Quantum oscillations in the magnetotransport of a finite two-dimensional Anderson model

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A formulation of the Hall effect in a strongly disordered system is presented, employing a Landauer-type picture. With use of this expression, the Hall effect and the magnetoconductance of a small, strongly disordered system of the type used by Nguyen, Spivak, and Shklovskii are analyzed, and a condition is derived for quantized values of the former.

I. INTRODUCTION

The modification of electronic phase relationships and the associated interference effects by magnetic fields is by now known to play an important role in the electronic transport in disordered conductors. These effects are quantitatively understood in the weakly disordered or weak localization regime (for recent reviews, see Bergmann[1] and Lee and Ramakrishnan[2]). However, there is no such understanding of magnetotransport in the strongly disordered regime where conduction occurs by hopping among localized states or by tunneling through barriers. In particular, one would like to have an understanding, in the fully quantum regime, of exactly what corresponds to the Lorentz force, which leads to the usual elementary picture of the Hall effect. The only model which addresses this issue is the one by Holstein and collaborators[3] for the Hall effect in the phonon-induced hopping regime. It should be of interest to have a theory for the magnetoconductance and the Hall effect employing a Landauer[4] picture, as well as its multichannel generalization.[5] In this picture (see Fig. 1), an arbitrary piece of a disordered system forms a barrier between two ideally conducting leads and the conductance is expressed in terms of the transmission coefficient (matrix of such coefficients in the multichannel case) through the system. In this representation all the scattering inside the sample is elastic (see, however, Büttiker[6]) and the inelastic scattering and dissipation occur in the outside electron reservoirs to which the leads are connected. Thus, this picture is ideally suited for calculating the purely quantum transport in a segment of a system in which essentially no inelastic scattering occurs. A generalization of this formulation to thermoelectric transport has recently been presented by Sivan and Imry.[7] The Landauer-type picture has been extremely useful in order to obtain interesting quantum interference effects such as the h/e periodic Aharonov-Bohm magnetoresistance oscillations in a small ring,[8] as well as the aperiodic fluctuations of the magnetoconductance in small wires.[9]

In this paper the issue of magnetotransport in a strongly disordered model is addressed. We present the formulation of the Hall effect in the Landauer picture. We believe that this theory should be useful in the context of the quantized Hall effect.[10] We apply our expression for the Hall effect to the model recently introduced by Nguyen, Spivak, and Shklovskii.[11] (Henceforth abbreviated by NSS.) These authors studied the Anderson model on a square lattice containing magnetic flux. They calculated the transmission between two opposite corners of the square and found that it oscillates with a period equal either to the normal flux quantum, $\phi_0=ch/e$, or to the "superconducting," i.e., two-electron-flux quantum, $\phi_0/2=ch/2e$. NSS have assumed that the magnetic flux is confined to a hole in the center of the square. Thus, their model attempts to mimic, except for treating the strongly localized regime, the Aharonov-Bohm cylindrical or ring configuration.[8,12,13] However, one may also study the behavior of the transmission and hence of the magnetoconductance, and the Hall-type effect, when the mag-

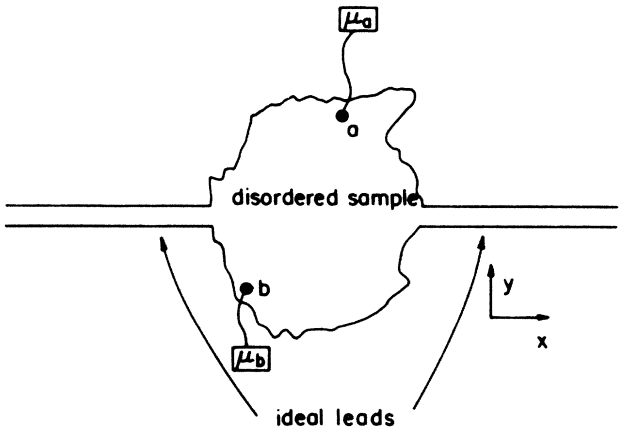

FIG. 1. Landauer-type geometry for calculating the chemical potential at sites a and b.

netic field is applied uniformly to the whole sample, as done in the following.

In Sec. II, we derive an expression for the Hall voltage of a disordered system, in terms, as typically done in the experiment, of the chemical potential difference perpendicular to the current driven through the sample. In Sec. III we study the NSS model in the presence of a uniform magnetic field, when an external current is flowing through the system. We focus on the smallest possible structure that shows the magnetic field effects and solve it analytically. The solution exhibits oscillations in the magnetoconductance and in the Hall-type resistance, with period equal to the normal flux quantum ch/e. It also reveals a special condition under which the transmission of the system is unity (namely, the resistance vanishes) and at the same time the Hall conductance equals $ne^2/\pi h$ (n being an integer). Finally, a combination of our solution and an effective medium argument reproduces a result resembling the main result of NSS—a transition in the transmission oscillation period between a normal and superconducting flux quantum. However, in our model the latter period is obtained only at a single point. Section IV includes a discussion and possible extensions of the model.

II. AN EXPRESSION FOR THE HALL VOLTAGE IN A QUANTUM CONDUCTOR

Here we derive a general expression for the chemical potential difference between two sites on a system through which a current is flowing. This will be used to obtain the Hall voltage, perpendicular to the current. We consider a general Laudauer-type[4] geometry, depicted in Fig. 1, where the arbitrarily complex system forms the barrier between two ideal single-channel wires that are along the x direction. The assumption of single channel leads is done only for definiteness. Our main result [Eq. (5)] is also easily generalizable for an arbitrary configuration of current feeding.

The wires in Fig. 1 are connected to electron reservoirs with chemical potentials μ_1 and μ_2 on the left-hand side (LHS) and the right-hand side (RHS), respectively. Denoting by T the transmission coefficient through the system (at energies around μ_1 or μ_2—remembering that for linear transport $\mu_1-\mu_2$ is very small) the net current in the wires is

$$I_x = en_0 v_F(\mu_1-\mu_2)T = \frac{e}{\pi\hbar}T(\mu_1-\mu_2)\ , \tag{1}$$

where v_F is the Fermi velocity and $n_0=1/\pi\hbar v_F$ is the density of states in the wires, including spin. We recall that (1) may be obtained by considering the scattering states coming from the left. The current is due to those states having energies between μ_2 and μ_1, of which there are $n_0(\mu_1-\mu_2)$, each carrying a current $ev_F T$, hence Eq. (1). The well-known Landauer formula for the conductance

$$G=\frac{e^2}{\pi\hbar}\frac{T}{1-T}\ , \tag{2}$$

is obtained by dividing (1) by the voltage difference V_{LR} between the LHS and the RHS wires, given by

$$\mu_L-\mu_R = eV_{LR} = (1-T)(\mu_1-\mu_2)\ . \tag{3}$$

An instructive derivation of Eq. (3) was given by Engquist and Anderson.[14] They defined μ_L and μ_R (which they denoted by μ_A and μ_B, respectively; note that here a and b will be used for something else) by weakly connecting measurement reservoirs to the wires and adjusting their chemical potentials so that zero currents flow between each measurement reservoir and the corresponding wire. The values thus obtained for these chemical potentials are *defined* as μ_L and μ_R. This definition is clearly valid for equilibrium states; it is a natural extension for nonequilibrium, current-carrying ones, and it mimics the four-terminal experimental measurement in the latter case.

The usual derivations of the Landauer formula count electrons in plane-wave states on the wires. It is also possible to use the exact scattering states. There are two kinds of these: ψ^l (ψ^r) coming from the left (right) and having amplitudes 1 in the moving to the right (left) plane-wave states on the LHS (RHS), r (r') in the reflected, moving to the left (right), states on the LHS (RHS), and t (t') in the transmitted, moving to the right (left), states on the RHS (LHS). The ψ^l states are taken to be filled up to μ_1 and the ψ^r states are filled up to μ_2. We now choose two "sites," a and b, on the system itself. (These can be tight-binding sites or other objects such as pieces of ideal wires, etc.) The isolated sites have wave functions ϕ_a and ϕ_b, respectively. We take the scattering states ψ^l (ψ^r) to have amplitudes ψ^l_a and ψ^l_b (ψ^r_a and ψ^r_b) around the sites. In other words, the wave function of the state ψ^l (ψ^r) is $\psi^l_a\phi_a$ ($\psi^r_a\phi_a$) on the a site and $\psi^l_b\phi_b$ ($\psi^r_b\phi_b$) on the b site.

We now derive an expression for the chemical potential difference between the sites a and b in terms of the scattering states $\psi^{l,r}$. As a check on the procedure, it is straightforward to derive Eq. (2) using the ψ^l,ψ^r states. The only precaution is, if the measurement reservoir idea[14] is used, that the coupling matrix element squared of the measurement reservoir to ψ^l is proportional to $|t|^2$ on the RHS and to $1+|r|^2$ on the LHS. The interference term, proportional to r, can be ignored if the measurement is done over a segment of the wire much longer than a wavelength. Similar precautions should be necessary if a similar spatial averaging is employed in the Hall case.

To define the chemical potential on site a, we connect to it a measurement reservoir with density of states n_r chemical potential μ_a and with a small coupling matrix elements squared $\epsilon|\psi^l_a|^2$ and $\epsilon|\psi^r_a|^2$ to the states coming from the left and right, respectively. Assuming (as will turn out to be the case) that $\mu_2\le\mu_a\le\mu_1$, each full reservoir state between μ_2 and μ_a can decay into the empty states at a, coming from the right, with a rate, given by the golden rule, of $\epsilon|\psi^r_a|^2 n_0$, where n_0 is the density of states of each lead wire. Multiplying this rate by the number of such reservoir states, $n_r(\mu_a-\mu_2)$ gives the current from the reservoir to a. Similarly, the current from the full left states at a into the reservoir is given by the rate $\epsilon|\psi^l_a|^2 n_r$, multiplied by the number of such states, $n_0(\mu_1-\mu_a)$. The condition of zero current between a and the measurement reservoir is obtained by equating these two currents and yields

$$\mu_a=\frac{\mu_1|\psi_a^l|^2+\mu_2|\psi_a^r|^2}{|\psi_a^l|^2+|\psi_a^r|^2}\ . \tag{4}$$

Likewise, one can define μ_b and obtain an expression analogous to (4) with a interchanged with b. We then find,

$$\mu_a-\mu_b=(\mu_1-\mu_2)\times\frac{|\psi_a^l|^2|\psi_b^r|^2-|\psi_b^l|^2|\psi_a^r|^2}{(|\psi_a^r|^2+|\psi_a^l|^2)(|\psi_b^r|^2+|\psi_b^l|^2)}\ . \tag{5}$$

We note that this result is different from simple expressions such as $(\Delta n_a-\Delta n_b)/n_0$, which one might be tempted to introduce, invoking a naive Einstein-type formula. To do that correctly, one has to find the dependence of μ_a on a density increment Δn_a. One way to do this[15] is by changing $\mu=\mu_1=\mu_2$ at equilibrium and finding $\Delta n_a/\Delta\mu$. This procedure leads, in fact, to Eq. (5). We emphasize the generality and potential usefulness of the result (5) (with appropriate averaging over a length of several electron wavelengths, if necessary) to calculate chemical potential differences in various directions and along various segments of any current-carrying system. In particular, no assumptions of symmetry or homogeneity are called for. In the following section, we employ Eq. (5) to derive the Hall-type voltage for a specific model.

III. ANDERSON-NSS MODEL FOR A TWO-DIMENSIONAL STRUCTURE

We consider a square lattice described by the Hamiltonian

$$\mathcal{H}=\sum_i E_i c_i^\dagger c_i+\sum_{i,j} V_{ij}c_i^\dagger c_j\ , \tag{6}$$

where c and $c^\dagger$ are the destruction and creation operators for the electrons and E_i are randomly distributed site energies. We assume that $V_{ij}=V$ for nearest-neighbor sites and zero otherwise. A constant magnetic field applied normal to the lattice imposes phase factors upon the transfer integrals V_{ij}

$$V_{ij}\rightarrow V_{ij}\exp\left[i\frac{e}{2c\hbar}(\mathbf{H}\times\mathbf{R}_i)\cdot\mathbf{R}_j\right]\ , \tag{7}$$

where $\mathbf{R}_i$ are the radius vectors to the lattice sites. The total phase change due to H around any closed loop is $2\pi/\phi_0$ times the flux enclosed in that loop.

To describe the external current driving the system we adopt the configuration proposed by NSS,[11] depicted in Fig. 2(a). We assume that the current leads connected to opposite corners of the system consist of tight-binding chains with equal site energies and transfer integrals.

In principle, this structure can be solved exactly. We study in this section the smallest possible system, consisting of a square of four sites [see Fig. 2(b)].

Suppose a Bloch wave of wave vector q is incident upon the structure of Fig. 2(b) from the left. Then the coefficient of the tight-binding wave function on the left chain is

$$\psi_n=e^{iqn}+re^{-iqn},\quad n=-1,-2,-3,\ldots\ , \tag{8a}$$

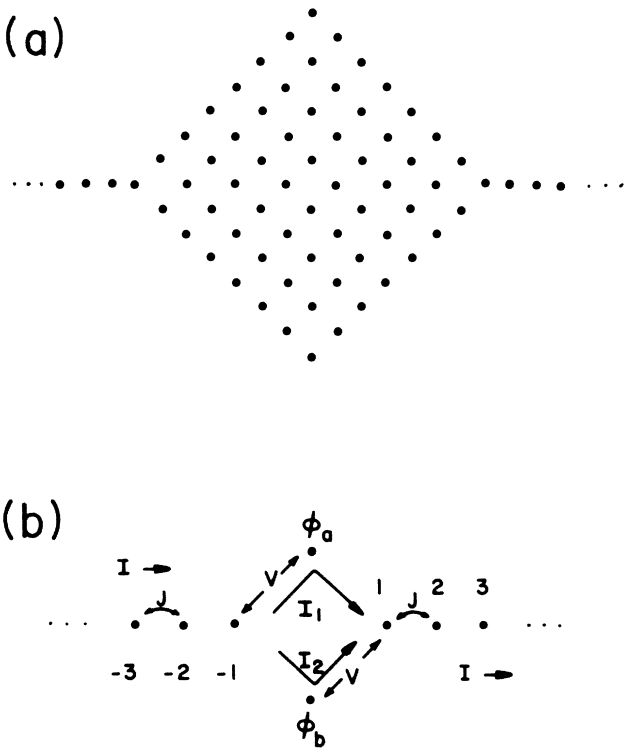

FIG. 2. (a) NSS structure. (b) The smallest possible NSS structure allowing for magnetic field effects.

and that on the right chain is

$$\psi_n=te^{iqn},\quad n=1,2,3,\ldots\ . \tag{8b}$$

The wave-function amplitudes of the two central sites are denoted by ϕ_a and ϕ_b. Using Hamiltonian (6), we obtain the equations

$$\begin{aligned}
&Ve^{-i(\varphi/4)}\phi_a+Ve^{i(\varphi/4)}\phi_b=J(1+r)\ ,\\
&Ve^{i(\varphi/4)}\phi_a+Ve^{-i(\varphi/4)}\phi_b=Jt\ ,\\
&Ve^{i(\varphi/4)}(e^{-iq}+re^{iq})+Ve^{-i(\varphi/4)}te^{iq}=(E-E_a)\phi_a\ ,\\
&Ve^{-i(\varphi/4)}(e^{-iq}+re^{iq})+Ve^{i(\varphi/4)}te^{iq}=(E-E_b)\phi_b\ ,
\end{aligned} \tag{9}$$

which determine r, t, ϕ_a, and ϕ_b. Here $\varphi=2\pi\phi/\phi_0$, where ϕ is the magnetic flux through the square and $\phi_0=2\pi c\hbar/e$ is the elementary flux quantum. The Schrödinger equation for the chain sites yields

$$E=2J\cos q\ , \tag{10}$$

where J denotes the transfer integral along the chains, and the site energies of the chains were chosen to be zero.

From the tight-binding model definition for the single-particle current from site l to site j

$$\tilde{I}_{lj}=-2\frac{e}{\hbar}\mathrm{Im}(V_{lj}c_l^\dagger c_j)\ , \tag{11}$$

and the solutions of Eqs. (9), we find for the single-particle current along the chains, $\tilde{I}$,

$$\tilde{I}=2\frac{e}{\hbar}J\sin q\,(1-|r|^2)$$
$$=2\frac{e}{\hbar}J\sin q\,|t|^2$$
$$=8\frac{e}{\hbar}J\sin^3 q\frac{\epsilon_1^2+\epsilon_2^2+2\epsilon_1\epsilon_2\cos\varphi}{|M|^2}\,, \tag{12a}$$

where

$$M=-1+2(\epsilon_1+\epsilon_2)e^{iq}-2\epsilon_1\epsilon_2(1-\cos\varphi)e^{2iq}\,. \tag{12b}$$

Here we have introduced dimensionless coupling constants for the two arms of the square,

$$\epsilon_1=\frac{V^2}{J(E-E_a)}\,,$$
$$\epsilon_2=\frac{V^2}{J(E-E_b)}\,. \tag{13}$$

The single-particle currents along the upper and lower arms of the square, denoted $\tilde{I}_1$ and $\tilde{I}_2$, respectively, are [see Eq. (11)]

$$\tilde{I}_1=8\frac{e}{\hbar}J\sin^2 q\frac{\epsilon_1^2\sin q-\epsilon_1\epsilon_2\sin(\varphi-q)+\epsilon_1\epsilon_2(\epsilon_1+\epsilon_2)\sin\varphi}{|M|^2}\,, \tag{14a}$$

$$\tilde{I}_2=8\frac{e}{\hbar}J\sin^2 q\frac{\epsilon_2^2\sin q+\epsilon_1\epsilon_2\sin(\varphi+q)-\epsilon_1\epsilon_2(\epsilon_1+\epsilon_2)\sin\varphi}{|M|^2}\,. \tag{14b}$$

The sum of the two currents, $\tilde{I}_1+\tilde{I}_2$ is of course equal to the external current $\tilde{I}$ [Eqs. (12)]. However, even for an ordered square in which $\epsilon_1=\epsilon_2$, there is a circulating current $\tilde{I}_c=\tilde{I}_1-\tilde{I}_2$, imposed by the magnetic field. This current may flow even when $\tilde{I}=0$ and is persistent[16] for small enough inelastic scattering.

We now use these results to find the magnetoconductance of the structure and its Hall-type voltage. Let us assume that the electrons incident upon the system from the left-hand side are coming from a reservoir held at a chemical potential μ_1. Similarly, electrons incident from the right are coming from a reservoir with a chemical potential μ_2. For $\mu_1>\mu_2$, the total current traversing the system from left to right is given by the single-state current $\tilde{I}$ [Eq. (12a)], multiplied by the density of states times $(\mu_1-\mu_2)$. With the density of states given by $1/2\pi J\sin q$ we find

$$I=\frac{e}{\pi\hbar}|t|^2(\mu_1-\mu_2)$$
$$=4\frac{e}{\pi\hbar}(\mu_1-\mu_2)$$
$$\times\sin^2 q\frac{\epsilon_1^2+\epsilon_2^2+2\epsilon_1\epsilon_2\cos\varphi}{|M|^2}\,. \tag{15}$$

The magnetoconductance G of the square is

$$G=\frac{I}{V_{LR}}\,, \tag{16}$$

where V_{LR} is the voltage difference across the square along the direction of the external current I. As discussed before, it is related to $\mu_1-\mu_2$ by the reflection coefficient $|r|^2$,

$$V_{LR}=\frac{1}{e}|r|^2(\mu_1-\mu_2)\,. \tag{17}$$

Thus,

$$G=\frac{e^2}{\pi\hbar}|t|^2/|r|^2$$
$$=\frac{e^2}{\pi\hbar}4\sin^2 q\frac{\epsilon_1^2+\epsilon_2^2+2\epsilon_1\epsilon_2\cos\varphi}{[1-2(\epsilon_1+\epsilon_2)\cos q+2\epsilon_1\epsilon_2(1-\cos\varphi)]^2}\,, \tag{18}$$

exhibiting oscillations with a periodic ch/e. In the small φ limit, we obtain

$$G=\frac{e^2}{\pi\hbar}4\sin^2 q\left[\frac{\epsilon_1+\epsilon_2}{1-2(\epsilon_1+\epsilon_2)\cos q}\right]^2\left[1-\varphi^2\epsilon_1\epsilon_2\left[\frac{1}{(\epsilon_1+\epsilon_2)^2}+\frac{2}{1-2(\epsilon_1+\epsilon_2)\cos q}\right]\right]\,. \tag{19}$$

Hence the sign of the magnetoconductance at small magnetic fields depends, in a complicated way, upon the sign of the coupling constants, i.e., upon the relationship between the site energies E_a, E_b, and the energy E (Eqs. (10) and (13)].

We next turn to the Hall-type effect. The expression for the chemical potential difference between the sites a and b was derived in the preceding section [Eq. (5)]. Inserting there the values of ϕ_a and ϕ_b obtained from Eqs. (9), together with the analogous values for states coming from the right, we find

$$\mu_a-\mu_b=(\mu_1-\mu_2)4\sin q\sin\varphi$$
$$\times\left[\frac{\epsilon_1}{1+2\epsilon_1(1-\cos\varphi)(\epsilon_1-\cos q)}\right.$$
$$\left.+\frac{\epsilon_2}{1+2\epsilon_2(1-\cos\varphi)(\epsilon_2-\cos q)}\right]\,. \tag{20}$$

Dividing this expression by the external current I [Eq. (15)] times the electronic charge e we obtain the effective Hall resistance R_{ab}

$$R_{ab}=\frac{\pi\hbar}{e^2}\frac{|M|^2}{\sin q}\frac{\sin\varphi}{\epsilon_1^2+\epsilon_2^2+2\epsilon_1\epsilon_2\cos\varphi}\times\left[\frac{\epsilon_1}{1+2\epsilon_1(1-\cos\varphi)(\epsilon_1-\cos q)}+\frac{\epsilon_2}{1+2\epsilon(1-\cos\varphi)(\epsilon_2-\cos q)}\right]. \quad (21)$$

This, again, oscillates with a period ch/e. Note that for small coupling $\epsilon_1\sim\epsilon_2\sim\epsilon$, the conductance G is of the order of ϵ^2 while R_{ab} is of the order ϵ^{-1}.

Let us now consider the situation where the transmission of the square is exactly unity. In this case the resistance of the sample vanishes. The condition under which this happens, from Eqs. (12) and (15) is

$$1-\cos\varphi=[2(\epsilon_1+\epsilon_2)\cos q-1]/2\epsilon_1\epsilon_2 . \quad (22)$$

Inserting Eq. (22) into expression (21) for R_{ab} yields

$$R_{ab}=\frac{4\pi\hbar}{e^2}\sin q\sin\varphi\left[\frac{\epsilon_1}{1+2\epsilon_1(1-\cos\varphi)(\epsilon_1-\cos q)}+\frac{\epsilon_2}{1+2\epsilon_2(1-\cos\varphi)(\epsilon_2-\cos q)}\right], \quad (23)$$

where $1-\cos\varphi$ is given by Eq. (22). Expression (23) does not reveal any special features of the Hall resistance. However, by requiring that

$$R_{ab}=\frac{\pi\hbar}{e^2}\frac{1}{n},\quad n-\text{integer}, \quad (24)$$

we obtain from (23) an expression for $\sin\varphi$. Combining this with (22), we find

$$\tan\frac{\varphi}{2}=2n\sin q[2(\epsilon_1+\epsilon_2)\cos q-1]\left[\frac{1}{\epsilon_2+[2(\epsilon_1+\epsilon_2)\cos q-1](\epsilon_1-\cos q)}+\frac{1}{\epsilon_2+[2(\epsilon_1+\epsilon_2)\cos q-1](\epsilon_2-\cos q)}\right], \quad (25)$$

as the condition under which the resistance is zero while the Hall resistance has the universal value $\pi\hbar/e^2$ divided by an integer n. This condition depends in a complicated way upon the sample parameters. But, when the model is completely ordered, in the sense that the square arms are made of the same tight-binding chains as those forming the leads, it becomes very simple. We put $V=J$ and $E_a=E_b=0$ and obtain [see Eqs. (10) and (13)]

$$\epsilon_1=\epsilon_2=1/2\cos q . \quad (26)$$

Then, using (26) in (25),

$$\tan\frac{\varphi}{2}=4n\cot q , \quad (27)$$

independent of the coupling. Thus we find that when our small structure is completely ordered, the Hall resistance has a constant value for field values for which the magnetoresistance vanishes. In the more general case, $T=1$ does not ensure quantized Hall conductance, possibly due to finite-size corrections to the latter.

Finally, we apply our results to the situation considered by NSS.[11] To this end we work, as they have done, through lowest order in the coupling parameters ϵ_1 and ϵ_2. Then the magnetoconductance [Eq. (18)] takes the simple form

$$g=\frac{G}{4e^2/\pi\hbar}=\sin^2q(\epsilon_1^2+\epsilon_2^2+2\epsilon_1\epsilon_2\cos\varphi) . \quad (28)$$

NSS have considered a large square with a double delta-function distribution for the site energies and computed the effective overlap integrals between the initial and final sites, where the external current enters and leaves the sample. In terms of our model, they have computed the coupling parameters ϵ_i for the various paths traversing the sample, when $E-E_i$ take the values $\pm W$, and correspondingly found that the ϵ_i take the values $\pm\epsilon$. They found that when negative and positive values of ϵ have the same probability, the total transmission of the structure exhibits oscillations with period $ch/2e$, i.e., the superconducting flux quantum.

We shall now invoke an effective medium argument for the conductance to obtain their result from Eq. (28). To this end we note that Eq. (28) yields

$$\begin{aligned} g_1&=\sin^2q\,2\epsilon^2(1+\cos\varphi),\quad \epsilon_1=\epsilon_2=\epsilon ,\\ g_2&=\sin^2q\,2\epsilon^2(1-\cos\varphi),\quad \epsilon_1=-\epsilon_2=\epsilon . \end{aligned} \quad (29)$$

Now let us assume that in our system there are p conductors with the conductance g_1, and 1-p conductors having g_2. Then, the effective-medium approximation[17] gives, for the effective conductance g_m of the system the following equation:

$$g_m^2-g_m\left[pg_1+(1-p)g_2-\frac{2}{z-2}[pg_2+(1-p)g_1]\right]-\frac{2}{z-2}g_1g_2=0 , \quad (30)$$

where z is the coordination number. The considerations of NSS as well as Eqs. (29) and (30) hold to lowest order in the coupling. This means that effectively, $z=2$, and consequently,

$$g_m=2\epsilon^2\sin^2q\frac{\sin^2\varphi}{1+(1-2p)\cos\varphi} . \quad (31)$$

This shows oscillations with the normal quantum flux period except for the case $p=\frac{1}{2}$. At this value of p, the oscillations have the period of the superconducting flux

quantum. Thus, to lowest order in the coupling, the effective-medium approximation reproduces for this model a feature resembling the numerical result of NSS, that under certain conditions a halving of the period is obtained except that here this is obtained only at a special value of p. This is of the same nature as the by now well-known property of Aharonov-Bohm oscillations in disordered rings, namely that the ensemble-averaged conductance exhibits $\phi_0/2$ periodicity (Buttiker *et al.*[5] and Refs. 18). In this model, $p=\frac{1}{2}$ is needed to have enough symmetry in order to average out the ϕ_0-periodic component.

IV. DISCUSSION

We have presented the behavior of the magnetoconductance and the Hall-type resistance of a small, disordered tight-binding model. There is no Lorentz force in the usual sense in this model. Rather, the wave vector q of the scattered state, i.e., the space dependence of the wave function, is crucial in order to obtain the Hall effect. In the case of the phonon-assisted hopping treated by Holstein and collaborators,[3] the inelastic scattering was crucial to obtain the Hall effect. A comparison of these two different cases is clearly called for, and we hope to be able to pursue this in the future.

The model presented is unique in the sense that it includes the effect of the current leads and their couplings to the system upon its transport properties. Usually, one considers the conductance of a system without including the leads in the Hamiltonian. Then, comparison with experimental data, especially for very small samples, invokes questions concerning the possible influence of the leads. It is extremely desirable to study this model for larger structures than the one considered in Sec. III. In particular, one would like to investigate the situation where the transmission is unity (i.e., the resistance vanishes) and explore the Hall resistance in those regions. Of particular interest is the question of whether taking a large disordered system will alter our result that the Hall conductance is quantized only at special values of the magnetic field or in a completely ordered system. Questions associated with the effects of ensemble and temperature averaging are also very relevant. Work on some of these aspects is in progress.

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[1]G. Bergmann, Phys. Rep. **101**, 1 (1984).

[2]P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 289 (1985).

[3]T. Holstein, Phys. Rev. **124**, 1329 (1961); T. Holstein and L. Friedman, *ibid.* **165**, 1019 (1968); L. Friedman and M. Pollak, Philos. Mag. B **38**, 173 (1978).

[4]R. Landauer, IBM J. Res. Dev. **1**, 223 (1957); Philos. Mag. **21**, 863 (1970).

[5]P. W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fisher, Phys. Rev. B **22**, 3519 (1980); M. Ya. Azbel, J. Phys. C **14**, L225 (1981); P. A. Lee and D. S. Fisher, Phys. Rev. Lett. **47**, 1546 (1981); D. C. Langreth and E. Abrahams, Phys. Rev. B **24**, 2924 (1981); M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, *ibid*, **31**, 6207 (1985).

[6]M. Büttiker, Phys. Rev. B **33**, 3020 (1986).

[7]U. Sivan and Y. Imry, Phys. Rev. B **33**, 551 (1986).

[8]Y. Gefen, Y. Imry, and M. Ya. Azbel, Phys. Rev. Lett. **52**, 129 (1984); R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, *ibid.* **54**, 2696 (1985).

[9]C. P. Umbach, S. Washburn, R. B. Laibowitz, and R. A. Webb, Phys. Rev. B **30**, 4048 (1984); A. D. Stone, Phys. Rev. Lett. **54**, 2692 (1985).

[10]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[11]V. L. Nguyen, B. Z. Spivak, and B. I. Shklovskii, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 35 (1985) [JETP Lett. **41**, 42 (1985)].

[12]B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].

[13]D. Yu. Sharvin and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 285 (1981) [JETP Lett. **34**, 272 (1981)].

[14]L. Engquist and P. W. Anderson, Phys. Rev. B **24**, 1151 (1981).

[15]R. Landauer (private communication).

[16]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[17]S. Kirkpatrick, Rev. Mod Phys. **45**, 574 (1973).

[18]Y. Gefen (private communication); D. A. Browne, J. P. Carini, K. A. Muttalib, and S. R. Nagel, Phys. Rev. B **30**, 6798 (1984); Y. Imry and N. Shiren, Phys. Rev. B **33**, 7992 (1986); M. Murat, Y. Gefen, and Y. Imry, Phys. Rev. B **34**, 659 (1986); A. D. Stone and Y. Imry, Phys. Rev. Lett. **56**, 189 (1986).