Surface-polariton solitons

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The propagation of surface or guided electromagnetic waves is considered for structures consisting of two or more dielectric media, at least one of which is nonlinear, that are separated by parallel, planar interfaces. The conditions are obtained under which bright envelope surface-polariton solitons can exist in these structures. Two physical systems are analyzed that can, in principle, sustain such nonlinear surface excitations.

Considerable interest has arisen in recent years[1–4] in the propagation of cw surface electromagnetic waves (surface polaritons) or guided electromagnetic waves in structures consisting of two or more dielectric media, at least one of which is characterized by a nonlinear dielectric tensor, that are separated by parallel, planar interfaces. These nonlinear surface polaritons or guided waves propagate in a wavelike fashion in directions parallel to the interface(s), and the nonlinearity of one or more of the dielectric media in the structure supporting these waves manifests itself in the spatial dependence of the associated electromagnetic fields through a nonexponential or nontrigonometric dependence of the field amplitudes on the coordinate normal to the interfaces.

It is not expected that the types of electromagnetic waves described in the preceding paragraph exhaust the waves possible in nonlinear waveguide systems. In particular, it is of considerable interest to look for surface polariton solitons. These are pulse envelope waves that have the nature of solitary waves propagating in directions parallel to the interfaces between the different media. The solitons can be bright, in which case they are pulses of finite height moving against a dark or low-intensity background. It is also possible, in principle, for dark solitons to exist, but these are of little practical interest.

In a recent search for surface polariton solitons,[5] it was argued that a bright surface polariton soliton can exist only in collisional solid-state plasmas and even then only in a narrow range of frequencies about the surface-plasmon frequency. This, of course, is overly restrictive. In general, bright solitons exist in a region where the sign of the group dispersion of the linear surface polariton is opposite to that of the effective nonlinear coefficient. In the present paper the more general Whitham method[6,7] is used and, with the aid of a mathematically rigorous perturbation analysis of the nonlinear dispersion relationship, two physical systems are investigated in which such nonlinear surface excitations can exist.

The (real) amplitude of an electric field component, or indeed the total field, at one of the interfaces in a nonlinear, planar waveguide can be denoted by $a(x,t)$ [a precise definition of $a(x,t)$ will be given below for each of the structures we will consider in detail], where x is the coordinate direction parallel to the waveguide and t is the time. The restriction to a one-dimensional formulation will imply that the *transverse* variation of the guiding structure is automatically included in the final pulse evolution equation.[5,8,9] It is effected through the use of the nonlinear guided wave dispersion equation that includes both material and waveguide effects. This is an adiabatic assumption in which, at any point (x,t), the transverse field solution is equal to the nonlinear cw guided wave solution. This means, physically, that as the pulse progresses down the guide the system adjusts to the self consistent nonlinear field distribution at a rate faster than the time scale of the pulse envelope.

The equations determining $a(x,t)$ are[6,7,10] *the energy equation*

$$\frac{\partial a^2}{\partial t}+\frac{\partial}{\partial x}\left[\frac{\partial\omega}{\partial k}a^2\right]=0\;, \tag{1}$$

the local dispersion relation

$$\omega=\omega_0+\left[\frac{\partial\omega}{\partial k}\right]_{k=k_0}(k-k_0)+\frac{1}{2}\left[\frac{\partial^2\omega}{\partial k^2}\right]_{k=k_0}(k-k_0)^2+\omega_2(k_0)a^2\;, \tag{2}$$

the consistency equation

$$k_t+\omega_x=0\;, \tag{3}$$

where the subscripts denote partial derivatives. In Eqs. (1)–(3), k and ω are the wave number and frequency of the nonlinear surface or guided wave polariton.

It is also convenient to introduce the phase function $\theta(x,t)$ in terms of which $\theta_t=-\omega$ and $\theta_x=k$. In a simple cw case $\theta(x,t)$ would be $(kx-\omega t)$, where k and ω are constants. For a pulse envelope the wave number and frequency $k(x,t)$ and $\omega(x,t)$ are now functions of position and time. They are, in fact, *local* quantities because they differ at different points on the pulse and, for a given point on the pulse, change as it progresses down the guide.

If we assume that $a(x,t)$ is a slowly varying envelope function then, to a first approximation $\omega_0[=\omega(k_0)]$ is $\omega_L(k_0)$, the cw linear dispersion relationship. Equation (2) is the form that is obtained by a Taylor expansion about the center coordinates (ω_0,k_0) of the pulse. The $\omega_2(k_0)a^2$ term arises from another expansion about the linear limit, to $O(a^2)$, of the cw nonlinear dispersion relationship. The replacement of $\omega(k_0)$ in Eq. (2) by the linear cw form means that the derivatives of $\omega(k_0)$ have not been correctly calculated at this stage.[7] It is convenient to leave Eq. (2) in this form until a further development of the theory has been presented. Later on it will be shown that a small correction to Eq. (2) will lead to the prediction that the pulse is a solution of the nonlinear Schrödinger equation.

The boundary conditions on the electromagnetic fields at the interface(s) in our nonlinear, planar waveguide system, and at infinity, enter the present formulation of the problem through the dispersion relation (2).

Suppose now that[7] $\omega=\omega_0+\bar{\omega}$ and $k=k_0+\bar{k}$, where ω_0 and k_0 are constants. The phase function θ then becomes $\theta=\theta_0+\bar{\theta}$, where $\theta_0=k_0x-\omega_0t$ and $\bar{\theta}_t=-\bar{\omega}$, $\bar{\theta}_x=\bar{k}$. Hence, since the cw linear dispersion relationship is $\omega_L(k_0)$,

$$\frac{\partial\omega}{\partial k}=\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}+\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}(k-k_0)=\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}+\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}\bar{\theta}_x \tag{4}$$

and

$$\frac{\partial}{\partial x}\left[\frac{\partial\omega}{\partial k}\right]=\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}\bar{\theta}_{xx}\ . \tag{5}$$

Therefore, from Eq. (1),

$$a_t+\frac{1}{2}\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}a\bar{\theta}_{xx}+\left[\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}+\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}\bar{\theta}_x\right]a_x=0\ . \tag{6}$$

Now, if $A(x,t)=a(x,t)\exp[i\bar{\theta}(x,t)]$, then

$$a_t=(A_t-iA\bar{\theta}_t)e^{-i\bar{\theta}}\ , \tag{7}$$

$$a_x=(A_x-iA\bar{\theta}_x)e^{-i\bar{\theta}}\ , \tag{8}$$

so that Eq. (6) becomes

$$A_t+\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}A_x+\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}\left[\frac{A\bar{\theta}_{xx}}{2}+A_x\bar{\theta}_x-\frac{iA\bar{\theta}_x^2}{2}\right]-iA\left[\bar{\theta}_t+\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}\bar{\theta}_x+\frac{1}{2}\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}\bar{\theta}_x^2\right]=0\ . \tag{9}$$

The dispersion relationship, Eq. (2), can be rewritten in the form

$$\bar{\theta}_t+\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}\bar{\theta}_x+\frac{1}{2}\left[\frac{\partial^2\omega_L}{\partial k_2}\right]_{k=k_0}\bar{\theta}_x^2=-\omega_2(k_0)\,|A|^2\ , \tag{10}$$

and the differentiation of Eq. (8) with respect to x gives

$$\frac{A\bar{\theta}_{xx}}{2}+A_x\bar{\theta}_x-\frac{iA\bar{\theta}_x^2}{2}=-\frac{iA_{xx}}{2}+\frac{ia_{xx}}{2}e^{i\bar{\theta}}\ . \tag{11}$$

Therefore, when this result is substituted into Eq. (9) we obtain the equation

$$A_t+\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}A_x-\frac{i}{2}\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}(A_{xx}-a_{xx}e^{i\bar{\theta}})+i\omega_2(k_0)\,|A|^2A=0\ . \tag{12}$$

Equation (12) is the nonlinear Schrödinger equation (NLS), provided that the term involving a_{xx} can be discarded. The question arises, therefore, as to whether the a_{xx} term is approximately absent on physical grounds or disappears on mathematical grounds. Considering the physical reason first, we note that the radius of curvature of the pulse envelope is $\rho=(1+a_x^2)^{3/2}/a_{xx}$. Hence if the envelope is sufficiently slowly varying then ρ is large. This in turn implies that a_{xx} is small within this approximation. Thus the NLS, arising from the absence of the a_{xx} term is simply a consequence of the slowly varying envelope approximation. Mathematically, if the derivatives of the local frequency ω in Eq. (2) are replaced by their linear cw equivalents, a term involving a_{xx}/a arises.[7] This term is precisely

$$(-\tfrac{1}{2})(\partial^2\omega_L/\partial k^2)_{k=k_0}(a_{xx}/a)\ ,$$

and causes the a_{xx} term in Eq. (12) to be exactly canceled. Because of the above argument, Eq. (12) reduces to the standard nonlinear Schrödinger equation

$$i\left[A_t+\left[\frac{\partial\omega_L}{\partial k}\right]_{k=k_0}A_x\right]+\frac{1}{2}\left[\frac{\partial^2\omega_L}{\partial k^2}\right]_{k=k_0}A_{xx}-\omega_2(k_0)\,|A|^2A=0\ . \tag{13}$$

Equation (13) can also be obtained from first principles by another method[8,9] which proceeds from a Taylor expansion of the field amplitude about a center, or carrier, frequency. Also, it is usual to introduce a general transformation that casts the envelope Eq. (13) into dimensionless form and allows the pulse to be studied in a coordinate frame moving with the group velocity v_g. Introducing the variables $\omega_0=\omega_L(k_0)$ and $\omega_0''=(\partial^2\omega_L/\partial k^2)_{k=k_0}$, the transformation in this case is

$$\begin{aligned}\tau&=s^2\omega_0 t\ ,\\ \xi&=s(\omega_0/\omega_0'')^{1/2}(x-tv_g)\ ,\\ q&=\frac{1}{s}(\omega_2/\omega_0)^{1/2}A\ ,\end{aligned} \tag{14}$$

where s is a constant scaling parameter introduced to ensure that τ, ξ, and q are of order unity. This is always a convenience in an actual calculation. This is a fairly standard transformation and leads to the nonlinear Schrödinger equation in the form

$$i\frac{\partial q}{\partial \tau}+\frac{1}{2}\frac{\partial^2 q}{\partial \xi^2}+|q|^2 q=0 . \tag{15}$$

An alternative form of the NLS is obtained by using the cw linear dispersion relationship in the form $k_L(\omega)$, after the transformations

$$\xi=s^2 k_0 x ,$$
$$\tau=s[k_0/(-k_0'')]^{1/2}\left[t-\frac{x}{v_g}\right], \tag{16}$$
$$q=\frac{1}{s}(k_2/k_0)^{1/2}A ,$$

where $k_0''=(\partial^2 k_L/\partial\omega^2)_{\omega=\omega_0}$, $k_2=(\partial k/\partial a^2)_{k=k_0}$, and s is not the same as the quantity introduced in Eq. (14). The result is

$$i\frac{\partial q}{\partial \xi}+\frac{1}{2}\frac{\partial^2 q}{\partial \tau^2}+|q|^2 q=0 . \tag{17}$$

If $\partial q/\partial\tau \propto A_\tau=0$, the pulse does not change shape as it moves down the guide and the last two terms in Eq. (15) are in balance. In a linear guide $A_\tau=0$ implies that the term $(\partial^2\omega_L/\partial k^2)_{k=k_0}A_{xx}=0$. In other words, the pulse does not disperse. In a nonlinear guide, $A_\tau=0$ implies that the dispersion is exactly balanced by the nonlinearity. Therefore, it is worth emphasizing that the term $(\partial^2\omega_L/\partial k^2)_{k=k_0}A_{xx}$, which arises from the a_{xx}/a term discussed earlier, is required both for linear pulse dispersion and for the formation of solitons.

The lowest-order solitary wave solution of Eq. (15), known as the single (or $N=1$) soliton solution, is

$$q=p\ \mathrm{sech}(p\xi)e^{ip^2\tau/2} , \tag{18}$$

where p is an arbitrary scaling factor. Higher-order solitons are also allowed ($N=2,3,4,\ldots$ solitons) which are solutions that are nonstationary, but evolve in such a way that they periodically return to their original condition. Notice that Eq. (18) represents a continuum of solutions having different values of p, in which large values of p correspond to a tall, narrow soliton while small values correspond to a low, wide soliton. Obviously, if the pulse width is given, the height of the single soliton solution is uniquely defined. It is always possible to transform the NLS equation using

$$\tau'=p^2\tau,\ \ \xi'=p\xi,\ \ q'=\frac{q}{p} \tag{19}$$

so that Eq. (18) becomes $q'=\mathrm{sech}(\xi')\exp(i\tau'/2)$. It becomes clear upon substitution of Eqs. (14) into Eqs. (19) that no loss of generality ensues from setting $p=1$, since we can use s to scale the results for any desired pulse width or height. In the rest of this paper it will be assumed that $p=1$ in Eq. (18).

The peak amplitude of this solitary wave is $|q_{\mathrm{peak}}|=1$. This, as will be shown later, gives directly the threshold power level necessary to launch this wave. In the coordinate system (x,t), the solution of Eq. (13) is of the form

$$A(x,t)=\left[-\frac{(\partial^2\omega_L/\partial k^2)_{k=k_0}}{\omega_2(k_0)}\right]^{1/2}\frac{e^{i\delta t}}{v_g t_1}\times\mathrm{sech}\left[\frac{t-(x/v_g)}{t_1}\right], \tag{20}$$

where $\delta=(\partial^2\omega_L/\partial k^2)_{k=k_0}/(2v_g^2 t_1^2)$, v_g is the linear group velocity, $v_g=(\partial\omega_L/\partial k)_{k=k_0}$, and t_1 is the pulse halfwidth. This can be shown from Eq. (20) and the transformations (14).

This solution describes a bright soliton if $-(\partial^2\omega_L/\partial k^2)_{k=k_0}/\omega_2(k_0)$ is positive. This is the case if either of the following conditions holds:

$$(\partial^2\omega_L/\partial k^2)_{k=k_0}>0,\ \ \omega_2(k_0)<0 , \tag{21a}$$

$$(\partial^2\omega_L/\partial k^2)_{k=k_0}<0,\ \ \omega_2(k_0)>0 . \tag{21b}$$

Only the second of these conditions was considered in Ref. 5.

We now turn to a consideration of two nonlinear, planar waveguide structures in which, according to Eqs. (21), bright surface polariton solitons can exist. The first consists of a (linear) metal characterized by a dielectric function $\epsilon_m(\omega)$ in the region $z>0$ interfaced to a nonlinear dielectric medium whose dielectric tensor has the form $\epsilon_{ij}^{\mathrm{NL}}=\delta_{ij}\epsilon_i^{\mathrm{NL}}$, where ω is the frequency and[4]

$$\epsilon_x^{\mathrm{NL}}=\epsilon_d+\alpha|E_x(\mathbf{x}|\omega)|^2+\beta[|E_y(\mathbf{x}|\omega)|^2+|E_z(\mathbf{x}|\omega)|^2] , \tag{22a}$$

$$\epsilon_y^{\mathrm{NL}}=\epsilon_d+\alpha|E_y(\mathbf{x}|\omega)|^2+\beta[|E_z(\mathbf{x}|\omega)|^2+|E_x(\mathbf{x}|\omega)|^2] , \tag{22b}$$

$$\epsilon_z^{\mathrm{NL}}=\epsilon_d+\alpha|E_z(\mathbf{x}|\omega)|^2+\beta[|E_x(\mathbf{x}|\omega)|^2+|E_y(\mathbf{x}|\omega)|^2] , \tag{22c}$$

in the region $z<0$ where ϵ_d is the linear part of the dielectric function and E_x, E_y, and E_z are the electric field components. Here α and β are the intrinsic nonlinear coefficients that are assumed to be constant and the electric field in our systems has been assumed to have the form $E(\mathbf{x},t)=E(\mathbf{x}|\omega)\exp(-i\omega t)$. The cw nonlinear dispersion relation for this structure, to lowest nonzero order in α and β, for a wave propagating in the x direction with a wave number k, and independent of y, is given by[11]

$$\frac{\epsilon_m}{K_m}+\frac{1}{K_d}\left\{\epsilon_d+\frac{a^2}{4}\left[\alpha\left[1+\frac{k^4}{K_d^4}\right]+\frac{2\beta k^2}{K_d^2}\right]\right\}=0 , \tag{23}$$

where $K_m^2=k^2-(\omega^2/c^2)\epsilon_m$, $K_d^2=k^2-(\omega^2/c^2)\epsilon_d$, and $a\equiv E_x(x,\ z=0|\omega)$ is assumed to be real.

The effective nonlinear coefficient $\omega_2(k_0)$ for the single interface between a nonlinear dielectric and a linear metal is, for $\alpha=\beta$,

$$\omega_2(k_0) = -\alpha \left[\frac{1}{4K_d} \left[1 + \frac{k^2}{K_d^2} \right]^2 \frac{1}{\partial G/\partial \omega} \right]_{\omega=\omega_0}, \tag{24}$$

where $\omega_0 = \omega_L(k_0)$,

$$\left[\frac{\partial G}{\partial \omega} \right]_{\omega=\omega_0} = \left\{ \frac{\epsilon_m'}{K_m} \left[\left[1 + \frac{1}{2} \frac{\omega^2}{c^2} \frac{\epsilon_m}{K_m^2} \right] + \frac{\omega}{c^2} \right] + \frac{\omega}{c^2} \left[\frac{\epsilon_m^2}{K_m^3} + \frac{\epsilon_d^2}{K_d^3} \right] \right\}_{\omega=\omega_0}, \tag{25}$$

where the prime denotes differentiation with respect to frequency, and

$$G(\omega,k) = \frac{\epsilon_m}{K_m} + \frac{\epsilon_d}{K_d} . \tag{26}$$

Note that $G[\omega_L(k),k]=0$ is the linear dispersion relation.

A second system in which Eqs. (21) are satisfied consists of a linear dielectric medium characterized by a dielectric constant ϵ_1 in the region $z > d$; a linear metal characterized by a dielectric function $\epsilon(\omega)$ in the region $0 < z < d$; and a nonlinear dielectric medium characterized by the dielectric tensor (22) in the region $z < 0$. The nonlinear dispersion relation for this structure is given implicitly by

$$G(\omega,k) = -\frac{a^2}{4\epsilon_d} \left[\frac{\alpha K_3^4 + 2\beta K_3^2 k^2 + \alpha k^4}{K_3^4} \right] \tag{27}$$

to lowest nonzero order in α and β, where

$$G(\omega,k) = \frac{\left[1 + \frac{\epsilon(\omega)}{\epsilon_1} \frac{K_1}{K_2} \right] + \left[1 - \frac{\epsilon(\omega)}{\epsilon_1} \frac{K_1}{K_2} \right] e^{-2K_2 d}}{\left[1 + \frac{\epsilon(\omega)}{\epsilon_1} \frac{K_1}{K_2} \right] - \left[1 - \frac{\epsilon(\omega)}{\epsilon_1} \frac{K_1}{K_2} \right] e^{-2K_2 d}} \times \frac{\epsilon(\omega) K_3}{\epsilon_d K_2} + 1 , \tag{28}$$

and a is again $E_x(x,z=0\,|\,\omega)$. In Eqs. (27) and (28), $K_1 = k^2 - \epsilon_1(\omega^2/c^2)$, $K_2^2 = k^2 - \epsilon(\omega)(\omega^2/c^2)$, and $K_3^2 = k^2 - \epsilon_d(\omega^2/c^2)$. The nonlinear coefficient $\omega_2(k_0)$ is given by

$$\omega_2(k_0) = -\frac{1}{4\epsilon_d} \left[\frac{\alpha K_3^4 + 2\beta K_3^2 k^2 + \alpha k^4}{K_3^4} \times \frac{1}{[\partial G(\omega,k)/\partial \omega]} \right]_{\omega=\omega_0}, \tag{29}$$

where $\omega_0 = \omega_L(k_0)$ are, of course, the solutions of $G(\omega_L, k) = 0$ evaluated at k_0. This equation has two solutions, shown in Fig. 1.

Now, since $-\omega_2$ is proportional to α, and $(\partial^2 \omega_L / \partial k^2)_{k=k_0} < 0$ for surface polaritons propagating along a single interface between a metal and a dielectric, bright solitons are possible only for a metal overcoated with a self-defocusing medium.

On the other hand, as shown in Fig. 2, a metal film with two or more bounding dielectric media has much richer dispersive properties in which there are regions of negative group dispersion. In principle, bright solitons exist in the following regions for $\alpha = \beta$ and for the upper branch of the dispersion relation, labeled A and B in Fig. 1,

$$k > 1.83\epsilon_d^{1/2} \frac{\omega_p}{c}, \quad \alpha > 0 \tag{30}$$

$$k < 1.83\epsilon_d^{1/2} \frac{\omega_p}{c}, \quad \alpha < 0 \tag{31}$$

where ω_p is the plasma frequency of the metal.

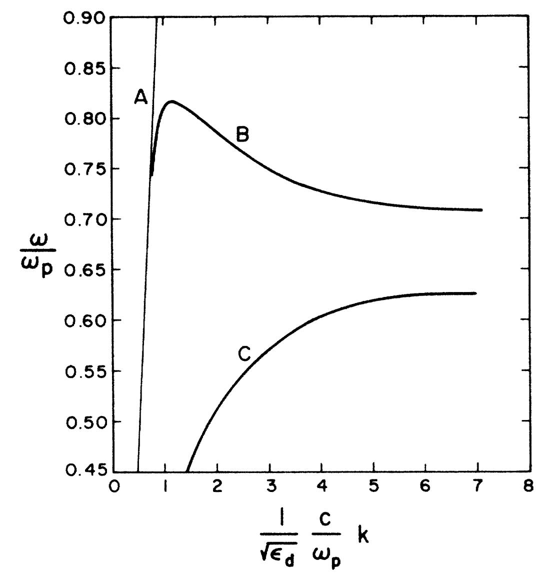

FIG. 1. Dispersion curves for surface polaritons in a thin metal film bounded by semi-infinite dielectric media. Data: $\epsilon_1 = 1.0$, $\epsilon_d = 1.5$, $\epsilon(\omega) = 1 - (\omega_p^2/\omega^2)$ is the dielectric function of the metal, $d = 0.5c/(\epsilon_d)^{1/2}\omega_p$ is the thickness of the metal film. Note that there are three distinct dispersion regions, namely, A and B, on either side of the maximum, and C.

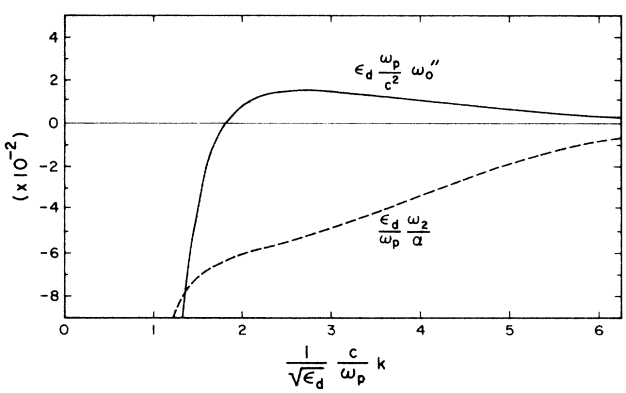

FIG. 2. Group velocity dispersion curve (region B) for a thin metal film bounded by semi-infinite dielectric media $(\epsilon_d \omega_p / c^2 \partial^2 \omega_0 / \partial k^2)$, and the effective nonlinear coefficient $(\epsilon_d \omega_2 / \omega_p \alpha)$. Data: $\epsilon_1 = 1.0$, $\epsilon_d = 1.5$, $\epsilon(\omega) = 1 - (\omega_p^2/\omega^2)$, $d = 0.5c/(\epsilon_d)^{1/2}\omega_p$.

It should be realized that the region defined by Eq. (30) [as well as part of the region defined by Eq. (31)] is also a region of negative group velocity. This means that any pulse propagating with a wave number in this region will consist of a carrier wave with frequency ω_0 traveling in the opposite direction to the pulse envelope in a manner similar to the backward wave devices of microwave technology. Such a "backward wave soliton" would at least require some rather unusual launch conditions.

Another cautionary point to note here is that the group velocity is also quite small in this region. This means that a fairly short pulse in space will, nevertheless, take a relatively long time to pass the observer and will therefore be quite a long pulse in time. Conversely, it is likely that, for the very short (picosecond) pulses predicted for solitons, the pulse length in space will be extremely short and care needs to be taken to ensure the validity of the slowly varying envelope approximation. For example, if a 2-ps pulse is traveling with a group velocity of 5×10^6 ms^{-1}, the spatial pulse width will be only 10 μm.

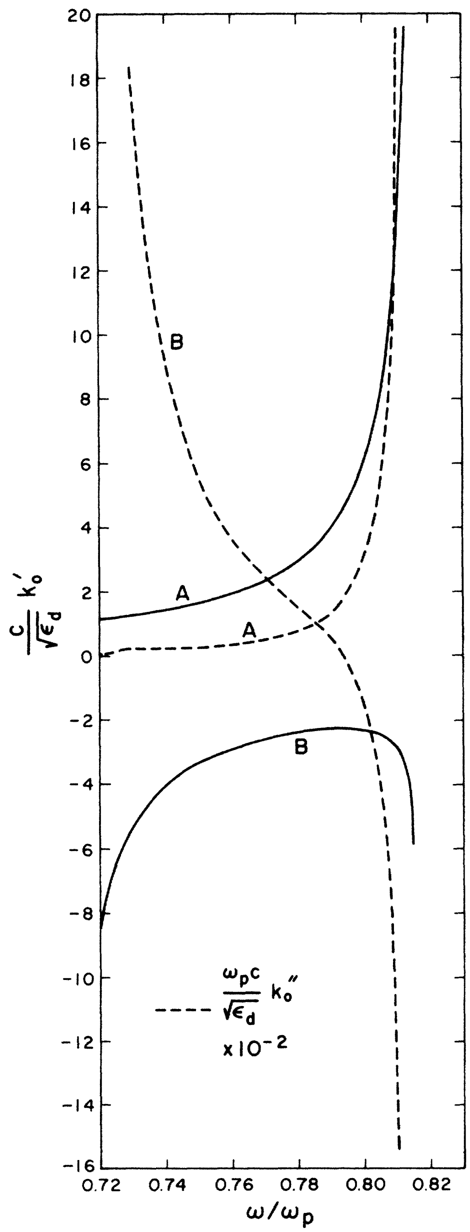

FIG. 3. Normalized inverse group velocity and its normalized second derivative calculated for the thin-film structure of Fig. 1 (regions A and B).

The definitions in Eqs. (14) and (16) can be used to calculate a characteristic length and time of the waveguiding structure. For example, suppose that a real pulse exists at $x=0$ with half width t_1. This is represented in the solutions of Eq. (17) by an initial pulse of dimensionless half width τ_1 so that

$$s^2=\left[\frac{\tau_1}{t_1}\right]^2\left[-\frac{k_0''}{k_0}\right]. \tag{32}$$

This fixes the scaling parameter to a value appropriate to the structure under study. If the peak amplitude at $\xi=0$ is q_1, say, then the actual field amplitude is

$$A=\left[\frac{k_0 c}{\omega}\right]\left[\frac{2}{\omega_2}\right]^{1/2}\left[\frac{\tau_1}{t_1}\right]\left[-\frac{k_0''}{k_0}\right]^{1/2} q_1 . \tag{33}$$

An estimate of the evolution distance for an arbitrary pulse can be found by setting $\xi=1$ in Eq. (16). This "rule of thumb" is very satisfactory when it is realized that the soliton period, in which the features of an $N>1$ soliton will repeat themselves, is $\xi=\pi/2$. The evolution distance is therefore

$$x=\frac{1}{s^2 k_0}=\left[\frac{t_1}{\tau_1}\right]^2\left[\frac{1}{-k_0''}\right]. \tag{34}$$

The nonlinearity is unbalanced in the absence of dispersion so that individual wave numbers now propagate with the same speed and interact for long periods. This leads to pulse distortion and eventual shock formation. Dispersion introduces unequal phase velocities and thus prevents this kind of breakup. It is, therefore, possible to devise a power threshold, based upon this balance, that will predict whether a soliton will evolve or not. This threshold follows from Eqs. (18) and (20). The peak amplitude of the lowest-order soliton, in dimensionless coordinates, is $|q_{\text{peak}}|=1$. This implies, from Eq. (20), that for a pulse of half width t_1 the magnitude of the peak amplitude $|A_{\text{peak}}|$ at the soliton launch is given by

$$|A_{\text{peak}}|^2 t_1^2=\left|\frac{1}{v_g^2}\frac{\omega_0''}{\omega_2(k_0)}\right|, \tag{35}$$

where $v_g=(\partial\omega_L/\partial k)_{k=k_0}=\omega_0'$. The minimum peak intensity required for soliton launching, with a pulse width t_1, is, since the energy is carried mainly outside the metal film,

$$I_{\text{peak}}\simeq\frac{\epsilon_0\epsilon_d}{2}v_g|A_{\max}|^2\ \text{W m}^{-2}. \tag{36}$$

In detail this expression reduces to

$$I_{\text{peak}}=4.425\times10^{-12}\left[\frac{\epsilon_d\omega_0''}{\omega_2\omega_0' t_1^2}\right]\text{W m}^{-2}. \tag{37}$$

Equations (34) and (36), can be illustrated for the thin-film structure discussed above. We assume the values $\omega_p = 8\times10^{15}\ s^{-1}$, $t_1 = 10$ ps, and $(t_1/\tau_1)^2 = 4.1\times10^{-23}$. Then if, for example, $\omega/\omega_p = 0.75$, $\epsilon_d = 1.5$, and $\alpha = 1.42\times10^{-17}\ m^2V^{-2}$, Fig. 2 shows that $\omega_2 = -3.8\times10^{-3}\ m^2s^{-1}V^{-2}$ and $\omega_0'' = (\partial^2\omega_L/\partial k^2)_{k=k_0} = 9.8\times10^{-2}\ m^2s^{-1}$, and Fig. 3 gives $k_0' = (\partial k_L/\partial\omega)_{\omega=\omega_0} = -1.34\times10^{-8}\ s\,m^{-1}$, $\omega_0' = v_g = 1/k_0' = -7.5\times10^{7}\ m\,s^{-1}$, and $k_0'' = (\partial^2 k_L/\partial\omega^2)_{\omega=\omega_0} = 21\times10^{-26}\ s^2m^{-1}$. These data give $x \sim 14$ cm and $I_{peak} \simeq 23$ mW/mm^2. Since the evolution distance in this case is very large, any threshold pulse that travels only a short distance will not disperse significantly, even in the absence of nonlinearity. The nonlinearity does not, therefore, have much impact unless the system is extremely long. A real integrated optics system will be $\ll 14$ cm in length.

The distance $x \sim 14$ cm is a yardstick or benchmark of the system. Another yardstick is the distance within which nonlinear effects would be observed on pulses of higher power. These higher power pulses result in higher-order solitons that are associated with pulse compression[12] within distances $x/(2^{N-1})$, where $N \geq 2$ is the soliton order. For $x \sim 14$ cm, $N = 8$ is required for nonlinear pulse compression to appear within ~ 1 mm. The intensity needed for this to occur would be ~ 11 W/mm^2. It should be emphasized that these figures for x, I_{peak}, and N are realistic estimates and show that solitons ought to be observable for this particular structure since $N = 13$ has been observed in optical fibers.[12]

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