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Correlation functions of order-parameter fluctuations in a Fermi system

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Inverse Fourier transforms of dynamic correlation functions of order-parameter fluctuations in the singlet-spin-pairing and equal-spin-pairing states of an interacting Fermi system are calculated in a mean-field approximation. The behavior of these functions in the corresponding critical regions, i.e., in the neighborhood of a critical point for an anisotropic, singlet superfluid (and for a superconductor) and for an equal-spin-pairing-type triplet superfluid, are examined. It is found that these functions do not conform to certain generalized homogeneity assumptions. The static counterparts of these functions are found to be nonsingular at corresponding critical points. A comparison of these findings with corresponding findings for a Bose system, reported in a previous work, highlights the role of Fermi statistics in the critical regions under consideration.

Static critical behavior of an interacting Bose system is known[1-3] to be independent of the microscopic detail in the Hamiltonian of the system. However, the same may not be true for the dynamic behavior. An indication is the oscillatory behavior[4] in time of the inverse spatial Fourier transform $S(\mathbf{q},t)$ of the dynamic correlation function of order-parameter fluctuations with time period $2\pi\hbar/E(\mathbf{q})$, where $E(\mathbf{q})$ is the single-particle excitation spectrum. The origin of this behavior is the microscopic, quantum-mechanical basis described in Ref. 4. It may be expected that such a behavior can also be found in the corresponding function for an interacting Fermi system. Moreover, effects, if any, of the statistics of particles in the system at criticality are likely to be more transparent in correlation functions, such as those of order-parameter fluctuations, rather than in thermodynamic functions, because while definitions of the former involve sums over Matsubara frequencies, definitions of the latter involve sums over these frequencies and integrations over momenta. For these reasons, the present report aims at deriving expressions for correlation functions of order-parameter fluctuations in the Fermi system. The functions will be calculated for two states of the system, viz., singlet-spin-pairing (SSP) and equal-spin-pairing (ESP) states, to ascertain whether states of the system influence their behavior in any significant manner at criticality. The functions will be calculated in a mean-field approximation (MFA) which is essentially the same as that in a theory[5] based on Bardeen-Cooper-Schrieffer wave functions for various types of spin pairing. The approximation enables one to regard the system as almost ideal in a critical region. Another reason for studying the correlation functions, which explains the interest in critical forms of these functions, is explained below.

In Ref. 4 (hereafter referred to as I), the function $S(\mathbf{q},t)$ was shown to comply with a dynamic scaling ansatz [see Eq. (108) in I] in the neighborhood of a critical point. It was possible to establish this because (i) the expressions for $S(\mathbf{q},0)$ [see Eqs. (48) and (57) in I] are proportional to terms $\coth[\beta E(\mathbf{q})/2]$ because of certain sums over the Matsubara frequencies $\omega_n=2n\pi/\beta\hbar$, with $n=0,\pm 1,\pm 2,\ldots$, and (ii) the magnitudes of the momenta $\mathbf{q}$ involved are small compared to the boson thermal momentum. A corresponding function $S(\mathbf{k},t)$, for the present system, is expected to involve terms due to sums over ω_n and those due to sums over $\nu_n=(2n+1)\pi/\beta\hbar$. It will therefore be interesting to see whether a dynamic scaling assumption [see Eq. (35)], similar to that in I, holds for $S(\mathbf{k},t)$.

In second-quantized notation, the mean-field Hamiltonians corresponding to SSP and ESP states, respectively, read

$$H_s=\sum_{\mathbf{k},\sigma}\varepsilon(\mathbf{k})a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}+\sum_{\mathbf{k}}[\Delta^\dagger(\mathbf{k})a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}+\Delta(\mathbf{k})a^\dagger_{\mathbf{k}\uparrow}a^\dagger_{-\mathbf{k}\downarrow}] \tag{1}$$

and

$$H_t=\sum_{\mathbf{k},\sigma}\varepsilon(\mathbf{k})a^\dagger_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}+\tfrac{1}{2}\sum_{\mathbf{k}}[\Delta^\dagger_{11}(\mathbf{k})a_{-\mathbf{k}\uparrow}a_{\mathbf{k}\uparrow}+\Delta_{11}(\mathbf{k})a^\dagger_{\mathbf{k}\uparrow}a^\dagger_{-\mathbf{k}\uparrow}+\Delta^\dagger_{22}(\mathbf{k})a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\downarrow}+\Delta_{22}(\mathbf{k})a^\dagger_{\mathbf{k}\downarrow}a^\dagger_{-\mathbf{k}\downarrow}] \ , \tag{2}$$

where

$$\varepsilon(\mathbf{k})=\left[\frac{\hbar^2k^2}{2m}-\mu\right] \ , \tag{3}$$

$$\Delta(\mathbf{k})=\tfrac{1}{2}\sum_{\mathbf{k}'}V_{\mathbf{k}\mathbf{k}'}\langle a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\uparrow}\rangle \ , \tag{4}$$

$$\Delta_{11}(\mathbf{k})=\sum_{\mathbf{k}'}V_{\mathbf{k}\mathbf{k}'}\langle a_{-\mathbf{k}\uparrow}a_{\mathbf{k}\uparrow}\rangle \ , \tag{5}$$

$$\Delta_{22}(\mathbf{k})=\sum_{\mathbf{k}'}V_{\mathbf{k}\mathbf{k}'}\langle a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\downarrow}\rangle \ . \tag{6}$$

Here, μ denotes the chemical potential of a fermion and $V_{\mathbf{k}\mathbf{k}'}$ an attractive interaction potential. Angular brackets $\langle\cdots\rangle$ in (4) denote the thermodynamic average calculated with H_s; those in (5) and (6) the averages calculated with H_t. The usual considerations[5,6] regarding attractions in pure s- and p-wave states imply that whereas $\Delta(\mathbf{k})$ is independent of magnitude of $\mathbf{k}$, $\Delta_{11}(\mathbf{k})$ and $\Delta_{22}(\mathbf{k})$ are odd functions of $\mathbf{k}$. Equations for these gap functions are well known.[5,6]

As already stated, the aim is to examine behavior at criticality of correlation functions of order-parameter fluctuations in SSP and ESP states. One needs to define these functions appropriately for the two states. To this end, one may first consider the operator

$$F(\mathbf{r},t)\equiv\int d^3R\,\psi_\uparrow(\mathbf{R}+\tfrac{1}{2}\mathbf{r},t)\psi_\downarrow(\mathbf{R}-\tfrac{1}{2}\mathbf{r},t)\ , \tag{7}$$

where

$$\psi(\mathbf{x},t)=\exp(iH_st/\hbar)\psi(\mathbf{x},0)\exp(-iH_st/\hbar)\ , \tag{8}$$

$$\psi(\mathbf{x},0)=V^{-1/2}\sum_{\mathbf{k}}a_{\mathbf{k}}(0)\exp(i\mathbf{k}\cdot\mathbf{x})\ , \tag{9}$$

and V denotes the volume enclosing the system. The thermodynamic average of $F(\mathbf{r},0)$ defined with H_s has the physical significance of a sort of wave function for Cooper pairs. It follows that

$$\Gamma_s(\mathbf{r},\mathbf{r}',t)\equiv\tfrac{1}{2}[\langle F^\dagger(\mathbf{r},t)F(\mathbf{r}',0)+F(\mathbf{r},t)F^\dagger(\mathbf{r}',0)\rangle-\langle F^\dagger(\mathbf{r},t)\rangle\langle F(\mathbf{r}',0)\rangle-\langle F(\mathbf{r},t)\rangle\langle F^\dagger(\mathbf{r}',0)\rangle]\ , \tag{10}$$

where angular brackets denote thermodynamic average calculated with H_s, may be taken as a measure of the dynamic correlation of order-parameter fluctuations in the SSP state. The corresponding function, for the ESP state, may be given by

$$\begin{aligned}\Gamma_t(\mathbf{r},\mathbf{r}'t)\equiv\tfrac{1}{4}[&\langle F_\uparrow^\dagger(\mathbf{r},t)F_\uparrow(\mathbf{r}',0)+F_\uparrow(\mathbf{r},t)F_\uparrow^\dagger(\mathbf{r}',0)+F_\downarrow^\dagger(\mathbf{r},t)F_\downarrow(\mathbf{r}',0)+F_\downarrow(\mathbf{r},t)F_\downarrow^\dagger(\mathbf{r}',0)\rangle\\&-\langle F_\uparrow^\dagger(\mathbf{r},t)\rangle\langle F_\uparrow(\mathbf{r}',0)\rangle-\langle F_\uparrow(\mathbf{r},t)\rangle\langle F_\uparrow^\dagger(\mathbf{r}',0)\rangle-\langle F_\downarrow^\dagger(\mathbf{r},t)\rangle\langle F_\downarrow(\mathbf{r}',0)\rangle-\langle F_\downarrow(\mathbf{r},t)\rangle\langle F_\downarrow^\dagger(\mathbf{r}',0)\rangle]\ ,\end{aligned} \tag{11}$$

where

$$F_\uparrow(\mathbf{r},t)\equiv\int d^3R\,\psi_\uparrow(\mathbf{R}+\tfrac{1}{2}\mathbf{r},t)\psi_\uparrow(\mathbf{R}-\tfrac{1}{2}\mathbf{r},t)\ , \tag{12}$$

$$\psi_\uparrow(\mathbf{x},t)=\exp(iH_tt/\hbar)\psi_\uparrow(\mathbf{x},0)\exp(-iH_tt/\hbar)\ , \tag{13}$$

etc., and angular brackets denote the thermodynamic average defined with H_t. If the system is spatially homogeneous, then $\Gamma(\mathbf{r},\mathbf{r}')$ in (10) and (11) depend on $(\mathbf{r}-\mathbf{r}')$ and it is useful to introduce the Fourier transform

$$S(\mathbf{k},\omega)\equiv\frac{1}{V^2}\int d^3r\,d^3r'\int dt\,e^{i[\omega t-\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')]}\Gamma(\mathbf{r}-\mathbf{r}',t)\ . \tag{14}$$

In what follows, first an expression for $S(\mathbf{k},\omega)$ will be obtained. This will lead to one for $S(\mathbf{k},t)$.

In view of (7)–(13) and equations[5,6] for gap functions, it is easy to see that

$$S_l(\mathbf{k},\omega)=S_{lc}(\mathbf{k},\omega)+S_{ld}(\mathbf{k},\omega),\ l=s,t\ , \tag{15}$$

where

$$S_{sc}(\mathbf{k},\omega)=\tfrac{1}{2}\int dt\,e^{i\omega t}[\langle b_{\mathbf{k}}^\dagger(t)b_{\mathbf{k}}(0)+b_{-\mathbf{k}}(t)b_{-\mathbf{k}}^\dagger(0)\rangle]\ , \tag{16}$$

$$S_{sd}(\mathbf{k},\omega)=\frac{-|\Delta|^2}{4E^2(\mathbf{k})}\tanh^2\left[\frac{\beta E(\mathbf{k})}{2}\right]\delta(\omega)\ , \tag{17}$$

$$\begin{aligned}S_{tc}(\mathbf{k},\omega)=\tfrac{1}{4}\int dt\,e^{i\omega t}[&\langle b_{\mathbf{k}\uparrow}^\dagger(t)b_{\mathbf{k}\uparrow}(0)+b_{-\mathbf{k}\uparrow}(t)b_{-\mathbf{k}\uparrow}^\dagger(0)\rangle\\&+\langle b_{\mathbf{k}\downarrow}^\dagger(t)b_{\mathbf{k}\downarrow}(0)+b_{-\mathbf{k}\downarrow}(t)b_{-\mathbf{k}\downarrow}^\dagger(0)\rangle]\ ,\end{aligned} \tag{18}$$

$$S_{td}(\mathbf{k},\omega)=\frac{-\delta(\omega)}{8}\sum_{j=1,2}\frac{|\Delta_{jj}(\mathbf{k})|^2}{E_{jj}^2(\mathbf{k})}\tanh^2\left[\frac{\beta E_{jj}(\mathbf{k})}{2}\right]\ , \tag{19}$$

$$b_{\mathbf{k}}=a_{-\mathbf{k}\uparrow}a_{\mathbf{k}\downarrow},\ b_{\mathbf{k}\uparrow}=a_{-\mathbf{k}\uparrow}a_{\mathbf{k}\uparrow},\ b_{\mathbf{k}\downarrow}=a_{-\mathbf{k}\downarrow}a_{\mathbf{k}\downarrow}\ , \tag{20}$$

$$E(\mathbf{k})=[\varepsilon^2(\mathbf{k})+|\Delta|^2]^{1/2}\ , \tag{21}$$

$$E_{jj}(\mathbf{k})=[\varepsilon^2(\mathbf{k})+|\Delta_{jj}(\mathbf{k})|^2]^{1/2}\ . \tag{22}$$

The Lehmann representations of Matsubara propagators

$$\mathcal{G}_I(\mathbf{k},\omega_n)=\int_0^{\beta\hbar}d\tau\,e^{i\omega_n\tau}\mathcal{G}_I(\mathbf{k}\tau,\mathbf{k}0),\ I=1,2,\ldots,6 \tag{23}$$

are useful to evaluate the averages in (16) and (18). Here, $\omega_n=2n\pi/\beta\hbar$, with $n=0,\pm1,\pm2,\ldots$, and

$$\begin{aligned}\mathcal{G}_1(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}}(\tau)b_{\mathbf{k}}^\dagger(\tau')]\rangle\ ,\\ \mathcal{G}_2(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}}^\dagger(\tau)b_{\mathbf{k}}(\tau')]\rangle\ ,\\ \mathcal{G}_3(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}\uparrow}(\tau)b_{\mathbf{k}\uparrow}^\dagger(\tau')]\rangle\ ,\\ \mathcal{G}_4(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}\uparrow}^\dagger(\tau)b_{\mathbf{k}\uparrow}(\tau')]\rangle\ ,\\ \mathcal{G}_5(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}\downarrow}(\tau)b_{\mathbf{k}\downarrow}^\dagger(\tau')]\rangle\ ,\\ \mathcal{G}_6(\mathbf{k}\tau,\mathbf{k}\tau')&=-\langle T[b_{\mathbf{k}\downarrow}^\dagger(\tau)b_{\mathbf{k}\downarrow}(\tau')]\rangle\ ,\end{aligned} \tag{24}$$

$$b(\tau)=\exp(H\tau/\hbar)b(0)\exp(-H\tau/\hbar)\ . \tag{25}$$

Angular brackets in the expressions for $(\mathcal{G}_1,\mathcal{G}_2)$ above denote thermodynamic averages calculated with H_s while those in the expressions for $(\mathcal{G}_3,\ldots,\mathcal{G}_6)$ with H_t. Using the Lehmann representations of $\mathcal{G}_I(\mathbf{k},\omega_n)$ and following a procedure similar to that in Sec. IV of I, it is quite straightforward to show that

$$S_{sc}(\mathbf{k},\omega)=\frac{(1-e^{-\beta\hbar\omega})^{-1}}{4\pi}[A_1(\mathbf{k},\omega)+A_2(\mathbf{k},\omega)]\ , \tag{26}$$

$$S_{tc}(\mathbf{k},\omega)=\frac{(1-e^{-\beta\hbar\omega})^{-1}}{8\pi}[A_3(\mathbf{k},\omega)+A_4(\mathbf{k},\omega)+A_5(\mathbf{k},\omega)+A_6(\mathbf{k},\omega)]\ , \tag{27}$$

where

$$A_I(\mathbf{k},\omega)\equiv i[\mathcal{G}_I(\mathbf{k},\omega_n)|_{i\omega_n=\omega+iO^+}-\mathcal{G}_I(\mathbf{k},\omega_n)|_{i\omega_n=\omega-iO^+}]\ . \tag{28}$$

The propagators in (28) can be calculated by solving the equations of motion for the temperature functions in (24). The equations of motion for the operators $b_{\mathbf{k}}(\tau)$, $b_{\mathbf{k}\uparrow}(\tau)$, etc., are required for this purpose. From (1), (2), and (25) one gets these equations. On solving equations of motion for the temperature functions one finds

$$\mathcal{G}_1(\mathbf{k},\omega_n)=\frac{1}{4}\left[1+\frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right]^2[i\omega_n-2E(\mathbf{k})\hbar^{-1}]^{-1}\times\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right]$$
$$-\frac{1}{4}\left[1-\frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right]^2[i\omega_n+2E(\mathbf{k})\hbar^{-1}]^{-1}\times\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right], \quad (29)$$

$$\mathcal{G}_2(\mathbf{k},\omega_n)=\frac{1}{4}\left[1-\frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right]^2[i\omega_n-2E(\mathbf{k})\hbar^{-1}]^{-1}\times\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right]$$
$$-\frac{1}{4}\left[1+\frac{\varepsilon(\mathbf{k})}{E(\mathbf{k})}\right]^2[i\omega_n+2E(\mathbf{k})\hbar^{-1}]^{-1}\times\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right]. \quad (30)$$

The propagators $\mathcal{G}_3(\mathbf{k},\omega_n)$ and $\mathcal{G}_4(\mathbf{k},\omega_n)$, respectively, are obtained replacing $E(\mathbf{k})$ by $E_{11}(\mathbf{k})$ in (29) and (30), whereas $\mathcal{G}_5(\mathbf{k},\omega_n)$ and $\mathcal{G}_6(\mathbf{k},\omega_n)$, respectively, are obtained replacing $E(\mathbf{k})$ by $E_{22}(\mathbf{k})$. In view of (28), one then gets

$$S_{sc}(\mathbf{k},\omega)=\frac{(1-e^{-\beta\hbar\omega})^{-1}}{4}\left[1+\frac{\varepsilon^2(\mathbf{k})}{E^2(\mathbf{k})}\right]\left[\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right]\right][\delta(\omega-2E(\mathbf{k})\hbar^{-1})-\delta(\omega+2E(\mathbf{k})\hbar^{-1})], \quad (31)$$

$$S_{tc}(\mathbf{k},\omega)=\frac{(1-e^{-\beta\hbar\omega})^{-1}}{8}\sum_{j=1,2}\left[1+\frac{\varepsilon^2(\mathbf{k})}{E_{jj}^2(\mathbf{k})}\right]\left[\tanh\left[\frac{\beta E_{jj}(\mathbf{k})}{2}\right]\right][\delta(\omega-2E_{jj}(\mathbf{k})\hbar^{-1})-\delta(\omega+2E_{jj}(\mathbf{k})\hbar^{-1})]. \quad (32)$$

From (31) and (32), one finds

$$S_{sc}(\mathbf{k},t)=\frac{1}{4}\left[1+\frac{\varepsilon^2(\mathbf{k})}{E^2(\mathbf{k})}\right]\left[\tanh\left[\frac{\beta E(\mathbf{k})}{2}\right]\right]\times\left[\frac{e^{-2iE(\mathbf{k})\hbar^{-1}t}}{1-e^{-2\beta E(\mathbf{k})}}+\frac{e^{2iE(\mathbf{k})\hbar^{-1}t}}{e^{2\beta E(\mathbf{k})}-1}\right], \quad (33)$$

$$S_{tc}(\mathbf{k},t)=\frac{1}{8}\sum_{j=1,2}\left[1+\frac{\varepsilon^2(\mathbf{k})}{E_{jj}^2(\mathbf{k})}\right]\left[\tanh\left[\frac{\beta E_{jj}(\mathbf{k})}{2}\right]\right]\times\left[\frac{e^{-2iE_{jj}(\mathbf{k})\hbar^{-1}t}}{1-e^{-2\beta E_{jj}(\mathbf{k})}}+\frac{e^{2iE_{jj}(\mathbf{k})\hbar^{-1}t}}{e^{2\beta E_{jj}(\mathbf{k})}-1}\right]. \quad (34)$$

These expressions are not similar to the expressions for $S(\mathbf{q},t)$ in I [see Eqs. (104) and (105) in I] inasmuch as the coefficient of the term in the large parentheses in (33) involves $\{\tanh[\beta E(\mathbf{k})/2]\}$, and that in (34) a similar term, arising due to sums over the Matsubara frequencies $\nu_n=(2n+1)\pi/\beta\hbar$, with $n=0,\pm1,\pm2,\ldots$, i.e., due to Fermi statistics. As will be seen, this dissimilarity has a significant effect on the critical forms of $S(\mathbf{k},t)$ and $S(\mathbf{k},0)$ for the present system.

A dynamic scaling ansatz, similar to that in Eq. (108) of I, for an anisotropic, spin-singlet superfluid (and for a superconductor) may be stated as follows: In the neighborhood of a critical temperature T_s of the singlet superfluid, the correlation function $S_s(\mathbf{k},t)$ satisfies the generalized homogeneity relation

$$S_s(l^{b_k}\varepsilon(\mathbf{k}),l^{b_\Delta}|\Delta|,l^{b_t}t)=lS_s(\varepsilon(\mathbf{k}),|\Delta|,t), \quad (35)$$

where l denotes an arbitrary positive number while the exponents b_k, b_Δ, etc., are unknown quantities, the inputs of the theory. In the neighborhood of a critical temperature T_t of an ESP-type, triplet superfluid also, a similar relation can be written down. We will now examine whether expressions for $S(\mathbf{k},t)$ calculated above satisfy these assumptions.

In the neighborhood of T_s, one may assume $|\Delta|\sim|\varepsilon(\mathbf{k})|$, for $|\mathbf{k}|\sim k_F$ (Fermi momentum), and $|\Delta|$ small compared to $|\varepsilon(\mathbf{k})|$ otherwise. A similar assumption may be made for $|\Delta_{11}(\mathbf{k})|$ and $|\Delta_{22}(\mathbf{k})|$ in the neighborhood of T_t. The quantities $\beta_s|\varepsilon(\mathbf{k})|$ and $\beta_t|\varepsilon(\mathbf{k})|$, on the other hand, may be regarded as small compared to unity, for $|\mathbf{k}|\sim k_F$, and large otherwise. It follows that, for $|\mathbf{k}|\sim k_F$, $S_{sc}(\mathbf{k},t)$ and $S_{tc}(\mathbf{k},t)$ (in a unitary ESP state[5]), respectively, may show oscillatory behavior in time in the critical regions corresponding to T_s and T_t. Furthermore, for a nonunitary ESP state, beats may occur. For a superconductor, taking (Δ_0/k_BT_s) of $O(1)$, where Δ_0 is the gap edge, and T_s of $O(1\text{ K})$, one finds that the oscillatory behavior is expected to occur at microwave frequencies. These are quantum-mechanical effects which have no counterpart in classical systems. Similar oscillatory behavior has also been observed for the effective Bose system (EBS), described by a mean-field Hamiltonian, in I. It may be noted that such a behavior is not special to the MFA. In a non-mean-field-theoretic description of the EBS (see, e.g., Ref. 7) this behavior can also be observed when suitable approximations are made.

The assumptions above, regarding the order of smallness of $|\Delta|$, $|\varepsilon(\mathbf{k})|$, etc., imply that (35) will hold near T_s, for $|\mathbf{k}|\sim k_F$, provided that $S_{sc}(\mathbf{k},0)$ is a generalized homogeneous function[8] (GHF) in this case. These assumptions also imply that a hypothesis similar to (35) will hold near T_t, if $S_{tc}(\mathbf{k},0)$ is a GHF. Since, for $|\mathbf{k}|\sim k_F$, $\beta_s|\varepsilon(\mathbf{k})|$ and $\beta_t|\varepsilon(\mathbf{k})|$ may be regarded small compared to unity, in view of (17), (19), (33), and (34) one may write in the critical regions corresponding to T_s and T_t, respectively,

$$S_s(\mathbf{k},0)\simeq\frac{1}{8}\left[1+\frac{\varepsilon^2(\mathbf{k})}{\varepsilon^2(\mathbf{k})+|\Delta|^2}\right], \quad (36)$$

$$S_t(\mathbf{k},0)\simeq\frac{1}{8}\left[1+\frac{\varepsilon^2(\mathbf{k})}{2[\varepsilon^2(\mathbf{k})+|\Delta_{11}(\mathbf{k})|^2]}+\frac{\varepsilon^2(\mathbf{k})}{2[\varepsilon^2(\mathbf{k})+|\Delta_{22}(\mathbf{k})|^2]}\right], \quad (37)$$

when $|\mathbf{k}| \sim k_F$. Evidently these are not GHF's. Therefore, the dynamic scaling assumptions do not hold, for $|\mathbf{k}| \sim k_F$. Similarly, when $|\mathbf{k}|$ is not of the same order as k_F, it can be seen that $S_s(\mathbf{k},0)$ and $S_t(\mathbf{k},0)$ are not GHF's. The same conclusions as above may be expected to be valid for corresponding function of a non-ESP state. It follows that for a Fermi system, capable of exhibiting momentum condensation, the microscopic exponents[4] ν and z cannot be defined. The results above are contrary to corresponding results for EBS,[4] the reason for which can be traced to $\{\tanh[\beta E(\mathbf{k})/2]\}$ in (33), and the similar terms in (34), arising due to Fermi statistics.

At a critical point for an anisotropic, singlet superfluid (and for a superconductor),

$$S_s(\mathbf{k},0) = \tfrac{1}{2}\tanh\left[\frac{\beta_s \varepsilon(\mathbf{k})}{2}\right]\coth[\beta_s \varepsilon(\mathbf{k})] \ . \qquad (38)$$

For an ESP-type, triplet superfluid, $S_t(\mathbf{k},0)$ at corresponding critical point is obtained replacing β_s by β_t in (38). This concurrence was expected, for at a critical point the system reduces to an ideal Fermi gas in the present MFA. The right-hand side of (38) is nonsingular. For an interacting Bose system,[4] however, the corresponding function $S(\mathbf{q},0)$ exhibits singular behavior: At a critical point, $S(\mathbf{q},0)$ depends on momenta $\mathbf{q}$ as $S(\mathbf{q},0) \sim |\mathbf{q}|^{-2}$, and in the neighborhood, as $S(\mathbf{q},0) \sim (|\mathbf{q}|^2+q_0^2)^{-1}$, where q_0^{-1} is the correlation length of order-parameter fluctuations. This singular behavior corroborates the fact[3] that an interacting Bose system behaves like a classical spin system at criticality. As is clear from above, this fact does not hold for the Fermi system. The role of the statistics in an interacting Fermi system at criticality is thus reemphasized.

[1] K. K. Singh, Phys. Rev. B **12**, 2819 (1975); **13**, 3192 (1976); **17**, 324 (1978).

[2] K. K. Singh and Partha Goswami, Phys. Rev. B **29**, 2558 (1984); **31**, 4285 (1985).

[3] P. B. Weichman, M. Rasolt, M. E. Fisher, and M. J. Stephen, Phys. Rev. B **33**, 4632 (1986).

[4] Partha Goswami, Phys. Rev. B **33**, 6094 (1986).

[5] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[6] R. Balian and N. R. Werthamer, Phys. Rev. **131**, 1553 (1963).

[7] K. K. Singh (unpublished).

[8] A. Hankey and H. E. Stanley, Phys. Rev. B **6**, 3515 (1972).