we predicted, on the basis of the three-dimensionality of the ferromagnet with random anisotropy, that there are three macroscopic-angle variables for this system, and therefore that the system possesses a longitudinal mode.[7] Further consideration of this point is necessary, in order to properly pursue the analogy to another three-dimensional spin system—spin glasses. In the case of spin glasses, the three-dimensionality is due to the dominant interaction, which is exchange. That interaction is also responsible for the susceptibility, which is associated with the energy of magnetization. The random anisotropy serves to produce the energy associated with the macroscopic-angle variables, and is a perturbation on the basic exchange-determined system. On the other hand, for ferromagnets with weak random anisotropy, the anisotropy is responsible for producing both the third-angle variable (i.e., the three-dimensionality of the system), and the energy associated with it. Thus, the analogy to the case of spin glasses (where the predicted longitudinal mode has been observed by Schultz and co-workers[8,9]) is not a firm one. Properly, it may be necessary to develop a microscopic theory for this system (perhaps employing computer simulations like those of Morgan-Pond,[10] and Walker and Walstedt[11] for spin glasses) to see if the mode predicted by Ref. 7 is well founded. Alternatively, one could perform the experiment, using for the theoretical value of the longitudinal resonance an expression [Eq. (42), which will be developed shortly] involving the shift in the conventional transverse resonance, the magnetic field, and the differential susceptibility.

The question of computer simulations brings up finite-size effects, which usually complicate the analysis. In the present case, such finite-size effects may be an advantage because, if the system is smaller than the correlation length for the low-field CSG regime, then it may be possible to study a regime in which it is not necessary to have a magnetic field to defeat the disordering effects of the random anisotropy. In fact, by the use of fine particles, it may be possible to study this case in the laboratory. We are not aware of previous work, either theoretical or experimental, directed toward spin resonance in fine particles of ferromagnets with weak random anisotropy.

Let us now return to a discussion of the FWA system, assuming a large sample size. We will compute the anisotropy constants $K_\perp$ and $K_{\parallel}$, and then apply these values to the predicted ESR frequencies. Their ratio has already been considered for the case of arbitrary random anisotropy, using symmetry arguments.[7] It takes the form $K_\perp/K_{\parallel}=(\frac{3}{2}+K_2/K_1)$, where $K_2/K_1=1$ for uniaxial random anisotropy. (Note that, due to a misprint, Ref. 7 has $\frac{1}{2}$ rather than $\frac{3}{2}$.) Thus Ref. 7 predicts that $K_\perp/K_{\parallel}=\frac{5}{2}$.

We will now explicitly compute both $K_\perp$ and $K_{\parallel}$. To do this, we begin by noting that, under a rigid rotation by the arbitrary angle ϕ about the arbitrary axis $\hat{\boldsymbol{\phi}}$, the magnetization $\mathbf{M}$ changes to

$$\mathbf{M}'=(\mathbf{M}\cdot\hat{\boldsymbol{\phi}})\hat{\boldsymbol{\phi}}+[\mathbf{M}-\hat{\boldsymbol{\phi}}(\mathbf{M}\cdot\hat{\boldsymbol{\phi}})]\cos\phi+(\hat{\boldsymbol{\phi}}\times\mathbf{M})\sin\phi \ . \tag{32}$$

The anisotropy constant for rigid rotations about $\hat{\boldsymbol{\phi}}$ is then given by

$$\begin{aligned} K_{\hat{\boldsymbol{\phi}}} &\equiv -\tfrac{1}{2}\beta_r\left\langle \frac{\partial^2(\mathbf{M}\cdot\hat{\mathbf{n}})^2}{\partial\phi^2}\right\rangle_{\phi=0} \\ &=\beta_r\langle(\mathbf{M}\cdot\hat{\mathbf{n}})^2-(\mathbf{M}\cdot\hat{\mathbf{n}})(\mathbf{M}\cdot\hat{\boldsymbol{\phi}})(\hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{n}})-(\mathbf{M}\times\hat{\mathbf{n}}\cdot\hat{\boldsymbol{\phi}})^2\rangle \ . \end{aligned} \tag{33}$$

For $\hat{\boldsymbol{\phi}}=\hat{\mathbf{H}}$, (33) can be shown to yield

$$\begin{aligned} K_{\parallel} &=\beta_r\langle(\mathbf{M}\cdot\hat{\mathbf{H}})(\hat{\mathbf{H}}\cdot\hat{\mathbf{n}})(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})+(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})^2-(\mathbf{M}_\perp\times\hat{\mathbf{H}}\cdot\hat{\mathbf{n}})^2\rangle \\ &\approx H_r\langle\hat{n}_{\parallel}(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})\rangle \ . \end{aligned} \tag{34}$$

In deriving this result, we employed the fact that $\hat{\mathbf{n}}$ varies in space much more rapidly than does $\mathbf{M}_\perp$, so that the last two terms of the first line nearly cancel. Using (23) and (24) it is straightforward to show that

$$\langle\hat{n}_{\parallel}(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})\rangle\approx(\tfrac{2}{15})(h_rM_0)(1+h) \ . \tag{35}$$

This result is rather curious, since for $h\ll1$ it is nearly independent of h, in contrast to (25). However, one should keep in mind that the correlation of $\mathbf{M}_\perp$ with the anisotropy involves the very short-range self-correlation of the anisotropy, whereas (25) involves the longer-range (and field-dependent) self-correlation of $\mathbf{M}_\perp$. Inserting (35) in (34) yields

$$K_{\parallel}\approx\frac{2}{15}\frac{H_r^2M_0}{H_{\mathrm{ex}}} \ . \tag{36}$$

Similarly, on using the equivalence of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, one finds that for $\hat{\boldsymbol{\phi}}=\hat{\mathbf{x}}$

$$\begin{aligned} K_\perp &=\tfrac{1}{2}\beta_r\langle 5(\mathbf{M}\cdot\hat{\mathbf{H}})(\hat{\mathbf{H}}\cdot\hat{\mathbf{n}})(\mathbf{M}_\perp\cdot\hat{\mathbf{n}}) \\ &\qquad +(\mathbf{M}_\perp\times\hat{\mathbf{n}}\cdot\hat{\mathbf{H}})^2+2(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})^2-3M_\perp^2(\hat{\mathbf{H}}\cdot\hat{\mathbf{n}})^2\rangle \\ &\approx\tfrac{5}{2}H_r\langle(\hat{\mathbf{H}}\cdot\hat{\mathbf{n}})(\mathbf{M}_\perp\cdot\hat{\mathbf{n}})\rangle\approx\frac{1}{3}\frac{H_r^2M_0}{H_{\mathrm{ex}}} \ . \end{aligned} \tag{37}$$

In deriving this, we used the fact that the last three terms on the first line nearly cancel. Thus, on comparing (37) and (34), we obtain the expected result that $K_\perp/K_{\parallel}=\frac{5}{2}$.

We will now apply (34) and (37) to the resonance frequencies of this system. From Ref. 7, the transverse resonance occurs at

$$\omega_\perp=\gamma(H+K_\perp/M) \ , \tag{38}$$

and the longitudinal resonance occurs at

$$\omega_{\parallel}=\gamma(K_{\parallel}/\chi)^{1/2} \ . \tag{39}$$

Since (26) implies that $\chi\sim h^{-3/2}$ for $h>h_r^2$, (39) predicts that $\omega_{\parallel}\sim H^{3/4}$ in that regime. More specifically, with (26) and (36), (39) yields

$$\omega_{\parallel}=2\gamma(H^3H_{\mathrm{ex}})^{1/4} \ . \tag{40}$$

Since the FWA corresponds to $h<1$, (40) predicts that the longitudinal resonance (if the associated macroscopic rotation angle is indeed a dynamical variable) should be somewhat lower than the transverse resonance frequency. Note that both anisotropy constants vary as H_r^2/H_{ex} in this regime (just as for a spin glass), an indication that the anisotropy is behaving as a perturbation. This is a favorable indication (but no proof) that the macroscopic rotation angle may be a dynamical variable, and therefore that (40) describes a realizable mode of the system. It should be remarked that $\omega_{\parallel}$ is independent of H_r in the FWA regime; this is because both $K_{\parallel}$ and χ [cf. (39)] are quadratic in H_r.

One may also obtain a relation which is even more general than (40), in that it reduces to (40) for the infinite system, and it includes the case of systems which are well aligned because their dimension is small compared to the correlation length (either in zero or finite field). In that case, (38) gives the anisotropy induced shift in the transverse ESR frequency as

$$\delta\omega_\perp\equiv\omega_\perp-\gamma H=\gamma K_\perp/M \ . \tag{41}$$

When this is combined with the relation $K_\perp/K_{\parallel}=\frac{5}{2}$ and (40), one finds that the longitudinal resonance frequency is given by

$$\omega_{\parallel}=(\delta\omega_{\perp})^{1/2}\left[\frac{5}{2}\frac{\gamma M}{\chi}\right]^{1/2} . \tag{42}$$

This relation should apply to large and small systems, in the laboratory, in the computer, and in analytic form, so long as these systems are well aligned.

V. SUMMARY AND DISCUSSION

We have found that there is a field regime where a continuous-symmetry ferromagnet in $d=3$ is well aligned, yet is not susceptible to perturbation theory about the well-aligned state. This is not merely a matter of stopping the perturbation expansion at too early a stage. It is that the system can be expected to have an infinite number of solutions at low enough fields, and the breakdown of the perturbation expansion is a manifestation of this. Let us consider this point in more detail.

It is difficult to determine very much about the number of solutions to the general Eqs. (16) and (17). Analogy to other disordered systems would indicate that, at low fields (so that one is in the Imry-Ma, or correlated-spin-glass, regime), the system has an infinite number of macroscopically similar solutions that have energy minima separated by rather small energy barriers. As the external field is increased, a number of these solutions should coalesce, thus decreasing the degeneracy of the system.[12] This process of coalescence should begin when the system becomes fairly polarized (i.e., near $h\approx h_r^4$). At a large enough field, on the other hand, all of the solutions will have coalesced into one, nondegenerate, solution, and Eq. (22) will apply. Let us now reverse the process. As the field is decreased, one would successively expect (28), which neglects fluctuations in $(\nabla M_\alpha)^2$, and then (27), which retains such fluctuations, to apply. In both of these cases, the self-consistency condition on $\langle(\nabla M_\alpha)^2\rangle$ could introduce the possibility of multiple solutions. This is enhanced when one permits this term to have significant fluctuations. As a consequence, analytic work would appear to be rather difficult. About all one can say is that when the high-field regime FWA begins to break down (i.e., near $h\approx h_r^2$), the system should start to develop multiple solutions. It is implicit in this description that there be hysteresis in the intermediate-field regime.

The neutron scattering data of Rhyne has produced results[13] which support the present work in a number of ways. First, besides finding a large-field correlation length varying approximately as $H^{-1/2}$, in agreement with previous predictions,[1] he finds a correction for smaller fields having a sign in agreement with (31). Moreover, there is hysteresis in the scattering intensity for intermediate fields, behaving qualitatively as one would expect: on increasing the field from zero, the system tends to retain a memory of the relatively disordered zero-field CSG state, which gives a relatively large amount of scattering; whereas on decreasing the field from a large value, the system tends to retain a memory of the relatively ordered high-field FWA state, which gives a relatively small amount of scattering.

Simulations have been hampered by the large sample sizes needed to accomodate the weak anisotropy limit. To date, the most relevant work is that of Serota and Lee,[14] who considered XY spins in one dimension. Unfortunately, in that case the intermediate regime does not occur. Nevertheless, their results are of interest, simply to indicate some of the possibilities of this system. Most important from our point of view is the history dependence that they observe: when they equilibrate in a field and then remove the field, the system develops a significant magnetization and a lower energy than the case where the system is equilibrated in zero field (where it develops very little magnetization). It would be interesting to see if this effect persists in higher spatial and spin dimension.

It is worth repeating that we have only considered the case of systems large compared to the zero-field correlation length $R_F\sim R_a h_r^{-2}$. It should be possible to find systems for which the characteristic size is comparable or smaller than R_F, such as fine particles, or systems with artificially introduced (and very weak) anisotropy. Such systems would not be totally decorrelated in zero field, and therefore they would have a net moment, yet not be collinear. Their properties might be rather interesting, as indicated by our analysis for ESR in Sec. IV.

In closing, we note that our discussion has not exhausted the possibilities associated with ferromagnets with weak random anisotropy. It should be clear, however, that this system is rich with possibilities and, due to the large number of materials from which such systems can be fabricated, there should be a wide variety of systems to which the considerations of the present paper apply.

ACKNOWLEDGMENTS

I would like to acknowledge valuable conversations with S. Serota and valuable correspondence with E. M. Chudnovsky. This work was supported in part by the National Science Foundation, through Grant No. DMR-82-09577.

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