Effect of electron-phonon interaction on spin susceptibility in $A15$ compounds

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We calculate the effect of electron-phonon interaction on the long-wavelength static spin susceptibility χ_{sp} in a normal metal for the case when the electronic density of states $N(\varepsilon)$ cannot be taken as constant within the range of several times the maximum phonon frequency $\Omega_{max}(\hbar=1)$ around the Fermi level. Such a situation may exist in some of the $A15$ compounds and in other narrow-band systems. It is found that the electron-phonon interaction can affect the temperature (T) dependence in $\chi_{sp}(T)$. The effect is dominated by the electron damping at high temperatures which effectively smears the peak in $N(\varepsilon)$, thereby reducing the value of $\chi_{sp}(T)$ at high T below its zero-temperature value. We conclude that the analysis of the magnetic susceptibility data on $A15$ materials based on a simple picture of noninteracting band electrons may overestimate the sharpness of the peak in $N(\varepsilon)$.

I. INTRODUCTION

Many vanadium-based $A15$ compounds have magnetic susceptibility χ which decreases with increasing temperature T (Ref. 1). The strongest temperature dependence is found in V_3Si and in V_3Ga which have a large superconducting transition temperature T_c. When either a high-T_c V_3Si (Ref. 2) or a low-T_c V_3Ge (Ref. 3) is disordered by radiation damage, the transition temperature, the magnitude of χ, and its temperature dependence are reduced [Figs. 1(a) and 1(b)]. A similar effect is observed in $(V_{1-x}Cr_x)_3Si$ (Ref. 4) where T_c, χ, and $|d\chi/dT|$ decrease with increasing x [Fig. 1(c)]. The conventional interpretation of T dependence in χ for $A15$ compounds is in terms of a sharp peak in the electronic band density of states $N(\varepsilon)$ near the Fermi level. In the simplest picture of noninteracting band electrons the spin susceptibility is given by

$$\chi_{sp}(T)=2\mu_B^2\int_{-\infty}^{+\infty}d\varepsilon N(\varepsilon)\left[-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\right] \quad (1)$$

and is temperature dependent on the scale given by the width of the peak in $N(\varepsilon)$ (μ_B is the Bohr magneton and $f(\varepsilon)$ is the Fermi function). In this model, which we will call the conventional model, disorder smears $N(\varepsilon)$, thereby reducing χ_{sp} and $|d\chi_{sp}/dT|$. More recently Yu and Anderson[5] have proposed a different mechanism for the unusual physical properties of $A15$ compounds, including the temperature dependence in χ. In their model it is the large electron-phonon coupling at high temperatures which causes the electrons to provide an effective double-well potential for the individual transition-metal atoms. The strength of the electron-phonon coupling and the depth of the double well decrease with decreasing temperature. The depth of the well is proportional to the concentration of the electrons near the minima in the potential. This in turn implies that at high T (large depth of the well and large electron concentration) the electron spins are more correlated than at low T (small depth of the well and small electron concentration). Therefore it is easier for an external magnetic field to polarize the spins at low T, i.e., $d\chi/dT<0$.

In this paper we consider the effect of the usual

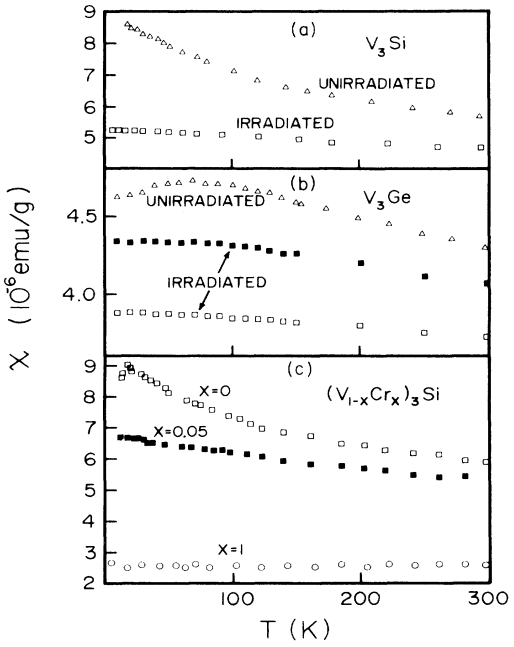

FIG. 1. Experimental magnetic susceptibilities for three vanadium-based $A15$ compounds. (a) V_3Si after Guha *et al.* (Ref. 2). (b) V_3Ge after Solleder *et al.* (Ref. 3). The lower of the two curves labeled "irradiated" corresponds to one order of magnitude larger irradiation doses. (c) $(V_{1-x}Cr_x)_3$ Si after Handstein *et al.* (Ref. 4).

electron-phonon interaction[6] on the spin susceptibility of $A15$ compounds within the conventional model. The basic physical idea is quite simple. At temperatures $k_BT \cong \Omega_{\max}$, where $\Omega_{\max}$ is the maximum phonon frequency (30—50 meV for $A15$ compounds) and k_B is the Boltzmann's constant, the electron lifetime $\tau_{e\text{-ph}}$ due to electron-phonon scattering is of the order of $\Omega_{\max}^{-1}$ (Ref. 6) even for the electrons at the Fermi surface. Thus if the zero-temperature band density of states $N(\varepsilon)$ has a structure on the scale $\Omega_{\max}$ near the Fermi level, at temperatures $k_BT \cong \Omega_{\max}$ this structure is washed out. This in turn has an effect on susceptibility which is analogous to the effect of electron-impurity scattering on $\chi_{\rm sp}$ (Fig. 1). However, we will also show that there are subtle effects of electron-phonon interaction on $\chi_{\rm sp}$ at low T.

The paper is organized as follows. In Sec. II we present the theory of $\chi_{\rm sp}$ for the case when $N(\varepsilon)$ varies on the scale of $\Omega_{\max}$. We will follow the work of Pickett[7] except for presenting the theory on the real frequency axis which enables us to follow the temperature dependence in the quasiparticle density of states

$$N_{\rm qp}(\omega)=\sum_k \left[-\frac{1}{\pi}\mathrm{Im}G(\varepsilon_k,\omega+i0^+)\right], \tag{2}$$

where G is the electron Green's function. In Sec. III we present the numerical results for $\chi_{\rm sp}(T)$ with a discussion. Although our numerical calculations are not intended to reproduce experimental results for $\chi(T)$ of any particular $A15$ compound, but rather to illustrate the effects of electron-phonon interaction on $\chi_{\rm sp}(T)$, we perform the calculations taking for $N(\varepsilon)$ and for the Eliashberg function $\alpha^2(\Omega)F(\Omega)$ the values appropriate for V_3Si. Section IV contains the conclusions.

II. THEORY

The Pauli contribution to the magnetization M at temperature T and at magnetic field H is

$$M=\mu_B[\mathcal{N}_+(H,T)-\mathcal{N}_-(H,T)], \tag{3}$$

where $\mathcal{N}_{+(-)}$ is the number of spin-down (-up) electrons (we take the Landé factor to be -2):

$$\mathcal{N}_\pm=\int_{-\infty}^{+\infty} d\omega\, e^{\omega 0^+} f(\omega)\sum_k \left[-\frac{1}{\pi}\mathrm{Im}G_\pm(k,\omega+i0^+)\right]$$
$$=\int_{-\infty}^{+\infty} d\omega\, e^{\omega 0^+} f(\omega) N_\pm(\omega)\ . \tag{4}$$

Here $f(\omega)=[\exp(\beta\omega)+1]^{-1}$ is the Fermi function, $\beta=(k_BT)^{-1}$, and $N_\pm(\omega)$ are the fully dressed densities of states for spin-down (-up) electrons

$$N_\pm(\omega)=\sum_k \left[-\frac{1}{\pi}\mathrm{Im}G_\pm(k,\omega+i0^+)\right]$$
$$=\sum_k \left[-\frac{1}{\pi}\mathrm{Im}\{\omega+i0^+-[E_k\mp\mu_BH-\mu(H,T)] -\Sigma^\pm(k,\omega+i0^+)\}^{-1}\right]. \tag{5}$$

In Eq. (5) H is the magnetic field, $\mu(H,T)$ is the chemical potential, E_k is the electronic band energy measured relative to the bottom of the band, and $\Sigma^\pm$ is the self-energy for spin-down (-up) electrons.

The spin susceptibility at temperature T is given by

$$\chi_{\rm sp}(T)\equiv\left[\frac{\partial M}{\partial H}\right]_{\substack{H=0\\ \mathcal{N}_+=\mathcal{N}_-}}$$
$$=-2\mu_B^2\int_{-\infty}^{+\infty} d\omega\, e^{\omega 0^+} f(\omega) \times\sum_k \left[-\frac{1}{\pi}\mathrm{Im}[\,G^2(k,\omega+i0^+) \times\Gamma(k,\omega+i0^+)]\right], \tag{6}$$

where by definition

$$\Gamma(k,\omega+i0^+)\equiv\left.\frac{\partial G_\pm^{-1}(k,\omega+i0^+)}{\partial(\pm\mu_BH)}\right|_{H=0}$$
$$=1-\left.\frac{\partial\Sigma^\pm(k,\omega+i0^+)}{\partial(\pm\mu_BH)}\right|_{H=0}. \tag{7}$$

Here we have assumed that

$$\left.\frac{\partial\Sigma^+(k,\omega+i0^+)}{\partial(+\mu_BH)}\right|_{H=0}=\left.\frac{\partial\Sigma^-(k,\omega+i0^+)}{\partial(-\mu_BH)}\right|_{H=0} \tag{8}$$

which in turn implies[8] $(\partial\mu/\partial H)|_{H=0}=0$. The latter equality was used in obtaining the second line of Eq. (7).

We adopt the isotropic approximation,[6,7] i.e., we assume that the k dependence of various quantities enters only via E_k. In that case Eq. (6) reduces to

$$\chi_{\rm sp}(T)=-2\mu_B^2\int_{-\infty}^{+\infty} d\omega\, e^{\omega 0^+} f(\omega)\int_{-\infty}^{+\infty} dE\, N(E)\,\mathrm{Im}\left[-\frac{1}{\pi}G^2(E,\omega+i0^+)\Gamma(E,\omega+i0^+)\right]. \tag{9}$$

In the next step we want to separate the contributions of various interactions (electron-phonon, Coulomb, and electron-impurity) to $\chi_{\rm sp}$. Let ε_{k0} be the electronic energy of noninteracting electrons measured relative to the chemical potential μ_b at $T=0$ for the case when only the Coulomb interaction among the electrons is considered. The corresponding density of states is

$$N_0(\varepsilon)=\sum_k \delta(\varepsilon-\varepsilon_{k0})\ . \tag{10}$$

If μ is the true chemical potential at $T=0$ for the case when all interactions are included, we define

$$\delta\mu_b=\mu-\mu_b\ . \tag{11}$$

We will assume that $\Omega_{\max} \ll \mu_b$ and that $\tau_i^{-1} \lesssim \Omega_{\max}$, where τ_i is the electron-impurity scattering time. Thus there is an energy $\bar{\varepsilon}$ such that

$$\tau_i^{-1}, \Omega_{\max} \ll \bar{\varepsilon} \ll \mu_b \tag{12}$$

(we take $\hbar = 1$). There are two types of $(\varepsilon_{k0}, \omega)$ regions:[9]

(i) the "near" region, where ε_{k0} *and* $\omega < \bar{\varepsilon}$, and

(ii) the "far" region, where ε_{k0} *or* $\omega > \bar{\varepsilon}$.

In the near region, the k dependence of the electron-phonon self-energy $\Sigma_{e\text{-ph},0}$ and of the electron-impurity self-energy $\Sigma_{i,0}$ can be ignored. Also, in the near region the Coulomb self-energy Σ_C can be treated in the quasiparticle approximation, i.e., we can take Σ_C as real and expand it around $\varepsilon_{ko} = 0, \omega = 0$

$$\Sigma_C(\varepsilon_{k0}, \omega + i0^+) = \Sigma_C(0) + \frac{\partial \Sigma_C(0)}{\partial \varepsilon_{k0}} \varepsilon_{k0} + \frac{\partial \Sigma_C(0)}{\partial \omega} \omega \ , \tag{13}$$

where the argument zero in various quantities on the left-hand side means that they are evaluated at $\varepsilon_{k0} = 0$ and $\omega = 0$. Thus in the near region the full Green's function

$$G(k, \omega + i0^+) = [\omega + i0^+ - \varepsilon_{k0} + \delta\mu_b - \Sigma_{e\text{-ph},0}(k, \omega + i0^+) - \Sigma_{i,0}(k, \omega + i0^+) - \Sigma_C(k, \omega + i0^+)]^{-1} \tag{14}$$

can be written as

$$G(k, \omega + i0^+) = Z_C^{-1} \left\{ \omega + i0^+ - \left[\frac{\varepsilon_{k0}}{Z_C} \left[1 + \frac{\partial \Sigma_C(0)}{\partial \varepsilon_{k0}} \right] + \frac{\Sigma_C(0)}{Z_C} \right] + \delta\mu - \Sigma_{e\text{-ph}}(\omega + i0^+) - \Sigma_i(\omega + i0^+) \right\}^{-1} , \tag{15}$$

where

$$Z_C \equiv 1 - \frac{\partial \Sigma_C(0)}{\partial \omega} \ , \tag{16a}$$

$$\delta\mu = \delta\mu_b / Z_C \ , \tag{16b}$$

$$\Sigma_{e\text{-ph}}(\omega + i0^+) = \Sigma_{e\text{-ph},0}(\omega + i0^+) / Z_C \ , \tag{16c}$$

$$\Sigma_i(\omega + i0^+) = \Sigma_{i,0}(\omega + i0^+) / Z_C \ . \tag{16d}$$

In the far region $\Sigma_{e\text{-ph},0}$ and $\Sigma_{i,0}$ can be ignored compared to other quantities in the expression for G, Eq. (14). For the purpose of calculating χ_{sp} we will use the quasiparticle approximation for Σ_C even in the far region and thus we will use the expression (15) for G for all k and ω. With this approximation the expression (6) for χ_{sp} becomes

$$\begin{aligned}
\chi_{sp}(T) = -2\mu_B^2 \int_{-\infty}^{+\infty} d\omega \, e^{\omega 0^+} f(\omega) \int_{-\infty}^{+\infty} d\varepsilon_0 N_0(\varepsilon_0) \\
\times \left[-\frac{1}{\pi} \mathrm{Im} \left[Z_C^{-2} \left\{ \omega + i0^+ - \left[\frac{\varepsilon_0}{Z_C} \left[1 + \frac{\partial \Sigma_C(0)}{\partial \varepsilon_{k0}} \right] + \frac{\Sigma_C(0)}{Z_C} \right] \right. \right. \right. \\
\left. \left. + \delta\mu - \Sigma_{e\text{-ph}}(\omega + i0^+) - \Sigma_i(\omega + i0^+) \right\}^{-2} \\
\times \left[1 - \frac{\partial \Sigma_{e\text{-ph},0}^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} - \frac{\partial \Sigma_{i,0}^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} - \frac{\partial \Sigma_C^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} \right] \Bigg] \Bigg] \\
= -2\mu_B^2 \int_{-\infty}^{+\infty} d\omega \, e^{\omega 0^+} f(\omega) \int_{-\infty}^{+\infty} d\varepsilon \, N(\varepsilon) \\
\times \left[-\frac{1}{\pi} \mathrm{Im} \left\{ [\omega + i0^+ - \varepsilon + \delta\mu - \Sigma_{e\text{-ph}}(\omega + i0^+) - \Sigma_i(\omega + i0^+)]^{-2} \right. \right. \\
\times \left[1 - \frac{\partial \Sigma_{e\text{-ph},0}^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} - \frac{\partial \Sigma_{e\text{-ph},0}^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} - \frac{\partial \Sigma_C^{\pm}}{\partial(\pm \mu_B H)} \bigg|_{H=0} \right] \\
\left. \left. \times \left[Z_C \left[1 + \frac{\partial \Sigma_C(0)}{\partial \varepsilon_{k0}} \right] \right]^{-1} \right\} \Bigg] ,
\end{aligned} \tag{17}$$

where

$$\varepsilon_k \equiv \frac{\varepsilon_{k0}}{Z_C} \left[1 + \frac{\partial \Sigma_C(0)}{\partial \varepsilon_{k0}} \right] + \frac{\Sigma_C(0)}{Z_C} \tag{18}$$

and

$$N_0\left[\left[\varepsilon-\frac{\Sigma_C(0)}{Z_C}\right]Z_C/\left[1+\frac{\partial\Sigma_C(0)}{\partial\varepsilon_{k0}}\right]\right]=N(\varepsilon) \tag{19}$$

is the *band Coulomb quasiparticle density of states.* If we introduce the quantity Λ_C by

$$\Lambda_C(\varepsilon,\omega+i0^+)\equiv\frac{1}{Z_C}\left[1-\frac{\partial\Sigma_C^{\pm}(\varepsilon,\omega+i0^+)}{\partial(\pm\mu_B H)}\right], \tag{20}$$

$\chi_{\rm sp}$ can be written as

$$\begin{aligned}\chi_{\rm sp}(T)=&-2\mu_B^2\int_{-\infty}^{+\infty}d\omega\, e^{\omega 0^+}f(\omega)\\ &\times\int_{-\infty}^{+\infty}d\varepsilon\, N(\varepsilon)\left\{-\frac{1}{\pi}\mathrm{Im}\left[\tilde{G}^{\,2}(\varepsilon,\omega+i0^+)\right.\right.\\ &\times\left[1-\frac{1}{\Lambda_C}\frac{\partial\Sigma_{e\text{-ph}}^{\pm}(\omega+i0^+)}{\partial(\pm\mu_B H)}\bigg|_{H=0}-\frac{1}{\Lambda_C}\frac{\partial\Sigma_i^{\pm}(\omega+i0^+)}{\partial(\pm\mu_B H)}\bigg|_{H=0}\right]\\ &\left.\left.\times\frac{\Lambda_C(\varepsilon,\omega+i0^+)}{1+\dfrac{\partial\Sigma_C(0)}{\partial\varepsilon_{k0}}}\right]\right\},\end{aligned} \tag{21}$$

where $\tilde{G}$ is the electron Green's function which includes only the Coulomb renormalized electron-phonon and the electron-impurity self-energies

$$\begin{aligned}\tilde{G}(\varepsilon,\omega+i0^+)=&[\,\omega+i0^+-\varepsilon+\delta\mu\\ &-\Sigma_{e\text{-ph}}(\omega+i0^+)-\Sigma_i(\omega+i0^+)]^{-1}\,.\end{aligned} \tag{22}$$

The quantity $\Lambda_C/1+\partial\Sigma_C(0)/\partial\varepsilon_{k0}$ leads to the Coulomb enhancement of $\chi_{\rm sp}$. If Λ_C is large compared to $\partial\Sigma_{e\text{-ph}}^{\pm}/\partial(\pm\mu_B H)\,|_{H=0}$ and $\partial\Sigma_{e\text{-ph}}^{\pm}/\partial(\pm\mu_B H)\,|_{H=0}$, only Coulomb interaction contributes to the susceptibility enhancement. In our numerical calculations we will evaluate expression (21) for $\chi_{\rm sp}$ with Λ_c and $1+\partial\Sigma_c(0)/\partial\varepsilon_{k=0}$ set equal to one. This will enable us to estimate the size of electron-phonon and electron-impurity contributions to the enhancement of $\chi_{\rm sp}$ as well as the importance of the self-energy effects entering via $\tilde{G}$. This means that our numerical values of $\chi_{\rm sp}(T)$ would have to be multiplied by the (possibly temperature-dependent) Coulomb enhancement factor if they are to be compared with the experimental results. Before proceeding with numerical calculations we want to point out the difference between the electron-phonon self-energies $\Sigma_{e\text{-ph},0}$ and $\Sigma_{e\text{-ph}}$, Eq. (16c). $\Sigma_{e\text{-ph},0}$ is obtained with screened and Coulomb vertex corrected electron-phonon matrix elements, while the matrix elements in $\Sigma_{e\text{-ph}}$ are also Coulomb renormalized.[9,10] The Eliashberg function $\alpha^2(\Omega)F(\Omega)$ obtained from inversion of the superconductive tunneling data contains the latter type of matrix elements.[10]

III. NUMERICAL RESULTS AND DISCUSSION

As explained in Sec. II we calculate the spin susceptibility by suppressing the Coulomb enhancement factor, i.e., by evaluating Eq. (9) with $\tilde{G}$, Eq. (22), instead of G and with Γ given by

$$\begin{aligned}\Gamma(\omega+i0^+)=&\,1-\frac{\partial\Sigma_{e\text{-ph}}^{\pm}(\omega+i0^+)}{\partial(\pm\mu_B H)}\bigg|_{H=0}\\ &-\frac{\partial\Sigma_i^{\pm}(\omega+i0^+)}{\partial(\pm\mu_B H)}\bigg|_{H=0}.\end{aligned} \tag{23}$$

The self-energies which enter $\tilde{G}$ were obtained by solving the integral equations[11]

$$\begin{aligned}\Sigma_{e\text{-ph}}(\omega+i0^+)=&\int_{-\infty}^{+\infty}d\omega'\frac{N_{\rm qp}(\omega')}{N(0)}\int_0^{\Omega_{\max}}d\Omega\,\alpha^2(\Omega)F(\Omega)\left[\frac{1}{\omega-\omega'-\Omega+i0^+}f(-\omega')+\frac{1}{\omega-\omega'+\Omega+i0^+}f(\omega')\right]\\ &+\int_{-\infty}^{+\infty}d\varepsilon\frac{N(\varepsilon)}{N(0)}\int_0^{\Omega_{\max}}d\Omega\frac{\alpha^2(\Omega)F(\Omega)}{e^{\beta\Omega}-1}[\tilde{G}(\varepsilon,\omega-\Omega+i0^+)+\tilde{G}(\varepsilon,\omega+\Omega+i0^+)]\,,\end{aligned} \tag{24}$$

$$\Sigma_i(\omega+i0^+)=\frac{1}{2\tau_i}\frac{1}{\pi}\int_{-\infty}^{+\infty}d\varepsilon\frac{N(\varepsilon)}{N(0)}\tilde{G}(\varepsilon,\omega+i0^+)\,, \tag{25}$$

together with the equation which expresses the particle number conservation

$$\int_{-\mu_b}^{0} d\varepsilon N(\varepsilon)=\int_{-\infty}^{+\infty} d\omega e^{\omega 0^+} f(\omega)N_{\rm qp}(\omega) . \quad (26)$$

Equation (26) determines the shift $\delta\mu$ in the chemical potential. Here $N_{\rm qp}(\omega)$ is the quasiparticle density of states defined by Eq. (2), with $\tilde{G}$ instead of G. We point out that in principle the Eliashberg function $\alpha^2(\Omega)F(\Omega)$ may vary with temperature because of the temperature dependence of the phonon frequencies. Also disorder may affect the electron-phonon matrix elements, the electronic density of states which enters the α^2F and the phonon frequencies.[12] The neutron-scattering experiments[13] on V_3Si and Nb_3Sn at $T=300$, 77, and 4 K suggest that the temperature dependence of the phonon density of states $F(\Omega)$ may be ignored for the purpose of evaluating $\Sigma_{e\text{-ph}}$ in the susceptibility calculation. Also it has been recently shown by Mitrović,[14] that much of the disorder dependence of α^2F in Nb_3Sn deduced[12] from the tunneling experiments can be accounted for within the conventional model for $A15$ compounds. This in turn implies that to a good approximation one can use the Eliashberg function for a "clean" compound in solving Eqs. (24)–(26) for different amounts of disorder, i.e., for different values of τ_i. In our numerical calculations we have taken $\alpha^2(\Omega)F(\Omega)=CG(\Omega)$, where $G(\Omega)$ is the generalized phonon density of states for V_3Si measured[13] at $T=77$ K. The value of constant C was chosen so that

$$\lambda\equiv 2\int_0^{\Omega_{\max}} d\Omega\,\alpha^2(\Omega)F(\Omega)/\Omega=1 . \quad (27)$$

This choice for α^2F in V_3Si gives the superconducting thermodynamic properties which are in excellent agreement with experiments.[15]

As far as the effect of electron-phonon interaction on $\chi_{\rm sp}$ is concerned, the detailed shape of $N(\varepsilon)$ is important only in the region of several times $\Omega_{\max}$ around the Fermi level (this will not be the case for the Coulomb interaction which "digs" more deeply into the Fermi sea). Thus we will approximate $N(\varepsilon)$ by a Lorentzian superimposed on a constant background

$$N(\varepsilon)=N_0\left[1+\frac{g}{\pi a}\frac{a^2}{a^2+(\varepsilon+b)^2}\right]\Theta(\varepsilon+\mu_b) , \quad (28)$$

where Θ is the step function and the bottom of the band, $-\mu_b$, was taken to be -0.6 eV. The area under $N(\varepsilon)/N(0)$ between -0.6 and 0 eV which appears in the condition (26) was conserved to 0.1%. Thus the estimated accuracy of the shift $\delta\mu$ in the chemical potential [see Eq. (22)] was 0.5 meV which is equal to the smallest ω'-mesh size in Eq. (25) for $\Sigma_{e\text{-ph}}$.

The vertex function Γ, Eq. (23), was obtained by solving the integral equation

$$\begin{aligned}\Gamma(\omega+i0^+)=1&+\int_{-\infty}^{+\infty}d\omega'\int_{-\infty}^{+\infty}d\varepsilon\frac{N(\varepsilon)}{N(0)}\left[-\frac{1}{\pi}\mathrm{Im}[\tilde{G}^2(\varepsilon,\omega'+i0^+)\Gamma(\omega'+i0^+)]\right]\\&\times\int_0^{\Omega_{\max}}d\Omega\,\alpha^2(\Omega)F(\Omega)\left[\frac{1}{\omega-\omega'-\Omega+i0^+}f(-\omega')+\frac{1}{\omega-\omega'+\Omega+i0^+}f(\omega')\right]\\&+\int_{-\infty}^{+\infty}d\varepsilon\frac{N(\varepsilon)}{N(0)}\int_0^{\Omega_{\max}}d\Omega\frac{\alpha^2(\Omega)F(\Omega)}{e^{\beta\Omega}-1}[\tilde{G}^2(\varepsilon,\omega-\Omega+i0^+)\Gamma(\omega-\Omega+i0^+)\\&\qquad+\tilde{G}^2(\varepsilon,\omega+\Omega+i0^+)\Gamma(\omega+\Omega+i0^+)]\\&+\frac{1}{2\tau_i}\frac{1}{\pi}\int_{-\infty}^{+\infty}d\varepsilon\frac{N(\varepsilon)}{N(0)}\tilde{G}^2(\varepsilon,\omega+i0^+)\Gamma(\omega+i0^+) . \end{aligned} \quad (29)$$

This integral equation was derived from Eq. (23) by differentiating Eqs. (24) and (25) in the presence of the magnetic field, and assuming that α^2F is field independent. We point out that if $N(\varepsilon)$ is taken as constant the integral Eq. (29) has solution $\Gamma=1$. Thus the electron-phonon and the electron-impurity interaction may contribute to the enhancement of the normal-state static spin susceptibility *only* if $N(\varepsilon)$ is varying on the scale of $\Omega_{\max}$ around $\varepsilon=0$.

The first set of calculations was done for

$$a=15 \text{ meV}, \quad b=0 \text{ meV}, \quad \text{and } g/(\pi a)=1$$

[see Eq. (28)] which were deduced by Handstein *et al.*[4] for V_3Si from the analysis of $\chi(T)$ based on Eq. (1). Also we have set $1/\tau_i=0$. In Fig. 2 we show the temperature dependence of the quasiparticle density of states $N_{\rm qp}(\omega)$, Eq. (2), together with the bare density of states $N(\omega)$. Note that *at low T* $N_{\rm qp}(\omega)$ *is narrowed* compared to $N(\omega)$ due to renormalization by the electron-phonon interaction.

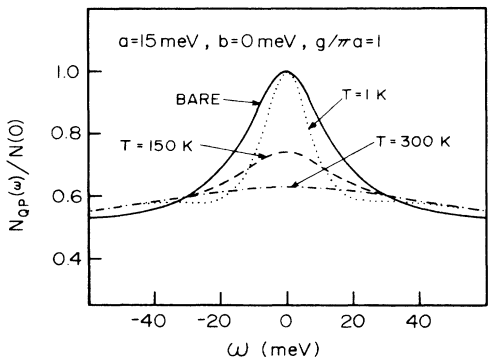

FIG. 2. Quasiparticle densities of states $N_{\rm qp}(\omega)$, Eq. (2), at three different temperatures. The bare $N(\omega)$, Eq. (28), is given by the solid line. The Lorentzian parameters are $a=15$ meV, $b=0$ meV, and $g/(\pi a)=1$.

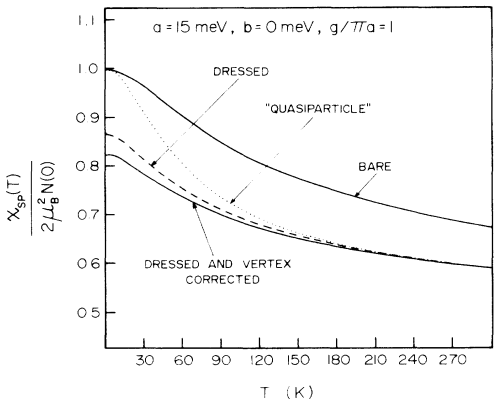

FIG. 3. Spin susceptibility $\chi_{sp}(T)$ calculated for $a=15$ meV, $b=0$ meV, and $g/(\pi a)=1$. The meaning of different curves is described in the text.

This effect is explained in great detail in Ref. 11. As the temperature is increased, $N_{qp}(\omega)$ becomes more and more smeared due to decrease of the electron lifetime (see Sec. I). In Fig. 3 we show the calculated $\chi_{sp}(T)$ (the bottom solid curve). We call this susceptibility "dressed and vertex corrected" because it contains the self-energy effects which enter the expression (9) for χ_{sp} via G, plus the vertex function Γ. In the same figure we show the "bare" $\chi_{sp}(T)$ which is obtained by using Eq. (1). The difference in magnitudes of these two susceptibilities should be noted. Also note a relatively small difference in $d\chi_{sp}(T)/dT$ between the two curves. Later on we will demonstrate that this latter feature is due to a particular choice of the band density of states $N(\varepsilon)$. To find the relative importance of the self-energy effects we have also evaluated Eq. (9) with Γ set equal to 1. The corresponding χ_{sp} is called "dressed" in Fig. 3. It is seen that the vertex function Γ due to electron-phonon interaction has a small effect on χ_{sp}. Finally we include in Fig. 3 the susceptibility which one would obtain from Eq. (1) by naively replacing $N(\varepsilon)$ with $N_{qp}(\varepsilon,T)$. That susceptibility will be called "quasiparticle" χ_{sp}.

In Fig. 4 we show the real and imaginary part of the vertex function $\Gamma(\omega+i0^+)$ at two different temperatures.

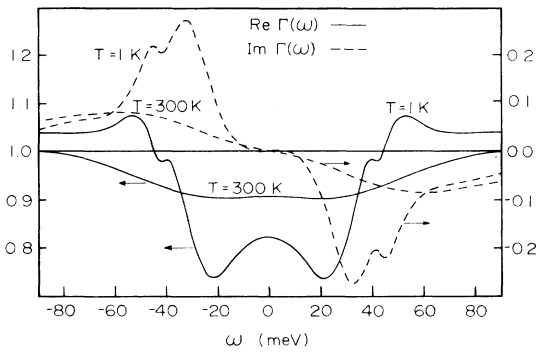

FIG. 4. Real and imaginary parts of the vertex function Γ due to electron-phonon interaction at two different temperatures, calculated for $a=15$ meV, $b=0$ meV, and $g/(\pi a)=1$.

Note that with $N(\varepsilon)$ symmetric about $\varepsilon=0$, $\mathrm{Re}\Gamma(\omega+i0^+)$ [$\mathrm{Im}\Gamma(\omega+i0^+)$] is an even (odd) function of ω. The small effect of Γ due to electron-phonon interaction on $\chi_{sp}(T)$ is due to a small deviation of $\mathrm{Re}\Gamma$ and $\mathrm{Im}\Gamma$ from 1 and 0, respectively, for our choice of $N(\varepsilon)$ and $\alpha^2(\Omega)F(\Omega)$. We believe that this will be the case for any of the $A15$ compounds. Note that the magnitude of $\Gamma-1$ decreases with increasing temperature. In the recent literature on the heavy-fermion compounds[16] there is a claim that there is no effect of electron-phonon interaction on the long-wavelength static spin susceptibility due to quite general reasons. It is claimed that the Ward identity (a consequence of the conservation laws of the system) requires the electron-phonon vertex corrections in $\chi_{sp}(T)$ to be equal to $[1-\partial\Sigma_{e\text{-ph}}(\omega)/\partial\omega]$ which is precisely canceled by $\Sigma_{e\text{-ph}}$ in the denominator of the Green's function, when $\Sigma_{e\text{-ph}}(k,\omega)$ is expanded about $k=k_F$, $\omega=0$ and when its weak momentum dependence is utilized. This argument is wrong for the following reason. To get the long-wavelength static susceptibility from the general momentum and frequency-dependent retarded susceptibility $\chi(\mathbf{q},\omega+i0^+)$, one has to take the limit $\omega\to0$ first, and only then take the limit $\mathbf{q}\to\mathbf{0}$ (Ref. 17). However, the above-mentioned Ward identity is obtained from the "generalized" Ward identity[18]

$$\omega\Gamma(\mathbf{q},\omega;p,\nu)-\mathbf{q}\cdot\boldsymbol{\Gamma}(\mathbf{q},\omega;\mathbf{p},\nu) = G^{-1}(\mathbf{p}+\mathbf{q},\nu+\omega)-G^{-1}(\mathbf{p},\nu) \quad (30)$$

by taking the limit $\mathbf{q}\to\mathbf{0}$ first, then dividing both sides of Eq. (30) by ω and finally taking the $\omega\to0$ limit (here ω and $\mathbf{q}$ are the frequency and the wave vector of the external field, while ν and $\mathbf{p}$ are the corresponding quantities for one of the electron lines which leaves the vertex). In other words, the Ward identity $\Gamma=1-\partial\Sigma/\partial\omega$ does not give the vertex function which is needed in calculation of $\chi_{sp}(T)$. This of course is not to say that the vertex function may violate the Ward identity. Our vertex function, Eq. (29), includes the ladder-type diagrams as explained in Ref. 7. Engelsberg and Schrieffer[18] have shown that this ladder approximation for the generalized vertex $\hat{\Gamma}\equiv(\Gamma,\boldsymbol{\Gamma})$ in the electron-phonon problem satisfies the Ward identity (30). Thus our susceptibility calculation does not violate the basic conservation laws. Later on we will show that the zero effect of electron-phonon interaction on $\chi_{sp}(T)$ for the case when $N(\varepsilon)$ can be taken as constant follows from our theory.

The puzzling feature of the dressed χ_{sp} in Fig. 3 is the reduction in its magnitude at low T compared to the bare χ_{sp}. One may be tempted to argue that near $T=0$ only the values of $N(\omega)$ [or $N_{qp}(\omega)$], $\Sigma_1(\omega+i0^+)\equiv\mathrm{Re}\Sigma_{e\text{-ph}}(\omega+i0^+)$, and $\Sigma_2(\omega+i0^+)\equiv\mathrm{Im}\Sigma_{e\text{-ph}}(\omega+i0^+)$ at $\omega=0$ should enter $\chi_{sp}(T)$. Then since $\Sigma_1(0)=\Sigma_2(0)=0$ at $T=0$, there should be no difference between the dressed χ_{sp} and the bare χ_{sp} near $T=0$. In fact, one may guess at first that there should be no difference between the dressed χ_{sp} and the quasiparticle χ_{sp} at all temperatures. To find the relative importance of Σ_1 and Σ_2 in determining the χ_{sp}, we have calculated the dressed χ_{sp} by suppressing Σ_1 and/or Σ_2. The results are shown in Fig. 5. We point out that $\chi_{sp}(T)$ calculated with both Σ_1 and

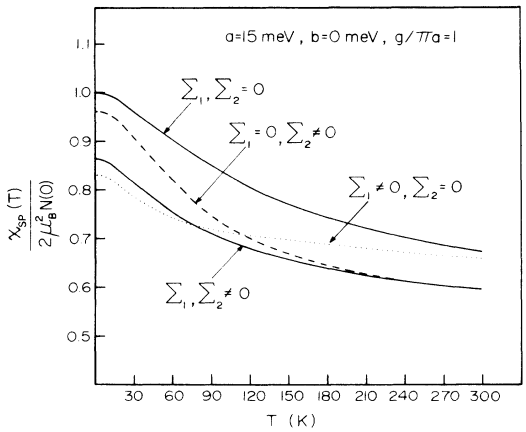

FIG. 5. The dressed susceptibility $\chi_{sp}(T)$ calculated with or without $\Sigma_1 \equiv \mathrm{Re}\Sigma_{e\text{-ph}}$ and/or $\Sigma_2 \equiv \mathrm{Im}\Sigma_{e\text{-ph}}$.

Σ_2 set equal to zero is identical to the bare susceptibility calculated with Eq. (1). Note that the curve calculated with $\Sigma_1=0$ and $\Sigma_2\neq 0$ meets the dressed susceptibility at about 200 K, as one would expect, since the effects of electron-phonon renormalization are gone at k_BT which is a sizable fraction of Ω_{max} (our α^2F had $\Omega_{max}=50$ meV). For the same reason the curve calculated with $\Sigma_1\neq 0$ and $\Sigma_2=0$ meets the bare susceptibility at high T. Thus the most dominant effect of the electron-phonon interaction on χ_{sp} at high T is via Σ_2 which smears the peak in $N_{qp}(\omega)$ [Fig. 2]. This conclusion is supported by Fig. 3, where one can see that the quasiparticle χ_{sp} is a good approximation to the dressed χ_{sp} at $T \gtrsim 150$ K. Figure 5 suggests that Σ_1 has the biggest effect on the value of the dressed χ_{sp} at low T. The question is why the renormalization due to electron-phonon interaction decreases the value of $\chi_{sp}(T)$ below $2\mu_B^2N(0)$ at low T. To answer that question we perform in the Appendix an approximate analytical calculation of $\chi_{sp}(0)$ for the case of $N(\varepsilon)$ symmetric about $T=0$ using the model

$$\Sigma_1(\omega) = -\tilde{\lambda}\omega\tau(\omega) + \Sigma_1(0) , \tag{31}$$

where $\tau(\omega)$ is some function of ω which is 1 for ω close to 0 and is zero for $|\omega| \gtrsim \Omega_{max}$; the parameter $\tilde{\lambda}$ represents $-\partial\,\mathrm{Re}\Sigma_{e\text{-ph}}(0)/\partial\omega$ [it is not equal to λ given by Eq. (27) in the case of a nonconstant $N(\varepsilon)$ near $\varepsilon=0$ (Ref. 11)]. The factor $\tau(\omega)$ in Eq. (31) simulates the fact that $\Sigma_1(\omega)$ becomes ineffective at large $|\omega|$. *This feature of* Σ_1 *is crucial for obtaining correct value of* $\chi_{sp}(0)$. The result of the calculation in the Appendix is

$$\chi_{sp}(0) \cong 2\mu_B^2 \frac{N_{qp}(0) + \tilde{\lambda}\bar{N}_{qp}}{1+\tilde{\lambda}} , \tag{32}$$

where $\bar{N}_{qp}$ is the average of $N_{qp}(\omega)$ over the interval $[-\Omega_{max}, 0]$

$$\bar{N}_{qp} = \frac{1}{\Omega_{max}} \int_{-\Omega_{max}}^{0} d\omega\, N_{qp}(\omega) . \tag{33}$$

Note that if $N(\varepsilon)$ was constant in the range of several times Ω_{max} around the chemical potential, we would have $\bar{N}_{qp} = N_{qp}(0) = N(0)$ and Eq. (32) would give the usual result

$$\chi_{sp}(0) = 2\mu_B^2 N(0) .$$

Thus in the limit of constant $N(\varepsilon)$ the electron-phonon interaction does not affect $\chi_{sp}(0)$. However, if we have a symmetric peak in $N(\varepsilon)$, then

$$\bar{N}_{qp} \cong \frac{N_{qp}(-\Omega_{max}) + N_{qp}(0)}{2}$$

and, since at $T=0$ $N_{qp}(0)=N(0)$, we get

$$\chi_{sp}(0) = 2\mu_B^2 \left[N(0) - \frac{\lambda}{1+\lambda} \frac{N_{qp}(0) - N_{qp}(-\Omega_{max})}{2} \right]$$

$$< 2\mu_B^2 N(0) .$$

In the numerical calculation $\tilde{\lambda} \equiv -\partial\,\mathrm{Re}\Sigma_{e\text{-ph}}(0)/\partial\omega = 0.7$ and $[N_{qp}(0) - N_{qp}(-\Omega_{max})]/2 \cong N(0)/4$ (see Fig. 2), which gives

$$\chi_{sp}(0)/[2\mu_B^2 N(0)] \cong 0.9$$

in agreement with our detailed numerical calculation of the dressed χ_{sp} (see Figs. 3 and 5).

We should point out that in our calculation of $\chi_{sp}(T)$ with a symmetric Lorentzian model for $N(\varepsilon)$ there was no shift in the chemical potential within the numerical accuracy (0.5 meV). Thus none of the results for $\chi_{sp}(T)$ with a symmetric $N(\varepsilon)$ is affected by $\delta\mu$.

Finally we address the temperature dependence in our calculated χ_{sp}. As noted before, there is little difference in the shapes of the two solid curves in Fig. 3 (or in Fig. 5). In particular we find a conjecture,[19] that the strong coupling effects (viz. the self-energy effects) increase the temperature dependence in χ_{sp} by $(1+\lambda)$ for $N(\varepsilon)\neq$const, to be incorrect. Also the quasiparticle χ_{sp} [the one obtained by replacing $N(\varepsilon)$ in Eq. (1) by $N_{qp}(\varepsilon)$] greatly overestimates the temperature dependence of the spin susceptibility. In the remainder of this section we show that a simi-

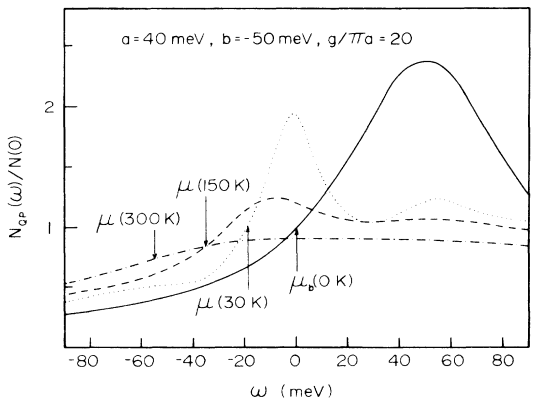

FIG. 6. Quasiparticle densities of states $N_{qp}(\omega)$, Eq. (2), at three different temperatures with a Lorentzian model for $N(\varepsilon)$ (solid curve). The arrows indicate the position of the chemical potential at each temperature.

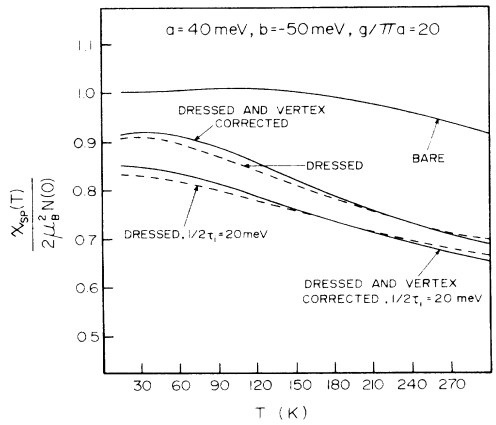

FIG. 7. Spin susceptibility $\chi_{sp}(T)$ calculated with an asymmetric model for $N(\omega)$ (Fig. 6). The meaning of various curves is explained in the text. The lowest pair of curves was calculated including the electron-impurity scattering with $1/(2\tau_i)=20$ meV.

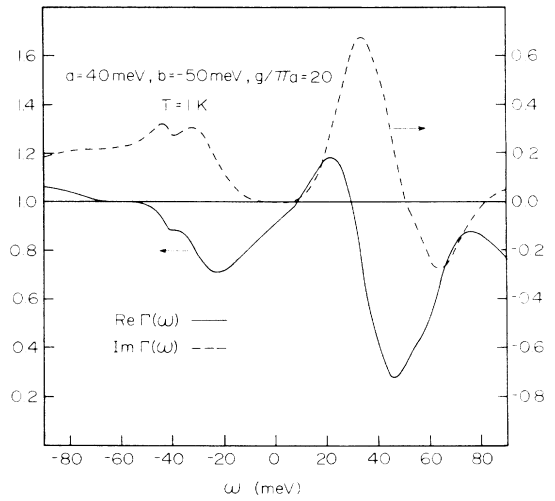

FIG. 9. Vertex function Γ due to electron-phonon interaction at $T=1$ K calculated with an asymmetric $N(\omega)$ (Fig. 6).

larity in shapes of the bare χ_{sp} and the dressed (or dressed and vertex corrected) χ_{sp} is not a general result. It is specific to our particular choice of the symmetric Lorentzian shape of $N(\varepsilon)$.

We have also calculated $\chi_{sp}(T)$ for a broader peak in $N(\varepsilon)$ with

$$a=40 \text{ meV}, \quad b=-50 \text{ meV}, \quad \text{and} \quad g/(\pi a)=20 .$$

The resulting $N(\varepsilon)$ is given by the solid curve in Fig. 6 and it approximates the detailed shape of $N(\varepsilon)$ for V_3Si as calculated by Mattheis and Weber.[20] In Fig. 6 we also show the corresponding $N_{qp}(\omega)$ at three different temperatures, indicating the position of the chemical potential. We point out that up to $T=60$ K $\delta\mu$ is canceled by $\mathrm{Re}\Sigma_{e\text{-ph}}(0)$ in the denominator of Eq. (22) (with $1/\tau_i=0$). This in turn implies very little difference between the value of N_{qp} at the chemical potential and $N(0)$ at these low temperatures (see Fig. 6). In Fig. 7 we give the results of our numerical calculations of χ_{sp}. Now the dressed (or the dressed and vertex corrected) χ_{sp} is much more temperature dependent for $T \gtrsim 60$ K than the bare χ_{sp}. As we have discussed earlier, this is mainly due to $\mathrm{Im}\Sigma_{e\text{-ph}}$ which effectively smears the density of states. In the same figure we show χ_{sp} calculated including both the electron-phonon and the electron-impurity interaction with $1/\tau_i=40$ meV. The electron-impurity scattering effectively smears the density of states even at low T (see Fig. 8), thereby reducing $\chi_{sp}(0)$ and the slope in $\chi_{sp}(T)$. Finally in Fig. 9 we show the electron-phonon vertex function which comes into the susceptibility calculation for the asymmetric Lorentzian $N(\varepsilon)$. Note that ReΓ (ImΓ) is not an even (odd) function of ω due to the asymmetry in $N(\varepsilon)$.

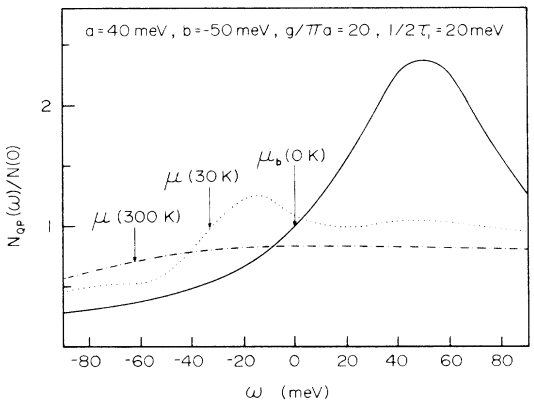

FIG. 8. Quasiparticle densities of states $N_{qp}(\omega)$, Eq. (2), calculated including electron-impurity scattering with $1/(2\tau_i)=20$ meV. The solid curve gives the bare $N(\omega)$.

IV. CONCLUSIONS

We have shown that the electron-phonon interaction affects the long-wavelength static spin susceptibility $\chi_{sp}(T)$ once the electronic band density of states $N(\varepsilon)$ varies on a scale of several times the maximum phonon frequency Ω_{max} near the Fermi level. At temperatures $k_BT \sim \Omega_{max}$ the damping of the electronic states near the chemical potential is so large that any structure in the density of states is washed out. This in turn leads to a decrease in χ_{sp} at high T. At low T the renormalization of electronic energies near the Fermi level affects the value of $\chi_{sp}(0)$. This effect is important only if $N(\varepsilon)$ has a structure in the interval $|\varepsilon| \lesssim \Omega_{max}$. Our calculation did not include all effects of the Coulomb interaction. The electronic band density of states $N(\varepsilon)$ was assumed to be dressed by the Coulomb interaction, but we left out the Coulomb enhancement of $\chi_{sp}(T)$. As discussed by Pickett[7] the electron-phonon self-energy effects may lead to a temperature-dependent Coulomb enhancement of χ_{sp}. However an accurate calculation of χ_{sp} including both the Coulomb and the electron-phonon interaction, as well as the details of the band structure, would be a formidable task. The main conclusion of our analysis is that the shape of $N(\varepsilon)$ deduced from $\chi_{sp}(T)$ by applying a simple

Eq. (1) cannot be trusted. In particular broader peaks in the band density of states $N(\varepsilon)$ may give the observed temperature dependence in the magnetic susceptibility of $A15$ compounds if the interactions are properly taken into account. Effects similar to those described in this work should be expected in other narrow-band systems if the electronic self-energy is strongly frequency dependent near the chemical potential.[16]

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we derive an approximate formula for the "dressed" $\chi_{sp}(0)$ taking for the electron self-energy at $T=0$ a model

$$\Sigma_1(\omega)=-\tilde{\lambda}\omega\tau(\omega)+\Sigma_1(0)\ , \tag{A1}$$

$$\Sigma_2(\omega)=0\ , \tag{A2}$$

where $\tau(\omega)$ is a function with properties

$$\tau(\omega)=\begin{cases}\cong 1 & \text{for } |\omega| \text{ close to } 0\ ,\\ \cong 0 & \text{for } |\omega| \gtrsim \Omega_{\max}\ .\end{cases} \tag{A3}$$

Also, we assume that $\tau(\omega)$ is flat for $\omega\to 0$ such that

$$\tilde{\lambda}=-\left.\frac{\partial\Sigma_1(\omega)}{\partial\omega}\right|_{\omega=0}$$

$\Sigma_1(0)$ will be canceled by $\delta\mu$ (Ref. 11) and Eq. (22) for the Green's function $\tilde{G}$ becomes

$$\tilde{G}(\varepsilon,\omega+i0^+)=\frac{1}{\omega[1+\tilde{\lambda}\tau(\omega)]-\varepsilon+i0^+} \tag{A4}$$

and we can write

$$-\frac{1}{\pi}\mathrm{Im}\tilde{G}^2=\frac{\partial}{\partial\varepsilon}\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)]) \tag{A5a}$$

$$=\left[-\frac{\partial}{\partial\omega}\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)])\right]\times\frac{1}{\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}}\ . \tag{A5b}$$

With (A5a) we get after integrating by parts

$$\int_{-\infty}^{+\infty}d\varepsilon N(\varepsilon)\left[-\frac{1}{\pi}\mathrm{Im}G^2(\varepsilon,\omega)\right]$$
$$=N(\varepsilon)\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)])\,|_{-\infty}^{+\infty}$$
$$-\int_{-\infty}^{+\infty}d\varepsilon\frac{\partial N(\varepsilon)}{\partial\varepsilon}\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)])\ . \tag{A6}$$

Now,

$$\lim_{\varepsilon\to+\infty}N(\varepsilon)\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)])=0$$

because the relevant values of ω in Eq. (9) are restricted from above by $f(\omega)$. Also

$$\lim_{\varepsilon\to-\infty}N(\varepsilon)\delta(\varepsilon-\omega[1+\tilde{\lambda}\tau(\omega)])=0\ ,$$

because $N(\varepsilon)=0$ for ε below the bottom of the band. Thus Eq. (A6) reduces to

$$\int_{-\infty}^{+\infty}d\varepsilon N(\varepsilon)\left[-\frac{1}{\pi}\mathrm{Im}G^2(\varepsilon,\omega)\right]=-\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega(1+\tilde{\lambda}\tau)}\ . \tag{A7}$$

Now, if $N(\varepsilon)$ is constant within a few times $\Omega_{\max}$ around $\varepsilon=0$, we have

$$-\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega(1+\tilde{\lambda}\tau)}=-\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega} \tag{A8}$$

because of the properties of $\tau(\omega)$ given by Eq. (A3). In that case

$$\chi_{sp(0)}=-2\mu_B^2\int_{-\infty}^{+\infty}d\omega\, e^{\omega 0^+}f(\omega)\times\int_{-\infty}^{+\infty}d\varepsilon N(\varepsilon)\left[-\frac{1}{\pi}\mathrm{Im}\tilde{G}^2(\varepsilon,\omega+i0^+)\right]$$
$$=2\mu_B^2\int_{-\infty}^{+\infty}d\omega\, e^{\omega 0^+}f(\omega)\frac{\partial N(\omega)}{\partial\omega}$$
$$=2\mu_B^2\int_{-\infty}^{+\infty}d\omega N(\omega)\left[-\frac{\partial f(\omega)}{\partial\omega}\right]\ , \tag{A9}$$

which is just Eq. (1). Thus we get the known result [for $N(\varepsilon)=$const. near $\varepsilon=0$] that the electron-phonon interaction does not affect χ_{sp}.

On the other hand, if $N(\varepsilon)$ does vary over the range of few times $\Omega_{\max}$ around $\varepsilon=0$, we get

$$\chi_{sp}(T)=2\mu_B^2\int_{-\infty}^{+\infty}d\omega\, e^{\omega 0^+}f(\omega)\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega(1+\tilde{\lambda}\tau)} \tag{A10}$$

Consider for the moment $N_{qp}(\omega)$, Eq. (2). With our model for the electron self-energy we get[11]

$$N_{qp}(\omega)=N(\omega[1+\tilde{\lambda}\tau(\omega)])\ . \tag{A11}$$

Then

$$\frac{\partial N_{qp}(\omega)}{\partial\omega}=\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega(1+\tilde{\lambda}\tau)}\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}$$

and

$$\left.\frac{\partial N(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\omega(1+\tilde{\lambda}\tau)}=\frac{1}{\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}}\frac{\partial N_{qp}(\omega)}{\partial\omega}\ . \tag{A12}$$

Using (A12) in (A10) we get, after integrating by parts,

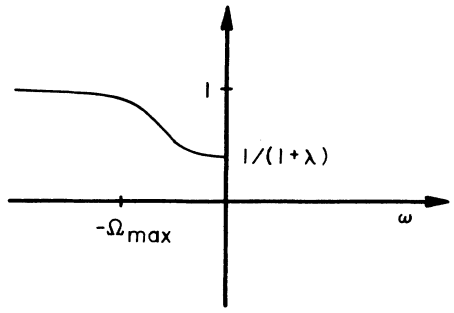

FIG. 10. Schematic diagram illustrating the shape of $1/\partial\{\omega[1+\tilde{\lambda}\tau(\omega)]\}/\partial\omega$ for $\omega \lesssim 0$.

$$\chi_{\rm sp}(T)=2\mu_B^2\int_{-\infty}^{+\infty}d\omega\left[-\frac{df(\omega)}{\partial\omega}\right]\times N_{\rm qp}(\omega)\frac{1}{\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}}-2\mu_B^2\int_{-\infty}^{+\infty}d\omega\, e^{\omega 0^+}f(\omega)N_{\rm qp}(\omega)\frac{\partial}{\partial\omega}\times\left[\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}\right]^{-1}. \quad \text{(A13)}$$

The last equation can be simplified by noting that at low T $-\partial f(\omega)/\partial\omega$ is sharply peaked at $\omega=0$ and therefore in the *first term* in Eq. (A13) we can set

$$\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}=\frac{\partial}{\partial\omega}[\omega(1+\tilde{\lambda})]=1+\tilde{\lambda}\ .$$

In the *second term* in Eq. (A13) we have to take into account the actual shape of $1/\partial\{\omega[1+\tilde{\lambda}\tau(\omega)]\}/\partial\omega$, which is illustrated in Fig. 10. We get

$$\frac{\partial}{\partial\omega}\left[\frac{\partial}{\partial\omega}\{\omega[1+\tilde{\lambda}\tau(\omega)]\}\right]^{-1}\cong\begin{cases}\dfrac{(1+\tilde{\lambda})^{-1}-1}{\Omega_{\rm max}}=-\dfrac{\tilde{\lambda}}{1+\tilde{\lambda}}\dfrac{1}{\Omega_{\rm max}} & \text{for } |\omega|\lesssim\Omega_{\rm max}\ ,\\ 0 \text{ for } |\omega|\gtrsim\Omega_{\rm max}\ ,\end{cases}$$

and the second term in Eq. (A13) can be approximated at $T=0$ by

$$2\mu_B^2\frac{\tilde{\lambda}}{1+\tilde{\lambda}}\left[\frac{1}{\Omega_{\rm max}}\int_{-\Omega_{\rm max}}^{0}d\omega\, N_{\rm qp}(\omega)\right]=2\mu_B^2\frac{\tilde{\lambda}}{1+\tilde{\lambda}}\bar{N}_{\rm qp}\ , \quad \text{(A14)}$$

where

$$\bar{N}_{\rm qp}\equiv\frac{1}{\Omega_{\rm max}}\int_{-\Omega_{\rm max}}^{0}d\omega\, N_{\rm qp}(\omega)\ .$$

Thus, at low T Eq. (A13) can be approximated by

$$\chi_{\rm sp}(0)=2\mu_B^2\frac{N_{\rm qp}(0)+\tilde{\lambda}\bar{N}_{\rm qp}}{1+\tilde{\lambda}}\ . \quad \text{(A15)}$$

We point out that this approximate derivation should be valid only for the case when $N(\varepsilon)$ peaks very close to $\varepsilon=0$. The point is that our assumption (A2) is adequate only for ω close to 0 even at $T=0$. If $N(\varepsilon)$ had a peak near $\varepsilon=\pm\Omega_{\rm max}$, Eq. (A11) would not be a reasonable approximation for $N_{\rm qp}(\omega)$.

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