Variation of the critical order-parameter phase difference with temperature from 0.4 to 1.9 K in the flow of superfluid ^{4}He through a tiny orifice

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The temperature dependence of the order-parameter phase difference at the critical rate of flow of superfluid ^{4}He through a tiny orifice has been studied between 0.4 and 1.9 K. Use was made of a low-frequency two-chamber acoustic resonator at frequencies between 400 and 2900 Hz. The orifice consisted of a hole ~ 170 nm in diameter in a free-standing foil ~ 100 nm thick. The critical phase difference was found in four runs to equal $31 \times 2\pi$ rad and in two later runs to equal $16 \times 2\pi$ rad at 0.95 K. In all cases the critical phase difference decreased linearly with increasing temperature between 0.4 and 1.9 K. The lines which fit the data extrapolate to zero at 2.45 ± 0.1 K.

I. INTRODUCTION

In the course of a search for analogs in superfluid ^{4}He of the Josephson effects in superconductors, we have measured the values of the superfluid order-parameter phase difference at the critical rate of flow of superfluid ^{4}He through a variety of tiny orifices in thin films.[1] In this article we report the results of measurements of the critical phase difference for a single orifice ~ 170 nm in diameter in a foil ~ 100 nm thick as a function of temperature between 0.4 and 1.9 K at several frequencies between 400 and 2900 Hz.

Relatively little is known about the critical rate of flow of superfluid ^{4}He through a single orifice or pore of short length. Most of the existing data concerning single pores involve those with diameters and lengths of the order of 10 μm at temperatures above 1 K.[2–11] In one influential study in this regime,[5] Hess showed that an orifice which ordinarily would have a low critical velocity of order 0.25 m/s independent of temperature had much higher critical velocities when shielded by superleak filters, these velocities having a temperature dependence in some respects consistent with thermal activation theory involving homogeneous nucleation of vortex rings.[12–14]

Only a few experiments have studied the critical rates of flow through single orifices with smaller dimensions or at temperatures below 1 K.[1,10,15–17] In particular, in one study restricted to temperatures below 1 K,[15] Schofield and Sanders found that 2.2- and 4.0-μm-diameter pores could show relatively large critical velocities which decreased linearly with increasing temperature, while pores larger in diameter tended to show lower critical velocities which were more nearly temperature independent.

In the present work we have measured the critical order-parameter phase difference for a single orifice which was considerably smaller than those used by others. In addition, these measurements were made over a broader range of temperature than has been studied for any one orifice. A more complete account of this work will be found in the Ph.D. thesis of one of the current authors (B.P.B.).[18] Additional details concerning the apparatus and the principles of its operation are given in a report of our search for Josephson effects.[19]

II. APPARATUS

Our cell consisted of a two-chamber low-frequency acoustic resonator, which is shown schematically in Fig. 1. The cell was completely filled with liquid ^{4}He. Its chambers were connected by two openings in parallel, a relatively open main passage and the tiny orifice under investigation.

The upper wall of the upper chamber was a flexible diaphragm of beryllium copper, which was driven by a cylindrical piezoelectric transducer. Oscillatory motion of the liquid between the chambers was excited by applying a sinusoidal voltage to this transducer. The lower wall of the lower chamber was also a flexible diaphragm of beryllium copper and carried the movable plate of a variable capacitor. This capacitor thus responded to pressure variations in the lower chamber.

The cell possessed two low-frequency resonances which resulted from the coupling of two simpler modes of oscil-

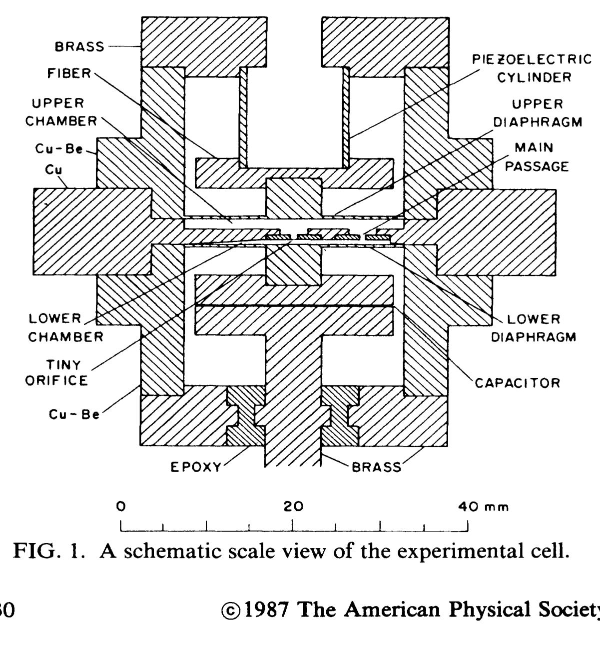

FIG. 1. A schematic scale view of the experimental cell.

lation. One of these simple modes was the pure Helmholtz oscillation of the fluid back and forth between chambers that would have occurred in the absence of diaphragm motion. The other of these modes was the oscillation of the lower diaphragm, which carried the relatively massive capacitor plate, that would have occurred in the absence of the fluid. There were, in fact, two Helmholtz modes, a first-sound mode involving comoving superfluid and normal-fluid components, and a second-sound mode involving countermoving components. The latter mode does not couple strongly to diaphragm motion, however, and is not expected to have been excited to a significant degree in this experiment. We therefore neglect it. The upper diaphragm carried less mass than the lower one and was stiffened by the piezoelectric element. Thus it had a much higher resonant frequency than the lower one. We neglect its motion in determining the two main low-frequency resonances.

The main passage in three of our runs consisted of a hole 0.3 or 0.5 mm in diameter drilled in a disk of copper or brass 0.5 mm thick which was sealed over a relatively large offset opening in the wall between chambers. In two other runs the large opening itself, 3.2 mm in diameter in a wall 0.8 mm thick, served as the main passage. Finally, in still another run, the main passage consisted of a nearly circular opening 29 μm in diameter in a copper foil 5 μm thick. This foil was sealed over a 0.8-mm-diameter hole in a 0.5-mm-thick disk, which in turn was sealed over the offset opening in the chamber wall. Different choices of main passage gave us different pairs of resonant frequencies.

The tiny orifice consisted of a nearly circular opening ~ 170 nm in diameter (155 nm$\times 185$ nm) in a nickel foil ~ 100 nm thick. This nickel foil was free standing over a diameter of ~ 27 μm. It was supported at the edges by a 5-μm-thick copper foil which in turn was sealed over a hole 0.8 mm in diameter in a disk of copper 0.5 mm thick. This disk was sealed over a relatively large central opening in the wall between chambers. The fabrication of this tiny orifice is described in Ref. 20, where it is pictured in Fig. 5 before its use in these experiments.

The variable capacitor associated with the lower diaphragm formed part of the tank circuit of a low-temperature back-diode rf oscillator which was attached to the cell, and variations in the capacitance served to modulate the frequency of the oscillator. The signal from this oscillator passed through an fm detector and then through a vector lock-in amplifier locked to the drive signal. The overall average pressure in the cell was determined by measuring the carrier frequency of the oscillator, which under one representative set of conditions varied from 8.8 to 6.6 MHz as the cell pressure increased from 0 to 50 kPa.

The central oxygen-free high-conductivity copper section of the cell body carried two independent germanium resistance thermometers which were calibrated *in situ* in terms of the $T62$ ^{3}He and $T58$ ^{4}He vapor-pressure scales of temperature. For this calibration the cell cavity was filled partially with ^{3}He or ^{4}He liquid, and the lower diaphragm transducer was used to make vapor pressure measurements.

The cell was cooled by a two-stage ^{3}He refrigerator. For vibration isolation the cell was suspended below the ^{3}He pot on springs, and the entire cryostat was mounted on a two-stage vibration isolation platform with flexible pumping connections.

III. PRINCIPLES OF THE EXPERIMENTAL METHOD

We use the two-fluid equations of motion for the liquid[21] together with the equation of motion for the diaphragm,

$$m\ddot{x} = -kx - P_1 A \ , \qquad (1)$$

to find the relationship between the difference $\mu_2 - \mu_1$ in chemical potential per atom between upper and lower chambers and the upward displacement x of the center of the lower diaphragm from equilibrium during oscillation. Here m is the effective mass of the diaphragm and capacitor plate, k is the effective spring constant of the diaphragm, A is its effective area, and P_1 is the pressure in the lower chamber.

In the low-temperature limit, in which we assume that the fluid consists entirely of the superfluid component, we find

$$\mu_{20} - \mu_{10} = \frac{m_4}{\rho}\frac{k}{A}\frac{\Omega_1+\Omega_2}{\Omega_2}\frac{\omega^2}{\omega_H^2}\frac{(1+\alpha_1)\omega_T^2 - \omega^2}{\omega_T^2} x_0 \ . \qquad (2)$$

Here and below we assume sinusoidal oscillation at angular frequency ω in proportion to $e^{i\omega t}$ and designate complex amplitudes by subscripts "0". The quantity m_4 is the mass of the ^{4}He atom, ρ is the mass density of the liquid, Ω_1 and Ω_2 are the volumes of the lower and upper chambers, respectively, ω_H is the resonant angular frequency of the Helmholtz oscillation that would occur in the absence of diaphragm motion, $\omega_T = \sqrt{k/m}$ is the resonant angular frequency of lower diaphragm oscillation in the absence of fluid, and $\alpha_1 \equiv A^2/(k\kappa\Omega_1)$, where κ is the compressibility of the liquid.

The actual resonant angular frequencies $\omega_\pm$, at which measurements are made, can be shown to obey the relations

$$\omega_+^2 + \omega_-^2 = \omega_T^2(1+\alpha_1) + \omega_H^2 \ , \qquad (3)$$

$$\omega_+^2 \omega_-^2 = \omega_T^2(1+\alpha_t)\omega_H^2 \ , \qquad (4)$$

where

$$\alpha_t \equiv A^2/[k\kappa(\Omega_1+\Omega_2)] \ .$$

For subcritical flow of the superfluid, in the absence of any superfluid vorticity or circulation, we may write

$$\mathbf{v}_s = (\hbar/m_4)\mathrm{grad}\phi \ . \qquad (5)$$

Then, without ambiguity, we can integrate the superfluid equation of motion along any path in the fluid from a quiescent point in the lower chamber to a similar point in the upper chamber to obtain

$$\phi_{20} - \phi_{10} = -\frac{1}{i\omega\hbar}(\mu_{20} - \mu_{10}) \ . \qquad (6)$$

Here $\mathbf{v}_s$ is the superfluid velocity, $\hbar$ is Planck's constant divided by 2π, and ϕ is the phase of the superfluid order parameter.

Finally, a small change in x gives rise to a proportionate small change in the frequency f of the rf oscillator, so that the corresponding amplitudes of oscillation x_0 and f_0 satisfy

$$x_0 = B f_0 , \tag{7}$$

where B is the constant of proportionality. By combining Eqs. (2), (6), and (7) we obtain a direct relationship between $\phi_{20}-\phi_{10}$ and f_0. The various apparatus parameters involved in this relationship may in principle be determined as follows. The quantity $(k/A)B$ can be obtained from static measurements of f versus cell pressure made with the aid of a pressure gauge external to the cryostat. The chamber volumes may be calculated from the cell dimensions. The angular frequency ω_T may be measured directly by exciting the cell when empty. The quantities α_1 and ω_H can then be determined from measurements of ω_+ and ω_- with the cell filled, using Eqs. (3) and (4) and the relationship $\alpha_1/\alpha_t = (\Omega_1+\Omega_2)/\Omega_1$.

The value $\delta\phi_c$ of $\phi_{20}-\phi_{10}$ at the critical rate of flow can be measured as follows. At subcritical rates of flow, the output voltage amplitude V_{r0} from the fm detector, which is proportional to f_0, will increase in proportion to the voltage amplitude V_{d0} applied to the driver. At resonance, the rate of increase of V_{r0} with V_{d0} will be relatively large, but it will remain finite due to the presence of small amounts of linear damping in the cell which have not been included in the equations. However, as V_{d0} is increased to the point where the flow through the tiny orifice becomes critical and additional, nonlinear dissipation begins to occur, a deviation from proportionality will set in, at a value of V_{r0} which we designate V_{r0c}. From V_{r0c}, the sensitivity of the fm detector, and Eqs. (2), (6), and (7), we can calculate the corresponding value of $\delta\phi_c$.

The foregoing expressions have been derived only for the low-temperature, pure-superfluid-component limit. However, a more complete analysis based on the two-fluid model shows that under our experimental conditions, with one possible exception, the expressions presented here can be used with negligible error up to temperatures at least as high as 2.0 K. The principal effect of the presence of the normal-fluid component is to introduce linear damping, which limits the Q values of the resonances. This situation follows from the relative openness of most of the main passages used in relation to the viscous penetration depth of the normal-fluid component. The one possible exception is provided by the 29-μm-diameter main passage, because of its smaller diameter.

IV. OBSERVATIONS AND RESULTS

An example of the rms response voltage $\tilde{V}_r = V_{r0}/\sqrt{2}$ measured as a function of the rms drive voltage $\tilde{V}_d = V_{d0}/\sqrt{2}$ is shown in Fig. 2, in which the onset of supercritical flow is clearly shown by an abrupt change in slope. Such curves were quite reproducible and independent of whether the data were taken as $\tilde{V}_d$ was being increased or decreased. As the temperature was increased, the quality factor Q of the resonance tended to decrease from values of 1000 or greater for temperatures less than 0.95 K, and the change in slope at the critical rate of flow tended to diminish. As a result, the transition, although remaining sharp, became less and less distinct until it could no longer be discerned. This phenomenon determined the upper temperature limit of our observations in any particular run, the lower limit being determined by the cooling capacity of our refrigerator. These limits varied somewhat from run to run as conditions were changed.

Our results for the temperature dependence of $\delta\phi_c$, derived from six separate runs, are plotted in Fig. 3. Addi-

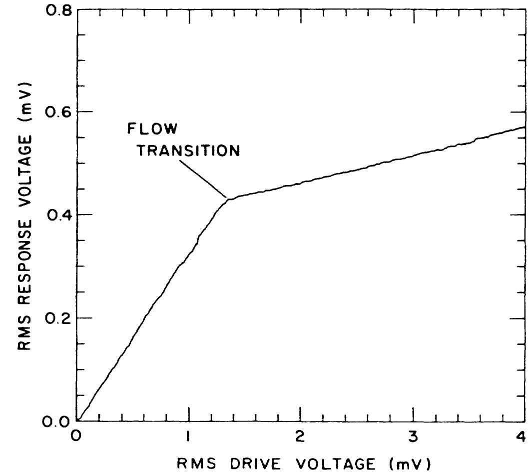

FIG. 2. An example of the dependence of the rms response voltage $\tilde{V}_r$ upon the rms drive voltage $\tilde{V}_d$ on resonance showing the transition from subcritical to supercritical flow in the tiny orifice. Run 6, $T=0.65$ K, $\nu_+ = 2009.3$ Hz.

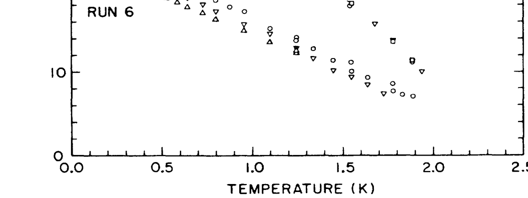

FIG. 3. The critical phase difference $\delta\phi_c$ in radians divided by 2π versus temperature T.

TABLE I. Results for the various runs at the lower (−) and upper (+) resonances for $T=0.95$ K and $P=25\pm2$ kPa. $\nu_{\pm}=\omega_{\pm}/2\pi$.

Run No.	ν_- (Hz)	ν_+ (Hz)	$\left[\frac{\delta\phi_c(\text{rad})}{2\pi}\right]_-$	$\left[\frac{\delta\phi_c(\text{rad})}{2\pi}\right]_+$	v_{sc-} (m/s)	v_{sc+} (m/s)
1	779.7		31.4		12.8	
2	1177.0		31.9		13.0	
3	601.6		31.0		12.7	
4	420.0		30.1		12.3	
5	1179.1	2853.4	17.0		6.9	
6	646.5	2007.8	15.7	15.0	6.4	6.1

tional information for these runs is given in Table I. The results fall into two groups, with values of $\delta\phi_c$ at any given temperature in one group differing from the values in the other group by very nearly a factor of 2. As shown in Table I, the data from the first four runs were all taken at the lower resonant frequencies, and, as illustrated in Fig. 3, are particularly consistent with each other, thus showing no frequency dependence for $\delta\phi_c$ over the range of frequencies involved. Run 4 is the run which used the 29-μm-diameter main passage. The data from the last two runs are reasonably consistent with each other. The data from run 6 were taken at both the lower and upper resonant frequencies and provided further evidence that no strong frequency dependence exists for $\delta\phi_c$. We have been unable to discover, however, any reason for the difference between the results of the two groups. Electron microscope examinations of the orifice before run 1 and after run 6 showed no significant change in the orifice other than a small decrease in the average diameter, from 180 to 160 nm, a change which may have occurred during one of the microscope examinations itself. In addition, checks of the relationship observed between $\tilde{V}_r$ and $\tilde{V}_d$ showed that no change occurred in the response of the cell to excitation throughout this work.

It was important to be certain that the critical effects observed were due to the tiny orifice and not the main passage. Indeed, further decreases in the slopes of curves of $\tilde{V}_r$ versus $\tilde{V}_d$ were always seen at higher response amplitudes, and we ascribed these to the onset of supercritical flow in the main passage. In two other runs without a tiny orifice there were no sharp breaks in the response curves at lower response amplitudes, such as that shown in Fig. 2, and the curves remained steep and linear up to the points where the decreases in slope at higher amplitudes set in.

The transition at lower amplitudes differed from the transition at higher amplitudes in several ways. First, as illustrated in Fig. 2, it was quite sharp and reproducible. At this transition the shape of the resonance, as observed when the drive frequency was varied at constant drive amplitude, changed from a symmetrical Lorentzian form to a flattened asymmetrical form whose peak shifted to lower frequency as the amplitude was increased. This shift in frequency was consistent with the idea that dissipation at the weak link was tending, in effect, to block it off. On the other hand, the transition at higher amplitude was often quite rounded and ill defined when viewed in a plot such as Fig. 2. Above that transition the response was often unreproducible and hysteretic, sometimes showing intermittent spontaneous transitions between two different response levels. Above the transition a further flattening of the resonance peak occurred, which, however, was symmetric about the resonant frequency.

The importance of the resonance of the lower diaphragm was appreciated only after run 4. As a result, measurements of the upper resonant frequency $\nu_+=\omega_+/2\pi$ were made only during runs 5 and 6, and our determinations of α_1 and α_t used in the analysis of all of the runs were based on these runs. For run 5 it proved impossible to find a solution for α_1 and α_t following the procedure described in Sec. III, in which the ratio $\alpha_1/\alpha_t=(\Omega_1+\Omega_2)/\Omega_1$ is regarded as known. In the face of this discrepancy, we proceeded as if the small volume Ω_1 were unknown. By use of $\nu_{\pm}=\omega_{\pm}/2\pi$ values from runs 5 and 6 measured at the same temperature and pressure we determined separate values for α_1 and α_t. At $T=0.95$ K and $P=25$ kPa these equaled 1.85 and 0.52, respectively. The ratio of α_1 to α_t was very nearly independent of temperature and for these values is 3.6, whereas the ratio $(\Omega_1+\Omega_2)/\Omega_1$ for the volumes computed from cell dimensions is 5.2. We do not understand the reason for this discrepancy, which could conceivably have a significant effect on our absolute values for $\delta\phi_c$ but is less likely to influence the temperature dependence.

Another sign that our model for the cell is inadequate was the appearance of other resonances of unknown origin which coupled with the principal two cell resonances. These weaker resonances, which were relatively temperature independent, were particularly troublesome above 2000 Hz. As a result we decided not to trust the measurements of $\delta\phi_c$ at ν_+ in run 5, which yielded $\delta\phi_c=13.6\times2\pi$ rad at $T=0.95$ K. Nevertheless, these data showed approximately the same linearity with temperature shown by the rest of our data.

Although the experiment yields the quantity $\delta\phi_c$ more directly than it does v_{sc}, the superfluid velocity averaged over the minimal cross-sectional area of the orifice at the critical rate of flow, we may estimate v_{sc} approximately by use of the formula

$$v_{sc}=\frac{\hbar}{m_4}\frac{\delta\phi_c}{l_{\text{eff}}}\,, \tag{8}$$

where l_{eff} is an effective hydrodynamic length for the orifice. For a long circular-cylindrical pore of length l and diameter d, l_{eff} is approximately equal to $l+8d/3\pi$; for a circular orifice in an ideally thin wall, $l_{\rm eff}$ equals $\pi d/4$. Adopting the former, and assuming $l = 100$ nm and $d = 170$ nm, we obtain the values for v_{sc} at 0.95 K listed in Table I.

V. CONCLUSION AND DISCUSSION

The principal result of this work is that all of our data for the order-parameter phase difference $\delta\phi_c$ at the critical rate of flow of superfluid ^{4}He through a single tiny orifice show that $\delta\phi_c$ decreases very nearly linearly with increasing temperature between 0.4 and 1.9 K. The lines which fit the data in Fig. 3 from runs 1–4 extrapolate to $\delta\phi_c = 0$ at 2.41 K, while those from runs 5 and 6 extrapolate to zero between 2.45 and 2.55 K.

This behavior is consistent with the critical velocity data of Hess for his filtered orifices between 1.3 and 1.8 K, insofar as his data are quite linear in temperature in that interval and extrapolate to zero at ~ 2.56 K.[5] Furthermore, a linear dependence of critical velocity on temperature has been observed by Schofield and Sanders between 0.3 and 1.0 K for their smallest orifices, although their results extrapolate to zero at ≤ 2.1 K.[15] To this extent our measurements provide support for a unification of results obtained previously in separate temperature regions.

At present, a theoretical understanding of the linear temperature dependence of our results over the entire range of observation is lacking. Approximate fits to the data may be obtained using the homogeneous nucleation theory of Langer and Fisher,[13] if the temperature variation of the ratio of the radius of the critical vortex ring to the vortex core parameter is taken into account. In making such fits we have used the relation between $\delta\phi_c$ and v_{sc} given by Eq. (8) and treated the quantity $\ln(f_0/f_L)$ of Langer and Fisher as a temperature-independent fitting parameter. We find that the critical vortex ring radii calculated are all small compared to the diameter of the orifice. The theoretical curves for $\delta\phi_c$, however, have significant curvature, upward below 1 K and downward above 1 K, and the agreement between them and the data is not satisfactory.

Roberts and Donnelly and Hess have pointed out that the temperature dependence predicted by the homogeneous nucleation theories will be modified by the presence of fluid boundaries and velocity nonuniformities.[9,22,23] In an extension of the work of Roberts and Donnelly, Donnelly, Hills, and Roberts have shown that when the free-energy barrier for thermal nucleation is of the form $E - pv_s$, where E and p are proportional to ρ_s but independent of v_s, it is possible to obtain a critical velocity which decreases approximately linearly with increasing temperature at sufficiently low temperature.[24] However, their model for such a free-energy barrier requires dimensional restrictions much smaller than the diameter of our orifice in order to yield critical velocities as large as ours.

Applying the ideas of Hess, we note that the superfluid velocity near the edge of our orifice might be a factor of 2 or so larger than the average velocity v_s in the orifice. As a result, when the critical radius of the vortex ring (or vortex arc, if nucleation is taking place at the wall) is appreciably smaller than the radius of the orifice, nucleation could occur at lower values of the average velocity in the orifice than would be the case for uniform flow. As the critical radius decreases with decreasing temperature, this effect should increase in importance, yielding a weaker temperature dependence for v_{sc} than predicted by homogeneous nucleation theories.

It remains to be seen, however, whether a model based on any of these effects can account for our results. Other effects which may need to be considered as the velocity becomes large, locally if not everywhere in the orifice, are the Landau critical velocity mechanism and the nucleation of vortices by quantum tunneling.

It is of course not clear whether our results are specific to one particular orifice or are more general. It will be of considerable interest to investigate the behavior of $\delta\phi_c$ for other orifices over as wide a range of temperature as possible. Indeed, linear behavior extending from 5 mK to 1.2 K and extrapolating to 0 at 2.46 K has just been reported for an orifice in the form of a rectangular slot 300 by 5000 nm in size in a foil 200 nm thick by Avenel and Varoquaux.[25]

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[1]B. J. Anderson, B. P. Beecken, and W. Zimmermann, Jr., in *Proceedings of the 17th International Conference on Low Temperature Physics*, edited by U. Eckern, A. Schmid, W. Weber, and H. Wühl (North-Holland, Amsterdam, 1984) Part I, p. 313. In that article, neither the role of the diaphragm resonance nor the temperature dependence of $\delta\phi_c$ as described in the present article were taken into account. When these factors are included in the analysis and $\delta\phi_c/2\pi$ and v_{sc} are referred to a common temperature of 0.95 K, Table 1 of that paper, expanded to include two additional entries, should read:

	Diameters (nm)	$\delta\phi_c$(rad)/2π	v_{sc} (m/s)
(no.1)	90×100	16	9
(no.2)	150×170	9	4
(no.3)	170×200	33	13
	240×240	32	10
	250×350	16	4
(no.4)	300×450	9	2
(no.5)	290×470	33	8

[2]P. L. Richards and P. W. Anderson, Phys. Rev. Lett. **14**, 540 (1965).
[3]B. M. Khorana, Phys. Rev. **185**, 299 (1969).
[4]P. L. Richards, Phys. Rev. A **2**, 1532 (1970).
[5]G. B. Hess, Phys. Rev. Lett. **27**, 977 (1971).
[6]J. P. Hulin, C. Laroche, A. Libchaber, and B. Perrin, Phys. Rev. A **5**, 1830 (1972).
[7]P. Leiderer and F. Pobell, Phys. Rev. A **7**, 1130 (1973).
[8]D. L. Musinski, J. Low Temp. Phys. **13**, 287 (1973).
[9]G. B. Hess, in *Low Temperature Physics-LT13,* edited by K. D. Timmerhaus, W. J. O'Sullivan, and E. F. Hammel (Plenum, New York, 1974) Vol. 1, p. 302.
[10]M. E. Banton, J. Low Temp. Phys. **16**, 211 (1974).
[11]G. B. Hess, Phys. Rev. B **15**, 5204 (1977).
[12]S. V. Iordanskii, Zh. Eksp. Teor. Fiz. **48**, 708 (1965) [Sov. Phys.—JETP **21**, 467 (1965)].
[13]J. S. Langer and M. E. Fisher, Phys. Rev. Lett. **19**, 560 (1967).
[14]J. S. Langer and J. D. Reppy, in *Progress in Low Temperature Physics, Vol. VI,* edited by C. J. Gorter (North-Holland, Amsterdam, 1970) Chap. 1.
[15]G. L. Schofield, Jr., Ph.D. thesis, University of Michigan, 1971 (available from University Microfilms International, Ann Arbor, MI 48106).
[16]B. Perrin, D. D'Humières, C. Laroche, J. P. Hulin, and A. Libchaber, Phys. Rev. Lett. **28**, 1551 (1972).
[17]O. Avenel and E. Varoquaux, Phys. Rev. Lett. **55**, 2704 (1985).
[18]B. P. Beecken, Ph. D. thesis, University of Minnesota, 1986.
[19]B. P. Beecken and W. Zimmermann, Jr., Phys. Rev. B **35**, 74 (1987).
[20]B. P. Beecken and W. Zimmermann, Jr., J. Vac. Sci. Technol. A **3**, 1839 (1985).
[21]I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).
[22]R. J. Donnelly and P. H. Roberts, Philos. Trans. R. Soc. London, Ser. A **271**, 41 (1971).
[23]G. B. Hess, Phys. Rev. Lett. **29**, 96 (1972).
[24]R. J. Donnelly, R. N. Hills, and P. H. Roberts, Phys. Rev. Lett. **42**, 725 (1979).
[25]O. Avenel, Bull. Am. Phys. Soc. **31**, 795 (1986); see E. Varoquaux, M. W. Meisel, and O. Avenel, Phys. Rev. Lett. **57**, 2291 (1986).