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Field-enhanced electronic transport in solids

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Electronic transport in the presence of trapping centers is considered by taking into account the local electric field contribution. A generalization of the previous models of field-assisted thermal ionization is proposed. The origin of the local electric field is discussed and a theoretical evaluation based on the electron-phonon interaction is given in agreement with the experimental results obtained in some differently doped CdF_2 crystals and in SiO films.

INTRODUCTION

Bulk-limited electronic conduction involving carrier emission from localized levels includes three effects: (a) quantum tunneling, (b) phonon-assisted tunneling, and (c) field-assisted thermal ionization (the Poole-Frenkel effect). This last phenomenon is currently attributed to the lowering of trapping Coulombic barriers by the applied electric field E and is commonly invoked to explain the electric current in several semiconducting compounds.[1] A classical calculation, which considers electron emission only in the direction of **E**, produces the following result for the emission probability $P(E)$:[2]

$$P(E)=P(0)\exp(\beta\sqrt{E}\,/kT)=P(0)\exp\alpha\ , \qquad (1)$$

where $\alpha=\beta\sqrt{E}\,/kT$ and $\beta=(e^3/\pi\epsilon)^{1/2}$ with ϵ the dielectric constant, α is the Poole-Frenkel constant, T the absolute temperature, and $P(0)$ the emission probability at zero applied field.

Experimental use of this formula often yields deviations, leading mainly to two discrepancies: (i) a value of β smaller than the theoretical one, and (ii) no prediction of a saturation effect at low fields (Ohmic behavior).

Some developments have been proposed to extend the formula to emission in three dimensions, for which

$$P(E)=P(0)\exp(\beta\sqrt{E\cos\theta}/kT)\ , \qquad (2)$$

where θ is the angle between the electron vector position **r** with respect to the trapping center and the applied electric field **E**. Up to now, most of the proposed models have considered different approximations of formula (2) by averaging over the angle θ and including backward emission under various assumptions.[3-6] Approaching the problem from another side, Pai has considered the details of the escape process from a Coulombic center through a diffusion equation.[7] In the present paper an extension of (2) based on the local electric field contribution is considered.

THEORY

In all the previously mentioned models, the effect of the local field L is neglected. By introducing it, the trapping potential is

$$U(\mathbf{r})=-e^2/4\pi\epsilon r-e\,|\,\mathbf{E}+\mathbf{L}\,|\,r\cos\theta\ , \qquad (3)$$

where θ is the angle between the sum vector $\mathbf{E}+\mathbf{L}$ and **r**. In analogy with Eq. (2) one has

$$P(E)=q\exp(\beta\sqrt{|\,\mathbf{E}+\mathbf{L}\,|\,\cos\theta}/kT)\ . \qquad (4)$$

As discussed in the following, the local electric field can be generated by uncompensated localized charges, like impurity centers, or by the displacement of lattice ions surrounding the trapping site from which the electron emission is taking place.

The internal field can be actually neglected whenever its effect is integrated over the space or the time; on the contrary, the local-field contribution must be taken into account when its local or instantaneous value is important, as for the ionization of the carrier from a trapping center. The local field increases the probability of thermal ionization, without giving rise to the transport of the ionized carriers, which is determined only by the applied field. On the average $\mathbf{L}=0$, but $L\neq 0$.

Previous theories which neglect such correction may be inadequate, since, as we shall see, L is usually comparable to the applied field. A modification of the conventional model according to a local-field correction has been recently considered, on phenomenological grounds, to explain the ohmic deviation at low applied fields in CdF_2:Y crystals.[8]

Since L is distributed either in space and/or time within the crystal, the simplest expression is obtained, as is usually done,[9] by assuming the largest contribution in Eq. (4), then

$$P(E)=q\exp(\beta\sqrt{E+L}\,/kT)\ , \qquad (5)$$

so that

$$\begin{aligned}P(E)/P(0)&=\exp[\beta(\sqrt{E+L}-\sqrt{L}\,)/kT]\\&=\exp[(\alpha^2+\gamma^2)^{1/2}-\gamma]\\&=\exp(\beta R/k)\ ,\end{aligned} \qquad (6)$$

if $\gamma=\beta\sqrt{L}\,/kT$ and $R=(\sqrt{E+L}-\sqrt{L}\,)/T$.

The dependence of $\ln P(E)/P(0)$ on α as deduced from the previous model[2-7] and from Eq. (6) for two γ values is plotted in Fig. 1, showing the effect of the inclusion of the local-field correction in the single-center theory. Assuming β of the order of 10^{-4} eV V$^{-1/2}$cm$^{1/2}$ and L of 10^4 V/cm, the two assumed γ values correspond to $T=10$ and 100 K. One must note that L and conse-

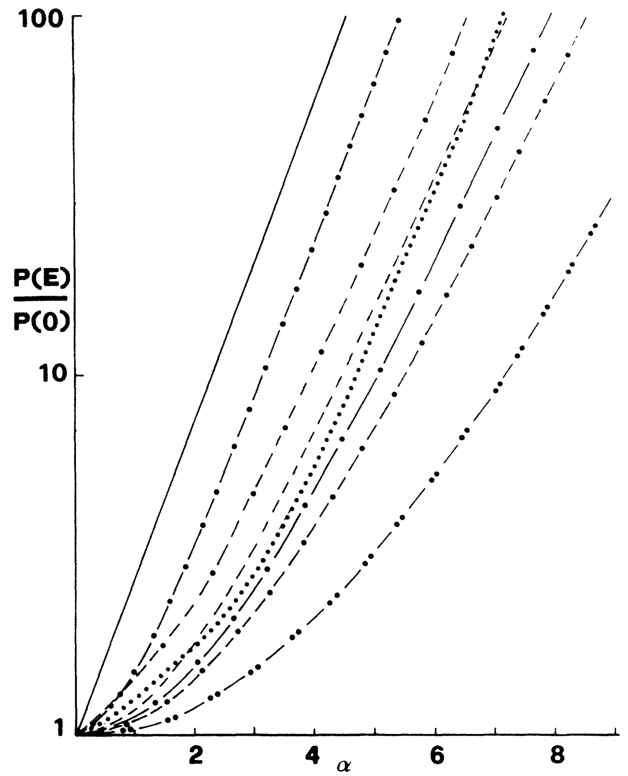

FIG. 1. $\ln[P(E)/P(0)]$ vs α for the traditional Poole-Frenkel formula and some of the subsequent models, including that here proposed. ——, Frenkel (Ref. 2); ·— — —·, Hartke (Ref. 3) and Connel *et al.* (Ref. 6); · · · ·, Jeda (truncation parameter, 0.5) (Ref. 5); — — —, Hill (emission in the direction of the applied field) (Ref. 4); ----, Hill (isotropic emission) (Ref. 4); ·——·, Pai (Ref. 7); —·—·—·, this work $\gamma = 1$; ··——··, this work $\gamma = 10$.

quently γ are not phenomenological parameters, but depend on the physical characteristics of the substance and can be evaluated, as indicated in the following, when the microscopic situation of the crystal is known.

EXPERIMENTAL RESULTS

Experimentally the ionization probability $P(E)$ is evaluated through the conductivity $\sigma(E)$, which is proportional to the ratio between the current and the applied field i/E. The relation between $P(E)$ and $\sigma(E)$ depends on the trapping probability t_p. Assuming that the free-carrier density is much smaller than the donor density, if t_p is inversely proportional to the number of ionized donors, one obtains[7] $\sigma(E)$ proportional to $[P(E)]^{0.5}$; on the contrary, by assuming t_p constant, one has $\sigma(E)$ proportional to $P(E)$.[2-6] These two possibilities depend on the level distribution of traps and donors in the energy gap.[10] The following results agree with the last assumption.

Thus according to Eq. (6), $\ln(i/E)$ plotted versus R is approximated by straight lines. The shape and the position of the $\ln(i/E)$ versus R curves will depend on the three parameters L, β, and $P(0)$. The "correct" value of L is the one which reduces the experimental points to a straight line; β is given by the slope of such a straight line and $P(0)$ is the intercept value with the vertical axis at $R=0$. Moreover, the plot of the $\ln P(0)$ obtained at different temperatures versus $1/T$ gives the activation energy ε of the conduction process.

Such plots have been used to interpret the current-voltage-temperature measurements obtained in some doped CdF_2 crystals where the electric conduction is attributed to the Poole-Frenkel effect.[11] CdF_2 crystals doped with some trivalent impurities can be converted to semiconductors by annealing in Cd vapor.[12] Different activation energies from 0.1 to 0.2 eV are obtained when the resistivity increases from 10 to 10^3 Ω cm.[13] Higher values of the activation energy, about 0.5 eV, are observed in conversion-incompatible impurities, like Eu.[14] Figure 2 shows the plot of $\ln[P(E)/P(0)]$ versus R for a CdF_2:Gd crystal. Table I gives the obtained values of L, ε, and β: the last two parameters are compared with those obtained in previous works,[13,14] where the measurements are examined according to Hill's theory.[4] A quite good agreement is obtained between the two methods and the Poole-Frenkel (PF) constant is close to the theoretical value, which is 2.6×10^{-4} eV $V^{-1/2}$ $cm^{1/2}$, by assuming the static dielectric constant for CdF_2 crystals $\epsilon=8.1$.[15] Despite the limited extension of the abscissa and ordinate values in Fig. 2, the voltage range applied to the sample includes

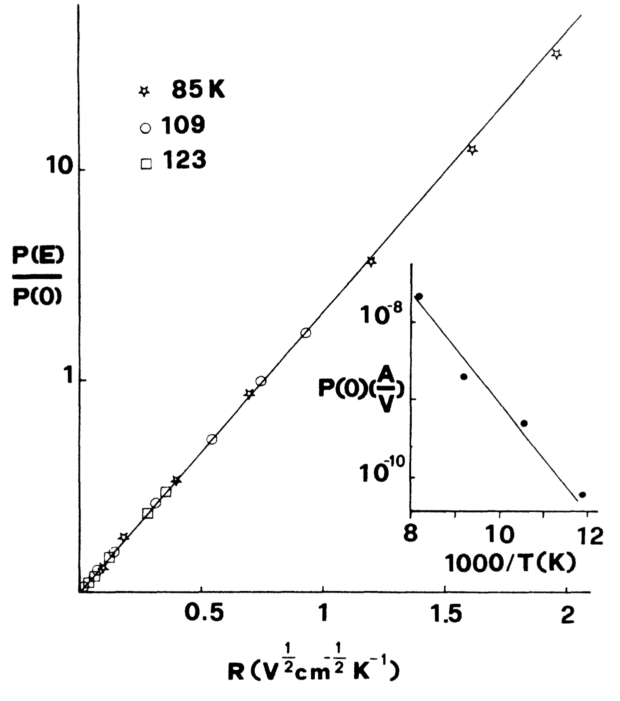

FIG. 2. $\ln[P(E)/P(0)]$ vs R for a CdF_2:Gd crystal at the indicated temperatures. Doping concentration is 0.1% mol; resistivity at 300 K is 6 10^2 Ω cm. The value of L giving the linear approximation and that of β, obtained from the slope, are reported in Table I. The inset shows the plot of $\ln P(0)$ vs $1/T$, giving the activation energy value ε (LF), which is also indicated in Table I. These parameters are compared with those evaluated by Hill's method and by the Arrhenius plot of the current at low applied fields.

TABLE I. Local field, activation energy, and Poole-Frenkel constant in different doped CdF_2 crystals. The results obtained with the present model are indicated with LF (local field).

Dopant	Concentration (mol %)	Resistivity (300 K) (Ω cm)	Local field (V/cm)	Activation energy (eV) LF	Hill	Arrhenius	PF constant LF	Hill
							10^{-4} eV $V^{-1/2}cm^{1/2}$	
Ga	1	10^2	1×10^4	0.21	0.20	0.19	2.4	2.4
Y	0.04	10	2×10^4	0.13	0.13	0.14	2.6	2.5
Gd	0.1	6×10^2	1×10^4	0.20	0.18	0.19	2.6	2.5
Gd	0.1	10	1×10^4	0.09	0.10	0.12	2.6	2.7
Eu	0.1	10^5	4×10^4	0.48	0.46	0.49	2.6	2.7

4 orders of magnitude, from 0.1 to 10^3 V, and the corresponding current extends from 10^{-10} to 10^{-5} A. Similar results obtained in other doped CdF_2 crystals are reported in Table I.

Equation (6) has been also used to plot the results obtained by Servini and Jonscher in silicon monoxide films.[16] Besides being a classical reference, these data cover a wide range of temperature, voltage, and current. Figure 3 shows $\ln[P(E)/P(0)]$ versus R for this case: A good linearity is obtained with $L=5\times10^4$ V/cm and the slope gives $\beta=1.69\times10^{-4}$ eV $V^{-1/2}cm^{1/2}$. The plot of $\ln P(0)$ versus $1/T$ in the inset gives $\epsilon=0.36$ eV. These results are in very good agreement with those of Servini and Jonscher.

DISCUSSION

The estimated values of L are about 10^4 V/cm in all the present cases. An evaluation of the same order of magnitude is obtained in $CaWO_4$ crystals from the broadening of paramagnetic-resonance lines.[17] In view of the relevance of the knowledge of the local fields it should be important to understand which sample properties influence the value of L. If the local field were due to uncompensated localized charges, impurity concentration would very likely be an important factor, but impurity distribution inside the lattice could also have a strong influence.

Alternatively, L may be due to the dipole of the lattice ions; in these conditions its value is independent of the impurity concentration and it depends on the lattice characteristics only. The nearly constant value of L obtained in CdF_2 crystals doped with different concentrations (0.04—1 mol %) and in SiO films suggests that the internal field may have such an origin. In this case an order-of-magnitude evaluation of L can be obtained on the basis of the energy shift of the bottom of the conduction band produced by the electron-phonon interaction.[18] For polar crystals at zero temperature, for example, the energy shift is $-a\hbar\omega$, which equalized to $\beta\sqrt{L}$, assuming $\omega\cong10^{13}$ s^{-1},[19] and $a=3.3$,[20] gives $L\cong10^4$ V/cm in agreement with the experimentally obtained results. Such an evaluation can be considered correct whenever the temperature is lower than the Debye temperature, as for example in the previously mentioned CdF_2 measurements where $T<<550$ K.[20]

According to such an evaluation, the barrier-lowering shift is attributed to the phonon emission from electrons at zero temperature, giving rise to the lattice distortion which generates the local electric field. On the other hand, the electron-phonon interaction at nonzero temperature should lead to a temperature dependence of the local electric field. The aforementioned measurements do not show any change of L with T. However, the temperature range may be too limited or the function variation too small to yield observable effects. Further studies are required for this purpose.

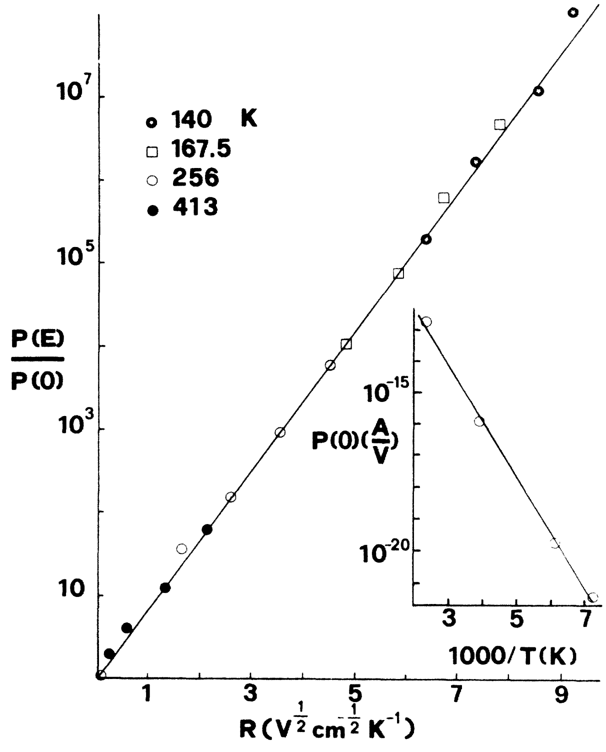

FIG. 3. $\ln[P(E)/P(0)]$ vs R for SiO films according to the results of Servini and Jonscher (Ref. 16). The inset shows $\ln P(0)$ vs $1/T$. The obtained values of L, β, and ϵ are reported in the text.

CONCLUSIONS

In conclusion, the local electric field contribution in the field-assisted thermal ionization process gives rise to an extension of the traditional formula which explains correctly the experimental results. Similar improvements should be expected also in other emission processes such as the temperature-independent and the phonon-assisted tunneling effects.

ACKNOWLEDGMENTS

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