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Electric-field-dependent average resistance and the resistance fluctuation in one-dimensional disordered systems

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Following an invariant-imbedding approach, we obtain analytical expressions for the ensemble-averaged resistance (ρ) and its Sinai's fluctuations for a one-dimensional disordered conductor in the presence of a finite electric field F. The mean resistance shows a crossover from the exponential to the power-law length dependence with increasing field strength in agreement with known numerical results. More importantly, unlike the zero-field case the resistance distribution saturates to a Poissonian-limiting form proportional to $A\,|F|\exp(-A\,|F|\rho)$ for large sample lengths, where A is constant.

It is now well established both numerically and analytically that almost all the electronic eigenstates of a one-dimensional (1D) disordered system are exponentially localized for arbitrarily weak static disorder.[1] This localized nature of the eigenstates manifests as an exponential increase of ensemble-averaged resistance $\langle\rho\rangle$ with the sample length (L).[2] Experiments on quasi-one-dimensional wires confirm this length-scale dependence of resistance, with the Thouless length $L_T=\sqrt{D\tau_{\rm in}}$ effectively replacing the sample length.[3–5] In addition to thus being nonadditive, the quantum Ohmic resistance is also known to be non-self-averaging[6–12] in that the resistance fluctuations over the ensemble of macroscopically identical samples dominate the ensemble average, i.e., there is no typical resistance. In point of fact the relative fluctuation of resistance is much larger than the relative fluctuation of the underlying random "impurity" concentration for large sample lengths. We call these generically Sinai's fluctuations.[13] These "Sinai" fluctuations, reflecting the sensitivity of the residual resistance to the microscopic details of the sample-dependent random potential, are due ultimately to the quantum-coherence effects of elastic scattering for $L \ll L_T$. Their most striking manifestation is the universal conductance fluctuation $\sim e^2/h$ in the metallic regime as function of chemical potential obtained recently by Lee and Stone.[12] Closely related to this is the magnitude of the flux-periodic conductance fluctuation.[12] (The ensemble fluctuation and the fluctuation for a given sample as function of chemical potential are expected to be related by some sort of ergodicity.) Our recent work[14] suggests that these fluctuations persist right up to the mobility edge on the insulating side in higher dimensions $(D>1)$ and are suspected to have important bearing on the physics at the mobility edge.

In the present work we have examined the influence of a finite electric field on these fluctuations for the case $D=1$, where analytical treatment turned out to be tractable. Our main result is that the fluctuations get harnessed by the field and the probability distribution of resistance saturates to a limiting Poissonian form for large L. The mean resistance shows a crossover from the exponential to the power-law length-scale (L) dependence. The crossover is in agreement with the numerical results on disordered (one-dimensional) 1D Kronig-Penney model giving weakly (algebraically) localized eigenstates in the presence of a strong electric field.[15] While there are a few recent studies on the influence of a finite electric field on the nature of the eigenstates and average resistance[16,17] and the spectrum,[18] there is none to our knowledge on its influence on the resistance fluctuation.

Our approach is based on "invariant imbedding" as used recently by one of us[9] and by Heinrichs,[19] but, in fact, can be traced to a much earlier work of Landauer.[20] This enables us to directly address the emergent quantity, namely, the reflection coefficient without having recourse to the wave function. The reflection coefficient is in turn related to the extrinsic resistance (conductance) via the Landauer formula.[2]

The model Hamiltonian for the 1D disordered system is

$$H=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\ , \tag{1}$$

where $V(x)$ for $0<x<L$ is the random potential assumed to be a delta-correlated Gaussian variable with

$$\langle V(x)\rangle = -\,|e\,|Fx\ ,$$

and (2)

$$\langle [V(x)-\langle V(x)\rangle][V(x')-\langle V(x')\rangle]\rangle = V_0{}^2\delta(x-x')\ ,$$

and $F\,(>0)$ is the external electric field directed along the $-ve$ x axis. The disordered sample extends from $x=0$ to $x=L$, the two ends being connected Ohmically to perfect leads maintained at a potential difference $|\,e\,|FL$. Consider an electron of wave number $k_1(L)$ incident at $x=L$ from the right. It is partially reflected with the complex amplitude reflection coefficient $R(L)$ and partially transmitted with wave number $k_0\neq k_1(L)$. Now, we must employ an imbedding procedure appropriate to this situation, namely of unequal incident and transmitted wave numbers.[19,21] The complex amplitude reflection coefficient $R(L)$ then obeys the stochastic Riccati equation which is conveniently represented as

$$R(L)=\frac{2k_0}{k_0+k_1(L)}r_1(L)+\frac{k_1(L)-k_0}{k_1(L)+k_0} \tag{2a}$$

with

$$\frac{dr_1}{dL}=\frac{1}{i[k_0+k_1(L)]}\left[\frac{k_1(L)}{k_0}[k_0^2-k^2(L)]-2[k^2(L)+k_0k_1(L)]r_1+\frac{k_0}{k_1(L)}[k_1^2(L)-k^2(L)]r_1^2\right] \tag{2b}$$

along with the boundary condition

$$r_1(L=0)=0\ ,\quad R(L=0)\equiv\frac{k_1(L=0)-k_0}{k_1(L=0)+k_0}=0\ .$$

Here we have defined the local wave numbers as $k_0^2=(2m/\hbar^2)E_F$, $k_1^2(L)=(2m/\hbar^2)(E_F+|e|FL)$, and $k^2(L)=(2m/\hbar^2)[E_F-V(L)]$ with $E_F=\hbar^2k_0^2/2m$. Differentiating Eq. (2a) with respect to L and substituting for dr_1/dL from Eq. (2b), we get the equation for $R(L)\equiv R_1(L)+iR_2(L)$:

$$\frac{dR_1}{dL}=\frac{2m}{\hbar^2}\frac{V(L)}{k_1(L)}R_1R_2-\frac{k^2(L)+k_1^2(L)}{k_1(L)}R_2+\frac{m}{\hbar^2}\frac{(1-R_1)|e|F}{k_1(L)(k_0+k_1(L))}\ , \tag{3a}$$

$$\frac{dR_2}{dL}=-\frac{m}{\hbar^2}\frac{V(L)}{k_1(L)}(1+R_1^2-R_2^2)+\frac{k^2(L)+k_1^2(L)}{k_1(L)}R_1-\frac{m}{\hbar^2}\frac{|e|FR_2}{k_1(L)(k_0+k_1(L))}\ . \tag{3b}$$

These nonlinear stochastic differential equations of the Riccati type generate the "Fokker-Planck" equation for the probability density $W_R(R_1,R_2;L)$ via the well-known van Kampen lemma.[9] We are, however, interested in less detailed information contained in the reduced probability density $W_r(r;L)$ for the reflection coefficient $r=R_1^2+R_2^2$. After a straightforward but tedious algebra we get

$$\frac{\partial W_r}{\partial L}=\frac{1}{\xi(1+|e|FL/E_F)}\left[r(1-r)^2\frac{\partial^2W_r}{\partial r^2}+(1-5r)(1-r)\frac{\partial W_r}{\partial r}+2(2r-1)W_r\right]+\frac{2m|e|F}{\hbar^2k_1^{(L)}(k_0+k_1(L))}\left[r\frac{\partial W_r}{\partial r}+W_r\right] \tag{4}$$

with

$$\xi=(\hbar^2/m)E_F/V_0^2\ .$$

This is our central equation. It is exact. One readily verifies that it correctly reduces to the zero-field limit as treated earlier by one of us.[9] We should note that by choice of direction of current flow, F in these expressions is positive.

At this stage it is convenient to introduce a change of variable,

$$\rho(L)=\frac{r(L)}{1-r(L)}\ . \tag{5}$$

Equation (5) is just the Landauer[2] expression for the resistance (extrinsic) in units of $(2\pi\hbar/e^2)$ in the limit of zero field. For a nonzero field the exact relation of $\rho(L)$ to $r(L)$ is somewhat complicated by the appearance of kinematic factors involving k_0 and $k_1(L)$, but these give only subdominant L dependence. We will, therefore, continue to identify $\rho(L)$ with the resistance of the 1D disordered conductor. The associated probability density $W_\rho(\rho;L)$ then obeys the equation

$$\frac{\partial W_\rho}{\partial l}=\rho(\rho+1)\frac{\partial^2W_\rho}{\partial\rho^2}+(2\rho+1)\frac{\partial W_\rho}{\partial\rho}+\frac{|e|F\xi}{E_F\{1+\exp[-(|e|F\xi l)/2E_F]\}}\left[(2\rho+1)W_\rho+\rho(\rho+1)\frac{\partial W_\rho}{\partial\rho}\right] \tag{6}$$

with

$$l=\left[\frac{E_F}{|e|F\xi}\right]\ln\left[1+\frac{|e|FL}{E_F}\right]\ .$$

For $|e|\xi F/E_F\ll 1$, that is in the weak-field limit, we can ignore the last term in the right-hand side (RHS) of Eq. (6). In this limit the equation for W_ρ formally reduces to the zero-field case except for replacement of L by l. Thus, one gets for the mean resistance

$$\rho_1\equiv\langle\rho\rangle=\tfrac{1}{2}\left[\left[1+\frac{|e|FL}{E_F}\right]^{2E_F/|e|F\xi}-1\right]\ , \tag{7}$$

which crosses over smoothly from the exponential L dependence for $F\rightarrow 0$ to a power-law ($L^{\alpha(F)}$) dependence for $(|e|FL)/E_F \gg 1$, with a field-dependent exponent $\alpha(F)=2E_F/(|e|\xi F)$. Equation (7) gives a power-series expansion for $\rho_1(F)$ for $L/\xi \ll 1$ and $F|e|L/E_F \ll 1$ as

$$\rho_1(F)=\rho_1(0)\left[1-\frac{|e|FL}{2E_F}+\frac{|e|^2F^2L^3}{3E_F^2\xi}+\cdots\right], \quad (8)$$

which is in agreement with the predictions of Soukoulis *et al.*[15]

Now we turn to the fluctuations. Equation (6) with the last term on the RHS neglected will give the well-known log-normal distribution for resistance, but with the length L replaced by l and, therefore, of diminished variance. The full "equation," however has the interesting feature of giving a limiting distribution as $L \to \infty$ obtained by setting $(\partial W_\rho/\partial L)=0$. The limiting solution is a Poissonian form

$$W_\rho(\rho,\infty)=\left[\frac{|e|F\xi}{E_F}\right]\exp\left[-\frac{|e|F\xi\rho}{E_F}\right]. \quad (9)$$

Thus the fluctuations get harnessed by the delocalizing influence of the electric field.

Equation (8) is of direct experimental interest and with the accessibility of length-dependent conductance measurement[4] on a submicrometer length-scale it should be possible and is desirable to probe this. Equation (9) is, on the other hand, at present best explored numerically by considering the dispersion of reflection coefficient and of related physical quantities.[22]

Finally, we would like to comment on these fluctuations in terms of the microscopic expression for the dimensionless resistance $\rho(L)$ (Thouless ratio) on any length scale given by[23]

$$\rho=\frac{\Delta E}{\delta E}, \quad (10)$$

where ΔE is a suitably defined level spacing (granularity) and δE the level shift (coupling) due to change of boundary conditions, for example, from periodic to antiperiodic. If one takes ΔE, which is essentially the reciprocal of density of states, to be a self-averaging quantity, then the distribution of $\rho(L)$ is directly related to the distribution of the reciprocal of δE, which measures the extent of delocalization. Thus, the zero-field log-normal distribution of $\rho(L)$ for large L reflects the normal distribution of the inverse localization length. Similarly, for the finite-field case, the Poissonian distribution of $\rho(L)$ should give a distribution for the exponent of the algebraic decay of the weakly localized states. We should, of course, recall that in this large length limit, $\rho(L)$ defined by Eq. (5) no longer represents the resistance accurately due to the neglect of the kinematic factors. However, the reflection coefficient $r(L)=\rho/(1+\rho)$ continues to have the operationally well-defined meaning, and from Eq. (4) given by

$$W_r(r,\infty)=\frac{|e|F\xi}{E_F(1-r)^2}\exp\left[-\frac{|e|F\xi}{E_F}\left[\frac{r}{1-r}\right]\right]. \quad (11)$$

These results apply to any scalar wave propagation in a random nondissipative 1D medium.

Given the rapidly developing submicrometer fabrication (microlitographic) techniques, it is clear that one is fast approaching the mesoscopic length scales where these consequences of the break down of "ensemble-averaging" are beginning to show. Of particular experimental interest will be the indirect observation of these Sinai's fluctuations in a single sample as an excess low-frequency, low-temperature noise when $L < L_T$ and the diffusion time is less than the period of dominant thermal phonons. Then the electron will become affected by the thermal phonon modulation of the random potential adiabatically, leading to a scintillation noise, i.e., a time translation of the Sinai's fluctuations.[24,25]

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