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Filamentation instability of an Alfvén wave in a compensated semiconductor

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This paper presents a theoretical investigation of the filamentation instability of a high-amplitude Alfvén wave in a compensated magnetoactive semiconductor. Fluid equations have been employed to find the nonlinear response of electrons and holes in the semiconductor plasma. The low-frequency nonlinearity arises through the ponderomotive force on electrons and holes, whereas the high-frequency nonlinearity has been taken through the nonlinear current densities of electrons and holes. For typical plasma parameters in compensated Ge $\epsilon_L=16$ at 77 K, $n_0^0=10^{17}$ cm^{-3}, $\nu_0=2\times10^{11}$ rad sec^{-1}, $B_s=100$ kG, and for the power density of the incident wave approximately 20 kW cm^{-2}, the growth rate of the filamentation instability turns out to be approximately 10^8 rad sec^{-1}.

I. INTRODUCTION

The use of low-frequency electromagnetic waves for the studies of various optical properties and diagnostics of semiconductors, semimetals, and metals has been well known for a long time.[1–8] The Alfvén waves whose frequencies are smaller than the plasma and cyclotron frequencies of electrons and holes, can be generated at high amplitudes in the laboratory. At the high-power level of the Alfvén waves propagating in semiconductors, various nonlinear effects must come into play. However, no attempt has been made so far to study such nonlinear effects in semiconductor plasmas.

In the realm of gaseous plasmas, people have exhaustively studied the excitation of ion acoustic waves, electron plasma waves, and other modes, which lead to various nonlinear phenomena. The semiconductor plasma differs from gaseous plasmas in two major ways: (i) the effective masses of carriers, the electrons and holes in a semiconductor, are much smaller than those of carriers in a gaseous plasma, and (ii) the semiconductor plasma is highly collisional. In a semiconductor the electrons and/or holes attain relatively high drift velocities in the presence of an external pump wave of moderate power density, and the plasma parameters in the semiconductor may also be varied over a wide range of values without much difficulty.

In this paper, we have studied the filamentation instability of a high-power beam of Alfvén waves in a compensated magnetoactive semiconductor, viz., germanium. A number of workers have studied the filamentation instability due to the parametric excitation and amplification of the ion acoustic wave in gaseous plasmas.[9–14] This instability results from parametric amplification of the low-frequency density perturbations which may be present due to the ion acoustic mode in the transverse direction of propagation of the pump wave. In compensated semiconductors where the densities of electrons and holes are equal, the plasma frequency of electrons and/or holes is usually quite large. Therefore, the microwave may propagate in such a plasma only in the Alfvén-wave mode in the presence of a considerably strong static magnetic field. We use a fluid model of homogeneous plasmas to find the response of electrons and holes. The low-frequency nonlinearity arises through the ponderomotive force on electrons and holes, while the nonlinearity in the high-frequency response has been taken through the nonlinear current densities of electrons and holes. It is observed that the hole nonlinearity dominates over the nonlinearity due to the motion of electrons in the compensated semiconductor for the microwave range of frequencies. It may be mentioned here that for a collisional plasma like a semiconductor and a nonuniform incident wave, viz., a Gaussian beam, the main source of nonlinearity arises through the nonuniform heating of electrons because of the transverse variation of the electric field along the wave front. This type of nonlinearity is the dominant cause for self-focusing of laser beams in the collisional plasma.[15]

In Sec. II we have derived the nonlinear dispersion relation for the low-frequency electrostatic perturbation when a left-hand circularly polarized Alfvén wave propagates transverse to the direction of propagation of the perturbation. In Sec. III the dispersion relation has been used to obtain expressions for the growth rates of the filamentation instability. Finally, a brief discussion of the results is presented in Sec. IV.

II. NONLINEAR DISPERSION RELATION

We consider the propagation of a left-hand circularly polarized Alfvén wave (pump) in a compensated semiconductor along the direction of an external static magnetic field,[16] $\mathbf{B}_s \| \hat{\mathbf{z}}$:

$$\begin{aligned} &\mathbf{E}_0=\mathbf{E}_0'\exp[-i(\omega_0 t-k_0 z)] ,\\ &E_{0x}=iE_{oy} ,\\ &k_0=(\omega_0/V_A)(1+V_A^2/c^2)^{1/2} ,\\ &V_A=B_s/(4\pi n_0^0 m_h)^{1/2} , \end{aligned} \tag{1}$$

where ω_0 is the angular frequency, V_A is the Alfvén speed, m_h is the average effective mass of a hole, n_0^0 is the density of electrons and holes, and c is the speed of light in a vacuum. The oscillatory magnetic field of the incident pump wave is contained in the xy plane

$$\mathbf{B}_0 = c\mathbf{k}_0 \times \mathbf{E}_0/\omega_0 . \tag{2}$$

Since the wave will be strongly damped for low frequencies in a collision-dominated plasma, we assume $\omega_0 > \nu_0$, where ν_0 is the average collision frequency of electrons and holes. The electrons and holes in the semiconductor plasma acquire the linear drift velocities given by

$$\begin{aligned} &\mathbf{V}_{0\perp} \simeq (e/m\omega_c^2)(\mathbf{E}_{0\perp}\times\boldsymbol{\omega}_c + i\omega_0\mathbf{E}_{0\perp}), \quad \omega_c > w_0 \\ &\mathbf{V}_{0h\perp} \simeq (-e/m_h\omega_{ch}^2)(-\mathbf{E}_{0\perp}\times\boldsymbol{\omega}_{ch} + i\omega_0\mathbf{E}_{0\perp}) , \quad \omega_{ch} > \omega_0 \\ &V_{0z} = V_{0hz} = 0 , \end{aligned} \tag{3}$$

where $-e$ is the electronic charge, m is the average effective mass of an electron, $\omega_c = eB_s/mc$ is the electron cyclotron frequency, and $\omega_{ch} = eB_s/m_h c$ is the cyclotron frequency of holes in the plasma.

We now assume a low-frequency perturbation $(\omega,\mathbf{k})$ which may be present in the semiconductor plasma due to an ion acoustic mode or some other reason. The oscillatory drift velocities of electrons and holes, and the magnetic field of the pump wave $(\omega_0,\mathbf{k}_0)$ interact parametrically with the perturbation $(\omega,\mathbf{k})$ and produce two high-frequency scattered sidebands[12] $(\omega_{1,2},\mathbf{k}_{1,2})$:

$$\begin{aligned} &\omega_{1,2} = \omega \mp \omega_0 , \\ &\mathbf{k}_{1,2} = \mathbf{k} \mp \mathbf{k}_0 . \end{aligned} \tag{4}$$

The sidebands in turn interact with the pump to produce a low-frequency ponderomotive force which then amplifies and drives the perturbation. The nonlinear growth of the electrostatic perturbation propagating transverse to the direction of propagation of the incident beam causes the breaking of the wave front of the incident homogeneous beam into filamentary structure. This phenomenon, known as the filamentation instability, has been extensively studied in gaseous plasmas.[9–14]

The response of electrons and holes of the compensated semiconductor plasma to this four-wave parametric process is governed by the following equations of motion and continuity:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\frac{e\mathbf{E}}{m} - \frac{e}{mc}(\mathbf{V}\times\mathbf{B}) - (\mathbf{V}\times\boldsymbol{\omega}_c) - \nu_0\mathbf{V} - \frac{V_{\text{th},e}^2}{n_0^0}\nabla n , \tag{5}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{V}) = 0 , \tag{6}$$

$$\frac{\partial \mathbf{V}_h}{\partial t} + (\mathbf{V}_h\cdot\nabla)\mathbf{V}_h = \frac{e\mathbf{E}}{m_h} + \frac{e}{m_h c}(\mathbf{V}_h\times\mathbf{B}) + (\mathbf{V}_h\times\boldsymbol{\omega}_{ch}) - \nu_0\mathbf{V}_h - \frac{V_{\text{th},h}^2}{n_0^0}\nabla n_h , \tag{7}$$

$$\frac{\partial n_h}{\partial t} + \nabla\cdot(n_h\mathbf{V}_h) = 0 , \tag{8}$$

where

$$\begin{aligned} &V_{\text{th},e} = (2k_B T_0/m)^{1/2} , \\ &V_{\text{th},h} = (2k_B T_0/m_h)^{1/2} , \end{aligned} \tag{9}$$

T_0 is the temperature of electrons and holes, and k_B is the Boltzmann constant.

We choose the low-frequency perturbation to be purely electrostatic and is propagating exclusively in the transverse direction $(\mathbf{k}||\hat{\mathbf{x}})$, that is, $\mathbf{E} = -\nabla\phi$, $\phi = \phi_0 \times \exp[-i(\omega t - kx)]$. Also, without loss of any generality we assume that the scattered sidebands $(\omega_1,\mathbf{k}_1)$ and $(\omega_2,\mathbf{k}_2)$ propagate in the xz plane, so that, $k_{1y} = k_{2y} = 0$.

Expressing

$$\begin{aligned} &\mathbf{E} = \mathbf{E}_0(\omega_0,\mathbf{k}_0) + \mathbf{E}(\omega,\mathbf{k}) + \mathbf{E}_1(\omega_1,\mathbf{k}_1) + \mathbf{E}_2(\omega_2,\mathbf{k}_2) , \\ &\mathbf{B} = c\mathbf{k}_0\times\mathbf{E}_0/\omega_0 + c\mathbf{k}_1\times\mathbf{E}_1/\omega_1 + c\mathbf{k}_2\times\mathbf{E}_2/\omega_2 , \\ &\mathbf{V} = \mathbf{V}_0(\omega_0,\mathbf{k}_0) + \mathbf{V}(\omega,\mathbf{k}) + \mathbf{V}_1(\omega_1,\mathbf{k}_1) + \mathbf{V}_2(\omega_2,\mathbf{k}_2) , \end{aligned} \tag{10}$$

$$\begin{aligned} &\mathbf{V}_h = \mathbf{V}_{0h}(\omega_0,\mathbf{k}_0) + \mathbf{V}_h(\omega,\mathbf{k}) + \mathbf{V}_{1h}(\omega_1,\mathbf{k}_1) + \mathbf{V}_{2h}(\omega_2,\mathbf{k}_2) , \\ &n = n_0^0 + n(\omega,\mathbf{k}) + n_1(\omega_1,\mathbf{k}_1) + n_2(\omega_2,\mathbf{k}_2) , \\ &n_h = n_0^0 + n_h(\omega,\mathbf{k}) + n_{1h}(\omega_1,\mathbf{k}_1) + n_{2h}(\omega_2,\mathbf{k}_2) , \end{aligned}$$

Eqs. (5)–(8) yield the following linear response:

$$\begin{aligned} &\mathbf{V}_{1\perp} = (e/m\omega_c^2)(\mathbf{E}_{1\perp}\times\boldsymbol{\omega}_c + i\omega_1\mathbf{E}_{1\perp}) , \\ &\mathbf{V}_{2\perp} = (e/m\omega_c^2)(\mathbf{E}_{2\perp}\times\boldsymbol{\omega}_c + i\omega_2\mathbf{E}_{2\perp}) , \\ &\mathbf{V}_{1h\perp} = (-e/m_h\omega_{ch}^2)(-\mathbf{E}_{1\perp}\times\boldsymbol{\omega}_{ch} + i\omega_1\mathbf{E}_{1\perp}) , \\ &\mathbf{V}_{2h\perp} = (-e/m_h\omega_{ch}^2)(-\mathbf{E}_{2\perp}\times\boldsymbol{\omega}_{ch} + i\omega_2\mathbf{E}_{2\perp}) , \\ &V_{1z} = eE_{1z}/mi\omega_1 , \\ &V_{2z} = eE_{2z}/mi\omega_2 , \\ &V_{1hz} = -eE_{1z}/m_h i\omega_1 , \\ &V_{2hz} = -eE_{2z}/mhi\omega_2 , \end{aligned} \tag{11}$$

where we have made use of the approximation ω_c, $\omega_{ch} > \omega_0$, ω_1, $\omega_2 > \nu_0$, $k_1 V_{\text{th},e}$, $k_2 V_{\text{th},e}$. Using Eqs. (3) and (11) in Eqs. (5)–(8), retaining all the components of the ponderomotive force, and neglecting small-order terms, we obtain the following expressions for the nonlinear electron and hole densities associated with the low frequency perturbation:

$$n = (\chi_e k^2/4\pi e)(\phi + \phi_p) , \tag{12}$$

$$n_h = (\chi_h k^2/4\pi e)(\phi + \phi_{Ph}) , \tag{13}$$

where

$$\begin{aligned} &\chi_e = (i\omega_P^2/\omega_c^2\omega^2)[\nu_0\omega - i(\omega^2 - k^2V_{\text{th},e}^2)] , \\ &\chi_h = (-i\omega_{Ph}^2/\omega_{ch}^2\omega^2)[\nu_0\omega - i(\omega^2 - k^2V_{\text{th},h}^2)] \end{aligned} \tag{14}$$

are the electron and hole susceptibilities and the ponderomotive potentials on electrons and holes are given by

$$\phi_P \simeq \frac{-e\omega_c\omega(Q_1E_{0x}+Q_2E_{0x}^*)}{2mk[\nu_0\omega-i(\omega^2-k^2V_{\mathrm{th},e}^2)]} ,$$
$$\phi_{Ph} \simeq \frac{-e\omega_{ch}\omega(Q_{h1}E_{0x}+Q_{h2}E_{0x}^*)}{2m_hk[\nu_0\omega-i(\omega^2-k^2V_{\mathrm{th},h}^2)]} ; \quad (15)$$
$$Q_1=(-ik_{1x}E_{1x}/\omega_c^2-k_{1x}E_{1y}/\omega_c\omega_1+ik_0E_{1z}/\omega_0\omega_1) ,$$
$$Q_2=(ik_{2x}E_{2x}/\omega_c^2+k_{2x}E_{2y}/\omega_c\omega_2-ik_0E_{2z}/\omega_0\omega_2) ,$$
$$Q_{h1}=(-ik_{1x}E_{1x}/\omega_{ch}^2+k_{1x}E_{1y}/\omega_{ch}\omega_1+ik_0E_{1z}/\omega_0\omega_1) ,$$
$$Q_{h2}=(ik_{2x}E_{2x}/\omega_{ch}^2-k_{2x}E_{2y}/\omega_{ch}\omega_2-ik_0E_{2z}/\omega_0\omega_2) .$$

Thus, the nonlinear current densities at the sideband frequencies can be obtained as

$$\mathbf{J}_1 \simeq -(\chi_e k^2/8\pi)(\phi+\phi_P)\mathbf{V}_{0\perp}^* + (\chi_h k^2/8\pi)(\phi+\phi_{Ph})\mathbf{V}_{oh\perp}^* ,$$
$$\mathbf{J}_2 \simeq -(\chi_e k^2/8\pi)(\phi+\phi_P)\mathbf{V}_{0\perp} + (\chi_h k^2/8\pi)(\phi+\phi_P)\mathbf{V}_{oh\perp} . \quad (16)$$

Using the nonlinear electron and hole density perturbations in the Poisson's equation and the linear parts of the current densities for the scattered sidebands in the wave equations, we obtain

$$\epsilon\phi=\chi_h\phi_{Ph}-\chi_e\phi_P , \quad (17)$$
$$\overleftrightarrow{\mathbf{D}}_1\cdot\mathbf{E}_1=\beta_1\phi(\hat{\mathbf{x}}+i\hat{\mathbf{y}}) , \quad (18)$$
$$\overleftrightarrow{\mathbf{D}}_2\cdot\mathbf{E}_2=\beta_2\phi(\hat{\mathbf{x}}-i\hat{\mathbf{y}}) , \quad (19)$$

where

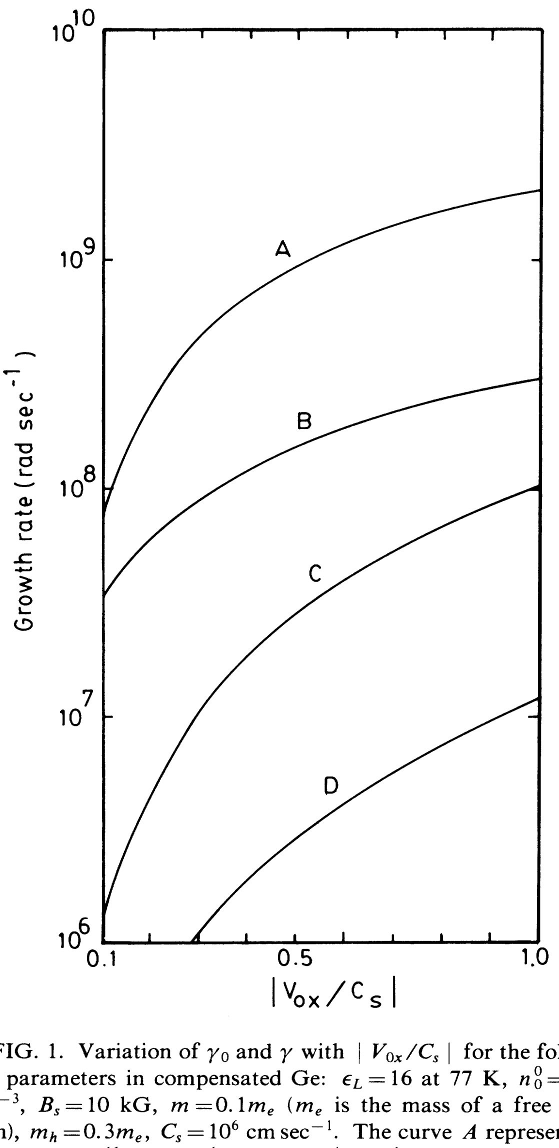

FIG. 1. Variation of γ_0 and γ with $|V_{0x}/C_s|$ for the following parameters in compensated Ge: $\epsilon_L=16$ at 77 K, $n_0^0=10^{17}$ cm^{-3}, $B_s=10$ kG, $m=0.1m_e$ (m_e is the mass of a free electron), $m_h=0.3m_e$, $C_s=10^6$ cm sec^{-1}. The curve A represents γ for $\nu_0=2\times10^{11}$ rad sec^{-1} and $k=10^4$ cm^{-1}, the curves B and C represent γ_0 and γ for $\nu_0=2\times10^{11}$ rad sec^{-1} and $k=10^3$ cm^{-1}, while the curve D represents γ for $\nu_0=2\times10^{12}$ rad sec^{-1} and $k=10^3$ cm^{-1}.

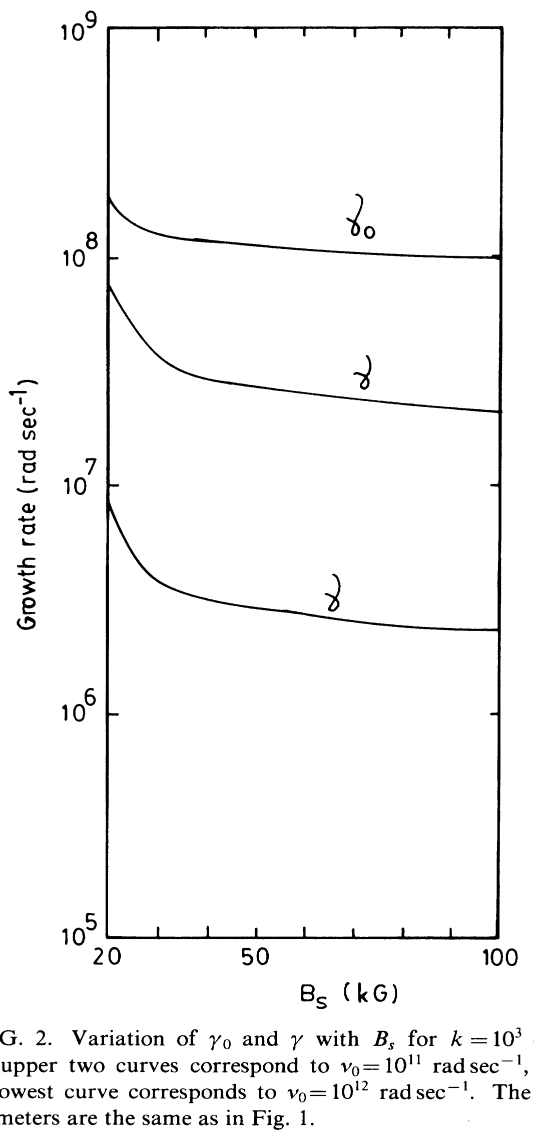

FIG. 2. Variation of γ_0 and γ with B_s for $k=10^3$ cm^{-1}. The upper two curves correspond to $\nu_0=10^{11}$ rad sec^{-1}, while the lowest curve corresponds to $\nu_0=10^{12}$ rad sec^{-1}. The other parameters are the same as in Fig. 1.

$$\beta_1=[eE^*_{0x}\omega_1 k^2(\chi_e-\chi_h)]/(2mc^2\omega_c)\ ,$$
$$\beta_2=[-eE_{0x}\omega_2 k^2(\chi_e-\chi_h)]/(2m\omega_c c^2)\ , \tag{20}$$
$$\overleftrightarrow{\mathbf{D}}_{1,2}=k^2_{1,2}\overleftrightarrow{\mathbf{I}}-\mathbf{k}_{1,2}\mathbf{k}_{1,2}-\omega^2_{1,2}\overleftrightarrow{\epsilon}_{1,2}/c^2\ ,$$

and $\overleftrightarrow{\mathbf{I}}$ is the unit dyadic. In Eq. (17) $\epsilon=1+\chi_e/\epsilon_L-\chi_h/\epsilon_L$ is the linear dielectric function for the low-frequency electrostatic mode,[7] and in Eqs. (20) $\epsilon_{1,2}$ are linear dielectric tensors for the high-frequency sidebands given by[14,15]

$$\overleftrightarrow{\epsilon}_{1,2}\simeq\begin{bmatrix}\epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right] & 0 & 0\\ 0 & \epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right] & 0\\ 0 & 0 & \epsilon_L-\frac{\omega_P^2}{\omega_{1,2}^2}\left[1+\frac{m}{m_h}\right]\end{bmatrix} . \tag{21}$$

Now, eliminating ϕ, $\mathbf{E}_1$, and $\mathbf{E}_2$ from Eqs. (17)—(19), we obtain the nonlinear dispersion relation for the low-frequency perturbation, neglecting the small—order terms, as follows:

$$\epsilon=\frac{\mu_1}{|\overleftrightarrow{\mathbf{D}}_1|}+\frac{\mu_2}{|\overleftrightarrow{\mathbf{D}}_2|}\ , \tag{22}$$

where $|\overleftrightarrow{\mathbf{D}}_{1,2}|$ are the determinants of the dispersion tensors $\overleftrightarrow{\mathbf{D}}_{1,2}$ and

$$\mu_1\simeq\left[\frac{|V_{0x}/C_s|^2\omega_P^4 C_s^2 V^2_{\mathrm{th},e}k^3 k_0 k_{1x}k_{1z}(1+m/m_h)}{2\omega_c^2\omega_0\omega^3 c^2}\right]\left\{k_1^2-\frac{\omega_1^2}{c^2}\left[\epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right]\right]\right\}\ , \tag{23}$$

$$\mu_2\simeq\left[\frac{|V_{0x}/C_s|^2\omega_P^4 C_s^2 V^2_{\mathrm{th},e}k^3 k_0 k_{2x}k_{2z}(1+m/m_h)}{2\omega_c^2\omega_0\omega^3 c^2}\right]\left\{k_2^2-\frac{\omega_2^2}{c_2}\left[\epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right]\right]\right\}\ , \tag{24}$$

$$|V_{0x}|=eE_{0x}/m\omega_c\ .$$

III. GROWTH RATES

When the resonance conditions, Eq. (4), are satisfied one can expand ϵ and $|\overleftrightarrow{\mathbf{D}}_{1,2}|$ around the resonance frequencies. Following Refs. 12 and 14, we finally obtain the expression for the growth rate of the four-wave parametric process, in the absence of the linear damping of the decay waves, as

$$\gamma_0=\left[-\frac{1}{(\partial\epsilon_r/\partial\omega)}\left[\frac{\mu_1}{\partial|\overleftrightarrow{\mathbf{D}}_1|_r/\partial\omega_1}+\frac{\mu_2}{\partial|\overleftrightarrow{\mathbf{D}}_2|_r/\partial\omega_2}\right]\right]^{1/2}\ , \tag{25}$$

where the subscript r denotes real part of the quantities involved. Hence, the undamped growth rate of the filamentation instability is given by

$$\gamma_0=\left[\frac{|V_{0x}/C_s|^2\omega_P^2\epsilon_L k_0 k C_s^2(1+m/m_h)}{16\omega_0}\left[\frac{k_{1x}k_{1z}}{\omega_1 D_1}\left\{k_1^2-\frac{\omega_1^2}{c^2}\left[\epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right]\right]\right\}+\frac{k_{2x}k_{2z}}{\omega_2 D_2}\left\{k_2^2-\frac{\omega_2^2}{c^2}\left[\epsilon_L+\frac{\omega_P^2}{\omega_c^2}\left[1+\frac{m_h}{m}\right]\right]\right\}\right]\right]^{1/2}\ , \tag{26}$$

where

$$D_1=\epsilon_{1xx}\left\{\left[k_{1x}^2-\frac{\omega_1^2}{c^2}\epsilon_{1zz}\right]\left[k_{1z}^2-\frac{\omega_1^2}{c^2}\epsilon_{1xx}\right]-k_{1x}^2k_{1z}^2+\left[k_1^2-\frac{\omega_1^2}{c^2}\epsilon_{1xx}\right]\left[\left[k_{1x}^2-\frac{\omega_1^2}{c^2}\epsilon_{1zz}\right]+\frac{\epsilon_L}{\epsilon_{1xx}}\left[k_{1z}^2-\frac{\omega_1^2}{c^2}\epsilon_{1xx}\right]\right]\right\}\ , \tag{27}$$

and D_2 is obtained by changing the subscript 1 by 2 in Eq. (27). Since the linear damping of the high-frequency scattered sidebands will be always much smaller than the linear damping rate, γ_L of the low-frequency electrostatic mode, the overall growth rate of the filamentation instability may be obtained from the relation

$$\gamma=[(\gamma_L^2+4\gamma_0^2)^{1/2}-\gamma_L]/2 , \qquad (28)$$

where

$$\gamma_L=\nu_0\omega^2(1+m_h/m)/4k^2V_{\text{th},e}^2 . \qquad (29)$$

In order to have some numerical appreciation of the order of magnitude of the growth rate of the filamentation instability of the incident Alfvén wave we have carried out the calculations of γ_0 and γ for the following typical plasma parameters in compensated Ge: $\epsilon_L=16$, $T_0=77$ K, $n_0^0=10^{17}$ cm^{-3}, $m=0.1m_e$ (m_e is the mass of a free electron), $m_h=0.3m_e$, $\nu_0=(0.1-5)\times10^{12}$ rad sec^{-1}, $\omega_0=10^{12}$ rad sec^{-1}, and $C_s=10^6$ cm sec^{-1}. The results of calculations are displayed in the form of curves in Figs. 1 and 2.

Figure 1 shows the variations of γ_0 and γ of the filamentation instability as a function of the pump-induced drift velocity of electrons ($|V_{ox}/C_s|$) for different parameters of interest. The growth rates increase with increasing $|V_{0x}/C_s|$. The high value of the collision frequency of carriers in the compensated semiconductor reduces the growth rate by one or two orders of magnitude. Again, the growth rate increases by about two orders when the wave number k of the perturbation is increased by one order of magnitude.

Figure 2 shows the variation of the growth rates γ_0 and γ as a function of the external static magnetic field B_s in the semiconductor. The growth rates decreases slowly with increasing magnetic field.

When we compare the growth rates of the filamentation instability[13,14] with the present investigation, we notice that for a comparable growth rate, the high-frequency laser radiation in a gaseous plasma needs higher power density than a microwave in a compensated semiconductor. This is because the semiconductor plasma is more unstable than the gaseous plasma. This may be possible as the nonlinearity in a compensated semiconductor plasma is much more due to the motion of light holes than that in a gaseous plasma where the motion of heavy ions may even be neglected relative to the motion of electrons.

IV. DISCUSSION

A high-amplitude left-hand circularly polarized Alfvén wave propagating along the direction of an external static magnetic field in a compensated semiconductor is effectively unstable against filamentation instability. It is observed that the nonlinearity due to the motion of holes is greater than that for the motion of electrons in the compensated semiconductor for the microwave range of frequencies. For a microwave of power density ~ 20 kW cm^{-2} in compensated Ge, the growth rate of the filamentation instability is quite large ($\sim 10^8$ rad sec^{-1}).

It may be further added from the present investigation that a spectrum of various low-frequency modes may be excited when a high-power microwave beam interacts nonlinearly in a semiconductor. These parametric excitations may in turn affect the scattering processes in semiconductors.

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