Macroscopic approach to the theory of one-dimensional disordered conductors

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We propose the following requirements that have to be satisfied by an ensemble of transfer matrices associated with disordered one-dimensional conductors of macroscopic length l: (1) flux conservation, (2) time-reversal invariance, (3) the appropriate combination law when two wires are put together, and (4) for samples of very small l, the average resistance is required to depend linearly in l and the statistical distribution is selected on the basis of a maximum information-entropy criterion. The present approach gives, for the resistance, results that are consistent with those of microscopic calculations.

I. INTRODUCTION

The probability distribution of the zero-temperature dc resistance of a one-dimensional conductor with static disorder has recently been studied by several authors.[1-9] When a microscopic approach is used, a statistical law is assumed for the potentials of the *individual* scatterers ("microscopic" quantities); that statistical law is then used to evaluate the various averages of interest. It is found, for instance, that the average resistance grows *exponentially* with $l=2L/L_c$, the length of the conductor measured in terms of the localization length. It is only in the limit $l \ll 1$ that a *linear* dependence, as in the ordinary Ohm's law, is recovered.

In Ref. 9 it was noted that many of the results do not depend on the details of the microscopic quantities, and the attempt was made to propose a statistical distribution directly for the transfer matrix R of the *full* conductor (a "macroscopic" quantity), by imposing on it a minimum set of physical requirements. In subsequent investigations[10,11] it became clear that such an approach can indeed be successful because an underlying central-limit theorem insures the stability of the results. In Ref. 9 we introduced a *combination requirement* for the statistical distribution of R that led to the scaling, with the length of the conductor, of certain "cumulantlike" quantities. However, the noncommutativity of the matrices R prevented us from scaling "cumulants" beyond the second one.

In the present paper we explicitly construct a statistical distribution that *fully* complies with the combination requirement. For this purpose, we first request that for small lengths a linear dependence of the average resistance with length (as in Ohm's law) be recovered; we then select, in that limit, the statistical distribution on the basis of a maximum-entropy criterion.[12,13]

In Sec. II we review some of the properties of the transfer matrices, as well as the combination requirement of Ref. 9. In Sec. III we show how the recovery of Ohm's law for $l \ll 1$, together with a maximum-entropy criterion, can be imposed. This leads to a diffusion equation for the probability distribution that is further studied in Sec. IV. In Sec. V we present a discussion of the results.

II. THE TRANSFER MATRIX AND THE COMBINATION REQUIREMENT

Consider the quantum-mechanical problem of scattering by a one-dimensional potential. The 2×2 transfer matrix[5] R relates the coefficients of the incoming-outgoing waves on the right of the potential to those on the left. It can be shown,[5] from *flux conservation* and *time-reversal invariance,* that R must be *pseudounitary,* i.e.,

$$R\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}R^{\dagger}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \tag{2.1}$$

and unimodular, i.e.,

$$\det R=1 \ . \tag{2.2}$$

A matrix R can be parametrized as[14,15]

$$R=\begin{bmatrix}\alpha & \beta\\ \beta^* & \alpha^*\end{bmatrix}, \quad |\alpha|^2-|\beta|^2=1 \tag{2.3a}$$

$$=\begin{bmatrix}e^{-i\mu} & 0\\ 0 & e^{i\mu}\end{bmatrix}\begin{bmatrix}\sqrt{1+\rho} & \sqrt{\rho}\\ \sqrt{\rho} & \sqrt{1+\rho}\end{bmatrix}\begin{bmatrix}e^{-i\nu} & 0\\ 0 & e^{i\nu}\end{bmatrix} . \tag{2.3b}$$

The parameters μ, ρ, and ν vary in the range

$$-\pi\le\mu,\nu<\pi, \quad 0\le\rho<\infty \ . \tag{2.3c}$$

The quantity ρ is the "dimensionless resistance" of the system,[1] i.e., its resistance measured in units of $\pi\hbar/e^2$. Calling T its transmission coefficient, we also have

$$-\ln T=\ln(1+\rho)\equiv z \ . \tag{2.4}$$

The collection of all pseudounitary and unimodular matrices R forms the group[14,15] SU(1,1), a noncompact group homomorphic to the Lorentz group SO(2,1). A collection or ensemble of random one-dimensional conductors of *macroscopic* (i.e., containing very many individual scattering centers) length l is then described by an ensemble of R matrices, whose differential probability we write as

$$dP(R)=p(R)d\mu(R) \ , \tag{2.5}$$

where $d\mu(R)$ is the invariant or Haar's measure[14] of

SU(1,1) (which remains invariant when all the R's are multiplied by a fixed one R_0) and $p(R)$ will be referred to as the "probability density." In terms of the parameters of Eq. (2.3b), $d\mu(R)$ can be written as[14]

$$d\mu(R)=(2\pi)^{-2}d\rho\, d\mu\, d\nu\ . \tag{2.6}$$

Our probability density will satisfy the two following requirements.

(i) We specify the type of material our wires are made of and consequently ask that the above probability density be only a function of R and the length l of the wires, i.e., $p_l(R)$.

(ii) The *combination requirement.* We first adopt the convention that the above R designates the transfer matrix of a wire of length l centered at the origin of coordinates. If that origin is displaced by the amount x_0, the new transfer matrix $\tilde{R}$ is related to R by [see Ref. 5, Eq. (3.31)]

$$\tilde{R}=U(x_0)RU^{-1}(x_0)\ , \tag{2.7}$$

where

$$U(x_0)=\begin{bmatrix} e^{ikx_0} & 0 \\ 0 & e^{-ikx_0} \end{bmatrix}\ , \tag{2.8}$$

k being the wave number of the electrons that are being scattered. Consider now a wire of length l divided in two subsystems of lengths l_1 and l_2, such that $l=l_1+l_2$ (see Fig. 1). If we denote by $\tilde{R}_1,\tilde{R}_2,\tilde{R}$ the transfer matrices (related to a common origin of coordinates 0) of systems 1,2 and the combined system, respectively, then we can write, approximately

$$\tilde{R}=\tilde{R}_2\tilde{R}_1\ . \tag{2.9}$$

Notice that under (2.9) resistances do *not* combine additively, but according to the more complicated law given in Eq. (3.5) below. If 0 is at the midpoint of the complete wire, (2.7) and (2.9) give (see Fig. 1)

$$R=[U(-l_1/2)R_2U(l_1/2)][U(l_2/2)R_1U(-l_2/2)]\ . \tag{2.10}$$

Notice that in each square bracket of (2.10) we have a mixture of the two indices, 1 and 2. However, (2.10) can also be written as

$$R'=R'_2R'_1\ , \tag{2.11}$$

where

$$R'=U(l/2)RU(l/2)\ , \tag{2.12a}$$

$$R'_2=U(l_2/2)R_2U(l_2/2)\ , \tag{2.12b}$$

$$R'_1=U(l_1/2)R_1U(l_1/2)\ . \tag{2.12c}$$

Since R and U belong to SU(1,1), so does R'. In the notation of Eq. (2.3a), $\alpha'=\alpha e^{ikl}$ and $\beta'=\beta$; since these relations can be trivially used to translate to R all the conclusions that we shall find below for R', from now on we drop the prime from R [in particular, in Eq. (2.11)], in order to simplify the notation.

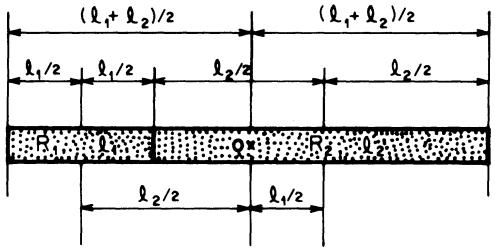

FIG. 1. A conductor of length l divided in two subsystems of lengths l_1,l_2 and transfer matrices (measured with respect to the corresponding midpoints) R_1,R_2.

Let us now go back to our ensemble of wires of length l. Imagine each wire divided into two subsystems as explained above. We assume that R_1 and R_2 are statistically independent. From a microscopic point of view, this could be achieved if the macroscopic quantities l_1,l_2 are large compared with the correlation length of the electronic potential.

From (2.11) (without primes) and the above assumed statistical independence, we can thus relate the probability density $p_l(R)$ of the whole system to the $p_{l_1}(R_1)$ and $p_{l_2}(R_2)$ associated with the two subsystems, according to the convolution[16]

$$p_l(R)=\int p_{l_2}(RR_1^{-1})p_{l_1}(R_1)d\mu(R_1)\ . \tag{2.13}$$

Equation (2.13) will be called the *combination requirement.* The operation involved in (2.13) is the generalization to the present group of the elementary idea of convoluton. We can arrive at (2.13) in the following way. Let $R=R_2R_1$. The average of any function $f(R)$ can be written as

$$\begin{aligned}\langle f(R)\rangle &= \int\int f(R_2R_1)dP_{l_1}(R_1)dP_{l_2}(R_2) \\ &= \int d\mu(R_1)p_{l_1}(R_1) \\ &\quad\times\int d\mu(R_2)p_{l_2}(R_2)f(R_2R_1)\ .\end{aligned} \tag{2.14}$$

The last integral in (2.14) is over R_2 for fixed R_1. We can write $R_2=RR_1^{-1}$ and integrate over R for fixed R_1. The measure is then $d\mu(R_2)=d\mu(RR_1^{-1})=d\mu(R)$, using the invariance property. We then have

$$\begin{aligned}\langle f(R)\rangle &= \int d\mu(R_1)p_{l_1}(R_1)\int d\mu(R)p_{l_2}(RR_1^{-1})f(R) \\ &= \int d\mu(R)f(R)\int d\mu(R_1)p_{l_2}(RR_1^{-1})p_{l_1}(R_1)\ .\end{aligned} \tag{2.15}$$

The last equation defines the differential probability

$$dP_l(R)=d\mu(R)\int d\mu(R_1)p_{l_2}(RR_1^{-1})p_{l_1}(R_1)\ , \tag{2.16}$$

in terms of which $\langle f(R)\rangle$ can be expressed as

$$\langle f(R)\rangle=\int f(R)dP_l(R)\ . \tag{2.17}$$

From (2.16), the probability density (2.13) follows. The convolution of two functions as defined in (2.13) is, in general, noncommutative if the group is not Abelian.[16] In the present problem, however, the resulting $p_l(R)$,

$l=l_1+l_2$, should be the same, no matter in which order we put together the two wires of lengths l_1 and l_2. In other words, we must have, from (2.13),

$$p_l(R)=\int p_{l_2}(RR_1^{-1})p_{l_1}(R_1)d\mu(R_1) \tag{2.18a}$$

$$=\int p_{l_1}(RR_2^{-1})p_{l_2}(R_2)d\mu(R_2) \tag{2.18b}$$

$$=\int p_{l_2}(R_1^{-1}R)p_{l_1}(R_1)d\mu(R_1)\ . \tag{2.18c}$$

Equation (2.18b) was obtained from (2.18a) by reversing l_1 and l_2 and changing the variable of integration from R_1 to R_2. Notice that, whereas in (2.18a) we have $R=R_2R_1$, in (2.18b) we have $R=R_1R_2$, indicating that the two pieces are combined in the opposite order. Equation (2.18c) was obtained from (2.18b) with the change of variable $R_1=RR_2^{-1}$; use is then made of the invariance property of Haar's measure, which implies $d\mu(R_1)=d\mu(R_2^{-1})$, for any fixed R. Finally, $d\mu(R_2^{-1})=d\mu(R_2)$ follows from the explicit expression (2.6) of Haar's measure and the relation

$$\{\mu,\rho,\nu\}\rightarrow\{-\nu-\pi/2,\rho,-\mu+\pi/2\} \tag{2.19}$$

between the parameters of a matrix of the group and those of its inverse.

Notice that RR_1^{-1} appears in (2.18a), whereas $R_1^{-1}R$ occurs in (2.18c). If the group were Abelian, the two expressions would be identical. In the present case, Eqs. (2.18) represent a requirement.

III. A DIFFUSION EQUATION FOR THE PROBABILITY DISTRIBUTION

In Eq. (2.13) we set $l_2=l$, $l_1=\delta l$, and consider δl as a small, but still macroscopic, length. We then have

$$p_{l+\delta l}(R)=\int p_l(RR_1^{-1})p_{\delta l}(R_1)d\mu(R_1)\ . \tag{3.1}$$

We see that $p_{\delta l}(R_1)$ plays the role of a "transition probability," in a way analogous to what occurs with the Smoluchowsky equation in connection with Markovian processes.[17,18]

This transition probability $p_{\delta l}(R_1)$ is now our building block. We request that it guarantee the recovery of Ohm's law as $\delta l\rightarrow 0$. In other words, calling ρ_1 the resistance, we request

$$\langle\rho_1\rangle_{\delta l}=\delta l\ , \tag{3.2}$$

where δl is measured in suitable units, so that the proportionality factor in (3.2) is unity. We will come back to this later.

Condition (3.2) is certainly not sufficient to determine $p_{\delta l}(R_1)$ uniquely. We shall show, however, that the ansatz obtained by *maximizing Shannon's information entropy*[12,13]

$$\mathcal{S}[p_{\delta l}]\equiv-\int p_{\delta l}(R_1)\ln[p_{\delta l}(R_1)]d\mu(R_1)\ , \tag{3.3}$$

constrained by (3.2), occupies a very special place, since the results that arise from it coincide with those of microscopic calculations. This suggests that the latter really contain the same physical information as the present macroscopic analysis.

The above-mentioned maximization yields the result[12]

$$p_{\delta l}(R_1)=\frac{e^{-\rho_1/\delta l}}{\delta l}\ . \tag{3.4}$$

Qualitatively speaking, we can say that we have constructed, for conductors of very small but still macroscopic length δl, an ensemble $p_{\delta l}(R_1)$ that is "as random as it is allowed" by the conditions of flux conservation, time-reversal invariance, and the linear dependence (3.2) of the average resistance with δl. In the absence of any other constraint, our ansatz (3.4) is thus *isotropic* in the angles μ_1,ν_1 of the preceding section [Eq. (2.3b)]. Additional requirements are needed in order to further restrict the choice of $p_{\delta l}(R_1)$ and thus provide a better treatment of the phases.

We now proceed to derive from (3.1) and (3.4) a "Fokker-Planck" equation for $p_l(R)$. Equation (3.1) implies that $p_l(R)$ can be obtained by successive convolutions of $p_{\delta l}$ with itself. It is proved in Ref. 10 that the convolution of two isotropic functions is again isotropic, so that the various probability densities occurring in (3.1) are functions of ρ only. One can easily prove[10] that if two group elements are multiplied, i.e.,

$$R(\mu,\rho,\nu)R'(\mu',\rho',\nu')=R''(\mu'',\rho'',\nu'')\ ,$$

then

$$\rho''=\rho+\rho'+2\rho\rho'+2[\rho(1+\rho)\rho'(1+\rho')]^{1/2}\cos[2(\nu+\mu')]\ . \tag{3.5}$$

Using (3.5), (2.19) and (2.6) in (3.1), we then find the explicit expression for the convolution

$$p_{l+\delta l}(\rho)=\int_0^\infty d\rho_1 p_{\delta l}(\rho_1)\int_{-\pi}^{\pi}\frac{d\nu_1}{2\pi}p_l\{\rho+\rho_1+2\rho\rho_1-2[\rho_1(\rho_1+1)\rho(\rho+1)]^{1/2}\cos(2\nu_1)\}\ . \tag{3.6}$$

If we interchange R_1 and R in the $p_l(RR_1^{-1})$ of (3.1), we obtain (3.6) with ν_1 replaced by μ_1, thus giving back the same result. For isotropic functions, the equality of (2.18a) and (2.18c) is therefore automatic.

We now expand both sides of (3.5) in powers of δl and eventually take the limit $\delta l\rightarrow 0$. The left-hand side gives

$$p_{l+\delta l}(\rho)=p_l(\rho)+\frac{\partial p_l(\rho)}{\partial l}\delta l+\cdots\ . \tag{3.7}$$

On the right-hand side (RHS) of (3.6) ρ_1 will effectively be small, because of (3.4), so that we expand the p_l occurring in the last integral of (3.6) in powers of $x=[\rho_1(\rho_1+1)]^{1/2}$. Denoting by $g(x)$ the argument of p_l, we have

$$p_l(g(x))=p_l(\rho)-\{2p_l'(\rho)[\rho(\rho+1)]^{1/2}\cos(2\nu_1)\}[\rho_1(\rho_1+1)]^{1/2}$$
$$+[2p_l''(\rho)\rho(\rho+1)\cos^2(2\nu_1)+p_l'(\rho)(1+2\rho)]\rho_1(\rho_1+1)+O((\rho_1(\rho_1+1))^{3/2}) . \quad (3.8)$$

The integral over ν_1 in (3.6) eliminates the second term of (3.8), so that the RHS of (3.6) becomes

$$\int_0^\infty d\rho_1 p_{\delta l}(\rho_1)\int_{-\pi}^{\pi}\frac{d\nu_1}{2\pi}p_l(g(x))$$
$$=p_l(\rho)+[p_l''(\rho)\rho(\rho+1)+p_l'(\rho)(1+2\rho)]\langle\rho_1\rangle_{\delta l}$$
$$+\langle O(\rho_1^2)\rangle_{\delta l} . \quad (3.9)$$

The term in (3.8) containing $[\rho_1(\rho_1+1)]^{3/2}$ vanishes upon averaging over ν_1, just as the second term does. That is why the last term in (3.9) is $\langle O(\rho_1^2)\rangle$.

Our ansatz (3.4) implies

$$\langle\rho_1^k\rangle=\Gamma(k+1)(\delta l)^k , \quad (3.10)$$

so that, combining (3.10) with (3.9), equating the result with (3.7), and taking the limit $\delta l\to 0$, we obtain the "Fokker-Planck, or diffusion-type" equation

$$\frac{\partial p_l(\rho)}{\partial l}=\frac{\partial}{\partial\rho}\left[\rho(\rho+1)\frac{\partial p_l(\rho)}{\partial\rho}\right] . \quad (3.11)$$

According to (3.4), the differential equation (3.11) has to be solved with the initial condition

$$p_0(\rho)=\delta(\rho) , \quad (3.12)$$

where $\delta(\rho)$ is the "one-sided" δ function that satisfies $\int_0^\infty\delta(\rho)d\rho=1$. From (3.11) we can easily find the form of the solution for $l\ll 1$, i.e., when $p_l(\rho)$ is still concentrated near $\rho=0$. We can then approximate (3.11) as

$$\frac{\partial p_l}{\partial l}\approx\frac{\partial}{\partial\rho}\rho\frac{\partial p_l}{\partial\rho} . \quad (3.13)$$

We can easily check that

$$p_l(\rho)=\frac{e^{-\rho/l}}{l} \quad (3.14)$$

is the solution of (3.13) that satisfies the initial condition (3.12). Result (3.14) is thus consistent with our initial ansatz (3.4).

We notice that the diffusion equation (3.11) is identical to the one found in Ref. 10 as a result of a central-limit theorem on the group SU(1,1), thus giving an interpretation of the latter from the standpoint of a maximum-entropy criterion. Equation (3.11) also coincides with the one obtained by Kumar in Ref. 6, where the potential "felt" by the electron is related to a Gaussian random process and with that found in Refs. 7 and 8. Some properties of the diffusion equation (3.11) will be studied in the following section.

IV. PROPERTIES OF THE DIFFUSION EQUATION

We can follow Ref. 6 and multiply both sides of (3.11) by ρ^n and integrate over ρ. This gives a recursion relation for the moments of ρ. For $n=0,1,2$ one has

$$\langle 1\rangle=1 , \quad (4.1)$$

$$\langle\rho\rangle=\tfrac{1}{2}(e^{2l}-1) , \quad (4.2)$$

$$\langle\rho^2\rangle=\tfrac{1}{12}(2e^{6l}-6e^{2l}+4) . \quad (4.3)$$

Equation (4.1) is the normalization condition, independent of l. Equation (4.2) is the well-known exponential increase of the resistance with length,[1-3] normally obtained from a "microscopic" approach. Comparing (4.2) with the results of Refs. 1–3, we find that l must be the length L of the conductor measured in terms of the localization length L_c

$$l=2\frac{L}{L_c} . \quad (4.4)$$

Equation (4.3) indicates that the width of the distribution of ρ increases more rapidly than the mean.

If we multiply both sides of Eq. (3.11) by z^n [where z is defined in Eq. (2.4)] and integrate, we have

$$\frac{\partial\langle z^n\rangle_l}{\partial l}=n(n-1)\left\langle\frac{\rho}{\rho+1}z^{n-2}\right\rangle_l+n\langle z^{n-1}\rangle_l . \quad (4.5)$$

In particular, we have, for $n=1$,

$$\frac{\partial\langle z\rangle_l}{\partial l}=1 . \quad (4.6)$$

The solution to (4.6) that has the property $\langle z\rangle_0=0$ is

$$\langle z\rangle_l=l, \quad 0\le l<\infty . \quad (4.7)$$

The variable z of (2.4) thus scales *exactly* with the length of the conductor.

For $n\ge 2$, the first term on the RHS of (4.5) gives a nonzero contribution and its value is unknown. We can, however, find an approximate expression valid when $l\gg 1$; indeed, the distribution is then pushed to large ρ, implying $\rho/(\rho+1)\approx 1$, so that we obtain the recursion relation

$$\frac{\partial\langle z^n\rangle_l}{\partial l}=n(n-1)\langle z^{n-2}\rangle_l+n\langle z^{n-1}\rangle_l, \quad l\gg 1 . \quad (4.8)$$

It is proved in the Appendix that (4.8) implies for the central moments

$$\mu_n=\langle(z-\bar{z})^n\rangle , \quad (4.9)$$

the recursion relation

$$\frac{\partial\mu_n}{\partial l}=n(n-1)\mu_{n-2} , \quad (4.10)$$

which connects only even, or only odd moments among themselves.

Starting from $\mu_0=1$, we easily see that (4.10) gives, for the term with the highest l-power in μ_{2k}, the expression

$$\mu_{2k}=\frac{(2k)!}{k!}l^k . \tag{4.11a}$$

We can easily prove by induction that $(2k)!/k!=(2k-1)!!2^k$, so that

$$\mu_{2k}=(2k-1)!!(2l)^k . \tag{4.11b}$$

Similarly, starting from $\mu_1=0$, we get $\mu_3=c$, where c is a constant of integration and, for μ_{2k+1}, the leading term

$$\mu_{2k+1}=\frac{(2k+1)!}{(k-1)!}\frac{c}{6}l^{k-1} . \tag{4.12}$$

If we measure μ_{2k+1} in units of $[(\mu_2)^{1/2}]^{2k+1}$, we thus get

$$\mu_{2k+1}/\mu_2^{(2k+1)/2}\sim l^{-3/2}\underset{l\to\infty}{\longrightarrow}0 .$$

This result, together with (4.11), thus proves that, in the limit $l\to\infty$, $(z-l)/\sqrt{2l}$ tends to a Gaussian variable with zero mean and variance 1. This property coincides with the results of Refs. 2 and 3.

Our fundamental diffusion equation (3.11) can actually be solved by quadratures. The hypergeometric functions

$$f_s(\rho)\equiv{}_2F_1(\tfrac{1}{2}+is,\tfrac{1}{2}-is;1;\rho) \tag{4.13}$$

satisfy the differential equation[14]

$$\frac{\partial}{\partial\rho}\left[\rho(1+\rho)\frac{\partial f_s(\rho)}{\partial\rho}\right]+(s^2+\tfrac{1}{4})f_s(\rho)=0 \tag{4.14}$$

and can be shown[14] to form a complete set in terms of which any function of ρ can be expanded; the measure to be used in the variable s is

$$w(s)=2s\tanh\pi s . \tag{4.15}$$

It is then easy to show that the function

$$p_l(\rho)=\int_0^\infty f_s(\rho)e^{-(s^2+1/4)l}w(s)ds \tag{4.16}$$

satisfies the differential equation (3.11). As for the initial condition, (4.16) gives

$$p_0(\rho)=\int_0^\infty f_s(\rho)w(s)ds . \tag{4.17}$$

Equation (4.17) is the expansion of $p_0(\rho)$ in terms of the complete set $f_s(\rho)$, the expansion coefficient being 1. But this means that

$$\int_0^\infty p_0(\rho)f_s(\rho)d\rho=1 . \tag{4.18}$$

We know, from (4.13), that $p_0(\rho)=\delta(\rho)$ [as in Eq. (3.12)] satisfies (4.18). From the uniqueness of the expansion we thus conclude that (4.17) coincides with $\delta(\rho)$. Therefore, our integral representation (4.16) satisfies (3.11) and (3.12).

We just mention in passing that the $f_s(\rho)$ of (4.13) is the $m=m'=0$ matrix element of a unitary irreducible representation of the group SU(1,1) belonging to the continuous class.[14,15] The solution (4.16) is thus the harmonic expansion[10,15] of $p_l(\rho)$ on SU(1,1).

For $l\gg1$ the integral (4.16) can be evaluated[10] using the saddle-point method, obtaining as a result the Gaussian distribution mentioned right after Eq. (4.12).

V. CONCLUSIONS

In this paper we have proposed the following set of conditions that have to be fulfilled by an ensemble $p_l(R)$ of transfer matrices associated with disordered one-dimensional conductors of length l.

(i) Flux conservation.
(ii) Time-reversal invariance of the electron Hamiltonian.
(iii) The combination requirement.
(iv) For samples of very small length δl, the average resistance is requested to depend linearly on δl.

Condition (iv) does not determine the statistical distribution of our "building block" $p_{\delta l}(R_1)$ uniquely, so that a maximum information-entropy criterion was invoked in the text.

The results so obtained for the resistance are consistent with those of microscopic calculations. This agreement suggests that, as far as the resistance goes, the microscopic calculations contain the same physical information as the present macroscopic approach.

It would be extremely interesting if one could find a set of physical conditions that, added to (iv), determined $p_{\delta l}(R_1)$ uniquely. In problems coming from other fields of physics[13] we have occasionally succeeded in finding such physical requirements. Should that be possible here, one would have an alternative approach to the one that employs the criterion of maximum entropy.

The analysis presented in this paper is so simple that it would be worthwhile trying to generalize it to treat samples with many channels.[19] Since a detailed knowledge of the group SU(1,1) was not needed in the present study, we expect a multichannel generalization to be feasible.

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APPENDIX: DERIVATION OF THE RECURSION RELATION (4.10)

We can write the central moment μ_n of Eq. (4.9) in terms of the moments $\langle z^m\rangle$ as

$$\mu_n=\sum_m\begin{bmatrix}n\\m\end{bmatrix}\langle z^m\rangle(-\bar{z})^{n-m}$$

$$=\sum_m(-1)^{n-m}l^{n-m}\begin{bmatrix}n\\m\end{bmatrix}\langle z^m\rangle , \tag{A1}$$

where (4.7) was used. Conversely, one can express $\langle z^k\rangle$ in terms of the central moments as

$$\langle z^k\rangle=\sum_r l^{k-r}\begin{bmatrix}k\\r\end{bmatrix}\mu_r . \tag{A2}$$

Differentiating with respect to l both sides of (A1), using (4.8) for $\partial\langle z^m\rangle/\partial l$ and then expressing all the $\langle z^k\rangle$ in terms of central moments through (A2), we have

$$\frac{\partial \mu_n}{\partial l} = \sum_r \frac{l^{n-r-1}}{r!}\mu_r \sum_m (-1)^{n-m}\frac{n!}{(n-m-1)!(m-r)!} + \sum_r \frac{l^{n-r-2}}{r!}\mu_r \sum_m (-1)^{n-m}\frac{n!}{(n-m)!(m-r-2)!} + \sum_r \frac{l^{n-r-1}}{r!}\mu_r \times \sum_m (-1)^{n-m}\frac{n!}{(n-m)!(m-r-1)!} . \quad \text{(A3)}$$

One can easily show the relations

$$\sum_m \frac{(-1)^{n-m}}{(n-m-1)!(m-r)!} = -\delta_{r+1,n} , \quad \text{(A4)}$$

$$\sum_m \frac{(-1)^{n-m}}{(n-m)!(m-r-2)!} = \delta_{r+2,n} , \quad \text{(A5)}$$

$$\sum_m \frac{(-1)^{n-m}}{(n-m)!(m-r-1)!} = \delta_{r+1,n} , \quad \text{(A6)}$$

which, substituted in (A3), give the required recursion relation (4.10).

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