Plasmon linewidth in metals and semiconductors: A memory-function approach

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The relation of the plasmon linewidth $1/\tau_p$ to the transport lifetime τ_t is investigated for an electron system interacting with charged impurities in three-dimensional (3D) and two-dimensional (2D) metals and semiconductors as well as in modulation-doped 2D systems. We calculate the dielectric function and the dynamical structure factor within the memory-function technique introduced by Götze and Wölfle, and read off the plasmon width directly from the structure factor. We find that for large electron densities τ_p and τ_t are roughly equal, while for low densities τ_p/τ_t may be as small as 0.2. For 3D systems, the plasmon line has a strongly non-Lorentzian shape, and an anomalously large width. All these results are interpreted as resulting from the screening properties of the electron gas. Together with a recent comparison of τ_t with the single-particle lifetime τ_s, they show how to extract information about the disorder in the system by observing experimentally the electron mobility or the plasmon resonance.

I. INTRODUCTION

The problem of electronic transport in disordered solids is of considerable interest form both academic and practical viewpoints. In particular, the study of two-dimensional (2D) electron systems has been very fruitful.[1] One particular problem one encounters in the theoretical analysis of experimental results is the fact that the relaxation rate for single-particle excitations $1/\tau_s$ and the transport relaxation rate $1/\tau_t$ are not simply related. The basic difference is that correlations between two different electrons scattering off the same impurity enter τ_t but not τ_s.[2] In the technical description of transport by perturbation theory, these correlations appear as vertex corrections. Despite this difference, it has been common knowledge (based on experience in metal physics) that $\tau_t \simeq \tau_s$ usually is a good approximation. τ_s cannot be measured directly, but it is an important parameter for the effects of disorder on thermodynamic quantities like the density of states, or the wave-vector-dependent compressibility,[3] which in turn describes the screening properties of the electrons and therefore influences τ_t. Recently it was found that in GaAs-based heterojunctions an estimate of τ_s from the above-mentioned approximate equality yields values far too large to be consistent with experimental findings.[4] These materials appear to have a much larger mobility than the amount of disorder observed by other means would suggest. On the other hand, in Si inversion layers, $\tau_t \simeq \tau_s$ holds to the usual accuracy,[5] which is about a factor of 2. These observations have been explained very recently by Das Sarma and Stern,[6] who showed that the suppression of backward scattering in the heterojunctions due to the peculiar microscopic geometry in these samples is sufficient to yield a τ_t 20 to 70 times larger than τ_s.

A natural question arising in this context is how relaxation times for other electronic excitations, e.g., plasma or cyclotron resonance, compare to τ_s and τ_t. This is particularly interesting from an experimental point of view, since these resonance lifetimes can be measured directly as widths of peaks in a spectrum, while to extract τ_t from data a theory is necessary, and for τ_s this task is even more difficult. On the other hand, theoretically the plasmon or cyclotron resonance linewidths cannot be obtained from the standard Boltzmann equation solved in collision time approximation, since in the region of interest the relaxation rates are strongly frequency dependent. The cyclotron resonance problem is further complicated by the fact that one has to deal with screening phenomena in a strong magnetic field. In this paper we confine ourselves to the plasma resonance which is simpler to deal with.

We consider an interacting electron gas in the presence of randomly distributed charged impurities. We want to investigate the plasmon linewidth $1/\tau_p$ in comparison with the scattering relaxation rate $1/\tau_t$ for two different kinds of systems. The conceptually simpler systems are homogeneous 3D metals or semiconductors. Of more recent interest is the case of modulation doped 2D electron systems, where the electrons are confined to one plane and the impurities to another, parallel to the first at a distance z_i. A strictly 2D system is obtained as the special case $z_i = 0$.

We will consider the plasma resonance at long wavelengths, where it is well defined, and consider broadening due to impurity scattering only. We will neglect other damping mechanisms, such as interband transitions, or multiple electron-hole pair production. As a tool we choose the memory-function technique developed by Götze and Wölfle,[7] supplemented by Götze's generalized relaxation time approximation[8] for the density correlation function. The latter is a generalization of Mermin's approach,[9] where the relaxation time was a phenomenological constant. The memory-function technique is ideally suited to handle effects at nonzero frequency and wave number, and its result for the relaxation time reduces in the static long-wavelength limit to the well-known Mott-

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Jones formula. Recently, a self-consistent version of this formalism[8] has been applied by Gold and Götze[10] to strongly disordered interacting electrons. While Ref. 10 focused on properties of systems near a metal insulator transition, we are interested in relatively clean systems. We will therefore not make the theory self-consistent, and instead keep the wave-number dependence of the memory function. We will also neglect all effects arising from the interplay between diffusive modes and the electron-electron interaction.[11] This is justified since the plasma resonance occurs at high frequencies well outside the diffusive regime. We find that for both 2D and 3D systems, τ_p/τ_t is mainly determined by the density parameter $y=k_F/q_{\rm TF}$, where k_F and $q_{\rm TF}$ are Fermi wave number and Thomas-Fermi screening wave number, respectively. For $y\gtrsim 0.5$, we find $\tau_p/\tau_t\simeq 1$. In $d=3$ dimensions, the sudden breakdown of screening at the plasma frequency leads to a strongly non-Lorentzian shape of the plasma resonance, but this does not spoil the approximate equality of τ_p and τ_t. On the other hand, for low density ($y\lesssim 0.2$), the strong frequency dependence of the screening leads to values of τ_p/τ_t as small as 0.2. τ_p/τ_t is essentially independent of the separation parameter z_i in the heterojunctions, in sharp contrast[6] to the ratio τ_t/τ_s.

II. THE MEMORY-FUNCTION FORMALISM

A. Random-phase approximation

We consider a system of electrons which interact mutually via a Coulomb potential $v(\mathbf{q})$, and with static impurities via some potential $u(\mathbf{q})$. We are interested in the electronic dynamical structure factor $S(\mathbf{q},\omega)$, which may be expressed in terms of the dielectric function $\epsilon(\mathbf{q},\omega)$ (Ref. 12)

$$S(\mathbf{q},\omega)=[1/\pi v(\mathbf{q})]\epsilon_2(\mathbf{q},\omega)\,|\,\epsilon(\mathbf{q},\omega)\,|^{-2}\,, \tag{2.1}$$

where ϵ_2 is the imaginary part of ϵ. By definition,[12] the dielectric function is determined by the response $\chi_{\rm sc}(\mathbf{q},\omega)$ of the electron system to a screened external field. In a clean system, the random-phase approximation[12] (RPA) is obtained by assuming $\chi_{\rm sc}$ to be identical to the response $\chi^{(0)}(q,\omega)$ of a free-electron gas. In the presence of impurities, it is natural to include the electron-impurity interaction into $\chi_{\rm sc}$. If we denote the irreducible polarizability of the disordered system by χ, our approximation for ϵ reads

$$\epsilon(\mathbf{q},\omega)=1-v(\mathbf{q})\chi(\mathbf{q},\omega)\,. \tag{2.2}$$

For χ, we use an approximation proposed by Götze,[8] who expressed χ in terms of $\chi^{(0)}$, and a generalized wave-vector- and frequency-dependent current relaxation rate $M(q,\omega)$. The result is

$$\chi(q,\omega)=\chi^{(0)}(q,\omega+M(q,\omega))[\omega+M(q,\omega)]\times[\omega+M(q,\omega)\times\chi^{(0)}(q,\omega+M(q,\omega))/\chi^{(0)}(q,0)]^{-1}\,. \tag{2.3}$$

If $M(q,\omega)$ is replaced by a simple relaxation time i/τ, Eq. (2.3) is familiar from kinetic theory, and has been derived in the present context by Mermin.[9] The current relaxation kernel $M(0,\omega)$ has been calculated to lowest order in the impurity concentration n_i by Götze and Wölfle.[7] The relaxation kernel or memory function technique as opposed to using diagrammatic methods or kinetic equations has been discussed at length in the literature,[13] so we use it here without any further comment. The zero-wave-number result of Ref. 7 is readily generalized to obtain

$$M(q,\omega)=\frac{1}{\omega}[\phi(q,\omega)-\phi(q,0)]\,, \tag{2.4a}$$

where

$$\phi(q,\omega)=-(n_i/nm)\sum_{\mathbf{k}}|\,u(\mathbf{q}-\mathbf{k})\,|^2 s(\mathbf{q}-\mathbf{k})\times[(\mathbf{q}-\mathbf{k})\mathbf{q}/q]^2\chi_c(\mathbf{k},\omega)\,. \tag{2.4b}$$

Here n_i, n, and m are impurity density, electron density, and electron mass, respectively, and $s(\mathbf{q})$ is the impurity structure factor. χ_c denotes the response function for the clean, interacting electron system, within RPA it reads as

$$\chi_c(\mathbf{k},\omega)=\chi^{(0)}(\mathbf{k},\omega)[1-v(\mathbf{k})\chi^{(0)}(\mathbf{k},\omega)]^{-1}\,. \tag{2.4c}$$

The functions $v(\mathbf{q})$, $u(\mathbf{q})$, and $s(\mathbf{q})$ determine the model. Once they are specified, Eqs. (2.1)—(2.4) reduce the task of calculating the dynamical structure factor to the evaluation of one d-dimensional integral in Eq. (2.4b).

Very recently Gold and Götze[10] have studied a self-consistent version of Eqs. (2.3)—(2.4b), which uses the self-consistency idea of Götze[8] to describe the disorder-induced metal-insulator transition in an interacting electron system. These authors also have included local field corrections to the RPA within a Hubbard approximation, and discussed, among other things, plasmon excitation anomalies due to strong disorder. In contrast, we are interested in the variation of the plasmon linewidth with electron density in relatively clean systems. We therefore neglect all self-consistency effects, but keep the q dependence of the current relaxation kernel, which for technical reasons had to be neglected in Ref. 10.

B. Basic properties of the model

In order to introduce some notation and provide a basis for the discussion in Sec. III, let us briefly recall the RPA result in the clean limit.

The electrons interact via a bare Coulomb potential

$$v(\mathbf{q})=(\pi e^2/K)(2/\mathbf{q})^{d-1}\,, \tag{2.5}$$

where K is a static background dielectric constant. Here and in the following d denotes the dimensionality of the system, and all formulas are valid for $d=2$ or 3 only. The dielectric function for the clean system is then given by Eq. (2.2), with χ replaced by the free-electron polarizability

$$\chi^{(0)}(\mathbf{q},\omega)=-\rho_F K_d^{(0)}(q/2k_F,\omega/4\varepsilon_F)\,, \tag{2.6a}$$

here $\rho_F=(g_v m/\pi)(k_F/\pi)^{d-2}$ is the density of states at the Fermi level with a possible valley degeneracy factor g_v, $\varepsilon_F=k_F^2/2m$ is the Fermi energy (we choose units such that $\hbar=1$), and $K^{(0)}$ is the Lindhard function. For $d=3$ it reads[12] as

$$K_3^{(0)}(x,z)=\frac{1}{2}+\frac{1}{8x}\left[[1-(x-z/x)^2]\ln\left[\frac{x+1-z/x}{x-1-z/x}\right]+[1-(x+z/x)^2]\ln\left[\frac{x+1+z/x}{x-1+z/x}\right]\right] \tag{2.6b}$$

and for $d=2$,[14]

$$K_2^{(0)}(x,z)=1+\frac{1}{2x^2}\{[z-x(x-1)]^{1/2}[z-x(x+1)]^{1/2}-[z+x(x-1)]^{1/2}\times[z+x(x+1)]^{1/2}\}\ . \tag{2.6c}$$

From Eqs. (2.2), (2.5), and (2.6) one easily finds

$$\lim_{q\to 0}\lim_{\omega\to 0}\epsilon^{(0)}(q,\omega)=1+(q_{\rm TF}/q)^{d-1} \tag{2.7}$$

with the Thomas-Fermi screening wave number $q_{\rm TF}=2(\pi e^2\rho_F/K)^{1/(d-1)}$ and

$$\lim_{q\to 0}\epsilon^{(0)}(q,\omega)=1-\omega_q^2/\omega^2\ . \tag{2.8a}$$

With the dimensionless electron density parameter $y=k_F/q_{\rm TF}$, the bare plasma frequency ω_q reads for $d=3$,

$$\omega_q=\omega_0\left[1+\frac{3}{10}\left[\frac{q/2k_F}{\omega_0/4\varepsilon_F}\right]^2+O(q^4)\right]\quad(d=3)\ , \tag{2.8b}$$

with $\omega_0=4\varepsilon_F/2\sqrt{3}y$, and for $d=2$,

$$\omega_q=4\varepsilon_F(q/2k_F)^{1/2}/2y^{1/2}+O(q)\quad(d=2)\ . \tag{2.8c}$$

Notice that the basic difference between $d=2$ and $d=3$ is the softening of the 2D plasmon in the long-wavelength limit. Apart from k_F and $q_{\rm TF}$, there is another length scale in the system, given by the wavelength q_c at which plasmon damping due to electron-hole production first becomes possible. For $d=3$, q_c has been obtained by Ferrell[15] as solution of the equation

$$q_c^{d=3}/2k_F=\frac{1}{y2\sqrt{2}}[(1+q_c/2k_F)\ln(1+2k_F/q_c)-1]^{1/2}\ . \tag{2.9a}$$

for $d=2$, one finds

$$q_c^{d=2}/2k_F=\frac{1}{2y}[(1+2k_F/q_c)^{1/2}-1]\ . \tag{2.9b}$$

C. The current relaxation kernel

The sharp plasma resonance in the clean system is due to the fact that for $q<q_c$ energy-momentum conservation cannot be fulfilled in processes where the plasmon decays into a single electron-hole pair. However, in the presence of impurities these can absorb excess recoil momentum, and the process becomes possible, leading to a finite dc conductivity as well as to a nonzero width of the plasma excitation. Within the present formulation, the damping is expressed in terms of a nonzero current relaxation kernel, Eqs. (2.4).

To further specify our model, we assume scattering off charged, uncorrelated impurities. Then we have $s(\mathbf{q})\equiv 1$, and the scattering potential can be written as[1]

$$u(\mathbf{q})=Zv(\mathbf{q})e^{q(d-3)|z_i|}\ . \tag{2.10}$$

Here Ze is the charge of the impurities, and the exponential describes "two-dimensional" systems where electrons and impurities are confined to two different planes which are separated by a distance z_i. With this spatial separation between electrons and impurities we simulate modulation doping of a 2D heterostructure within our model.

In a real 2D system such as an inversion layer or a heterostructure one has the additional complication of a finite width[1] of the electron layer which shows up as quantum form factors[1] modifying Eqs. (2.5) and (2.10). These form factors do not affect the long-wavelength ($q\to 0$) results, and change finite wave-number results only quantitatively. Consistent with the fact that we have neglected local field corrections in this paper we also ignore these form-factor effects. Their incorporation in the theory is straightforward with the right-hand side of Eqs. (2.5) and (2.10) being multiplied by form factors given in Ref. 1. In this paper we do not wish to do realistic calculations for any specific system, rather our goal is to provide a generic comparison of relative magnitudes of τ_p and τ_t which is qualitatively unaffected by form factors. Our 2D results are therefore strictly valid only for systems of zero thickness. If detailed experimental results on τ_p become available it will be simple to apply this formalism to obtain results for a specific system.

It is convenient to introduce dimensionless units. We will measure wave numbers in units of $2k_F$, energies in units of $4\varepsilon_F$, and the polarizability in units of ρ_F. Then we have from Eq. (2.4b)

$$\phi(q,\omega)=\frac{Z^2n_i}{g_v n}\frac{1}{2^d y^{2(d-1)}}\int_0^\infty dk\,k^{d-1}V_d(q,k)\chi_c(k,\omega) \tag{2.11}$$

with the vertex function

$$V_3(q,k)=(2/q^2)K_3^{(0)}(k/q,0)\ , \tag{2.12a}$$

$$V_2(q,k)=\frac{1}{\pi}\int_0^{2\pi}d\phi\frac{(q-k\cos\phi)^2}{q^2+k^2-2kq\cos\phi}\times\exp[-(|z_i|q_{\rm TF})\times 4y(q^2+k^2-2qk\cos\phi)^{1/2}]\ . \tag{2.12b}$$

Before we evaluate Eq. (2.11) numerically, we discuss a few results which can be obtained analytically.

In the static long-wavelength limit, the current relaxa-

tion kernel yields the transport relaxation time in the Drude formula:[7] $\tau_t = i/M(0,i0)$. From Eq. (2.11) one obtains

$$1/\tau_t = (Z^2 n_i/g_v n) f_d(y) \tag{2.13a}$$

with[7,10]

$$f_3(y) = (2\pi/3)\int_0^1 dk\, k^3 [4y^2k^2 + K_3^{(0)}(k,0)]^{-2}\,, \tag{2.13b}$$

$$f_2(y) = \int_0^1 dk \frac{1}{(1-k^2)^{1/2}} \frac{k^2}{(4y^2k^2+1)^2} e^{-(|z_i|q_{\rm TF})4yk}\,. \tag{2.13c}$$

Equation (2.13b) is a generalized Mott-Jones formula,[7] and (2.13c) is its 2D counterpart.[10]

At nonzero frequency, the integration in Eq. (2.11) may pick up a singular part because of the plasma pole inherent in χ_c. In $d=3$, this will happen only for frequencies $\omega_0 > \omega$. For frequencies near the threshold, we can use long-wavelength expansions to obtain for the singular part of χ_c:

$$\mathrm{Im}\chi_c^s(k\to 0,\omega) = -8\pi y^4 \omega^3 k^2 10\omega_0 \delta(k^2 - \tfrac{10}{3}\omega_0(\omega-\omega_0)) \quad (d=3)\,.$$

This yields for the singular part of the current relaxation kernel

$$\mathrm{Im}M^s(q,\omega\simeq\omega_0) = \frac{Z^2 n_i}{g_v n} 10\pi (\tfrac{10}{3})^{3/2} \frac{\omega_0^{9/2} y}{q^2} (\omega-\omega_0)^{3/2} \quad (d=3)\,. \tag{2.14}$$

This formula is valid only for $0<\omega-\omega_0 \ll q^2/\omega_0$; for $q^2/\omega_0^2 \ll \omega/\omega_0 - 1 \ll 1$ it crosses over to an $(\omega-\omega_0)^{1/2}$ behavior and for $q=0$ the square root singularity is found down to $\omega=\omega_0$.[7] For $d=2$, there is no threshold, and the nonanalyticity has to occur at zero frequency. Indeed we have for small frequencies

$$\mathrm{Im}\chi_c^s(k\to 0,\omega) = \pi y k^{3/2} \delta(k^{1/2} - 2y^{1/2}\omega) \quad (d=2)\,,$$

which leads to

$$\mathrm{Im}M^s(q,\omega\simeq\omega_q\to 0) = \frac{Z^2 n_i}{g_v n} 2^6 \pi y^2 |\omega|^5 \quad (d=2)\,. \tag{2.15}$$

Contrary to $d=3$, there is no difference between the leading result for nonzero q and the one for $q=0$.[10] The qualitative difference between the threshold behavior in $d=3$ and the singularity at $\omega=0$ in $d=2$ will have a pronounced effect on the dynamical structure factor.

As a last point of our general discussion we consider the plasmon pole itself. For small wave numbers, particle number conservation quite generally enforces the following expression for the polarizability [which can also be obtained from Eq. (2.3)]:

$$\chi(q,\omega) = \frac{1}{d} \frac{q^2}{\omega^2 + \omega M(q,\omega)}\,. \tag{2.16}$$

Using Eqs. (2.2) and (2.1) this leads to the following pole structure of the dynamical structure factor:[10]

$$S(q,\omega\simeq\Omega_q) = \frac{1}{\pi}(q/q_{\rm TF})^{d-2} \frac{\omega_q^2 \omega\, \mathrm{Im}M(q,\omega)}{(\omega^2-\Omega_q^2)^2 + (\omega 2\Gamma_q)^2}\,, \tag{2.17a}$$

where the dressed plasma frequency Ω_q is obtained as solution of the equation

$$\Omega_q^2 = \omega_q^2 - \omega_q \mathrm{Re}M(q,\Omega_q)\,. \tag{2.17b}$$

The plasmon lifetime $\tau_p = (2\Gamma_q)^{-1}$ is given by

$$2\Gamma_q = \mathrm{Im}M(q,\Omega_q)\left[1 + \frac{1}{2\Omega_q}\left[\mathrm{Re}M(q,\Omega_q) - \Omega_q \frac{\partial}{\partial x}\mathrm{Re}M(q,x)\Big|_{x=\Omega_q}\right]\right]^{-1}\,. \tag{2.17c}$$

Equation (2.17c) shows that the plasmon lifetime is not just given by the imaginary part of M, but rather the reactive part gives a substantial contribution. It has been emphasized in Ref. 10 that for this reason the Drude results, which are obtained by putting $\mathrm{Im}M(q,\Omega_q) \equiv i/\tau_t$ and $\mathrm{Re}M(q,\Omega_q)\equiv 0$, are not correct even in the limit $q\to 0$. Exactly this is the reason why we will obtain nontrivial results for the ratio τ_p/τ_t: As we will see, for different electron densities the frequency dependence of $M(q,\omega)$ is rather drastically different, which via Eq. (2.17c) yields different results for τ_p/τ_t. Since Eqs. (2.17) [as well as Eqs. (2.5) and (2.10)] are valid for small wave numbers only, they may be quantitatively misleading in $d=2$, though they describe the phenomena qualitatively correctly. To be closer to what is actually done in experiment, we will therefore calculate $S(q,\omega)$ from Eqs. (2.1)–(2.4), and read off τ_p directly in Sec. III. For a discussion of these results, we will come back to Eqs. (2.17).

III. RESULTS AND DISCUSSION

We have calculated the dynamical structure factor from Eqs. (2.1)–(2.4), and determined the plasma lifetime τ_p as the inverse width at half height of the resonance in $S(q,\omega)$. We discuss first the results for $d=3$. There are three length scales in the system: k_F, $q_{\rm TF}$, and q_c. We have chosen $q = 0.1 q_{\rm TF}$, other choices of q do not yield significantly different results. In Fig. 1 we show τ_p/τ_t as a function of the density parameter y, in Fig. 2 we show $\mathrm{Im}M$, and $\mathrm{Re}M$ for two different densities. The most interesting features are that for low densities, τ_p/τ_t it significantly smaller than 1, and that the plasma resonance has a strongly non-Lorentzian shape. To understand this behavior, we start at low densities. At low frequencies, there is strong screening, and $S(q,\omega)$ is strongly suppressed. However, due to the low electron density the screening is frequency dependent even for frequencies

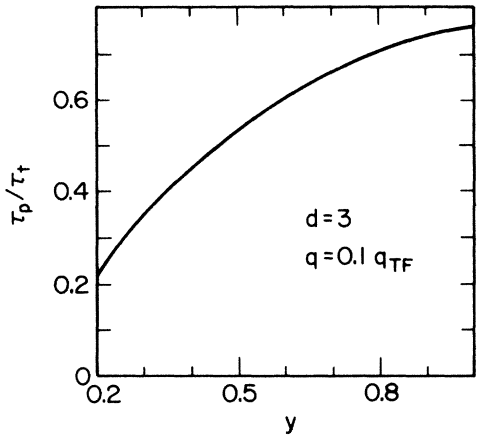

FIG. 1. Ratio of plasmon and transport lifetime, τ_p/τ_t, versus electron density parameter y, for $d=3$. See text for further explanation.

smaller than the plasma frequency. At $y=0.2$, Fig. 2(a), ImM rises by a factor of 3 if ω varies between 0 and ω_0. This ω dependence of ImM, which tends to decrease τ_p, also results in a nonzero ReM, which counteracts the decrease of τ_p according to Eq. (2.17c), but this is a comparatively small effect. At the plasma frequency, the screening breaks down completely, and the electrons see the bare Coulomb potential. Consequently, the dissipative part of M decreases dramatically, and its real part shows the corresponding resonance behavior. This increase of absorption manifests itself also in $S(q,\omega)$, which decreases much more slowly towards high frequencies than towards low frequencies. The resulting resonance is triangle shaped rather than Lorentzian, cf. the insert of Fig. 2(a). This anomalous shape is the counterpart of the peak in the homogeneous conductivity discussed by Götze and Wölfle.[7] The larger width of the triangle curve as compared to a Lorentzian further decreases τ_p, and the resulting τ_p is smaller than τ_t by a factor of 5. Within the present formalism, the current decays into bare (with respect to disorder) density modes, Eq. (2.4b). Consequently, the breakdown of screening occurs at the bare plasma frequency ω_0, rather than at the dressed one, $\Omega_{q=0}$. This somewhat unphysical feature is cured by the self-consistent formalism of Ref. 10. However, since we are working at nonzero wave numbers $q\neq 0$, the peak in $S(q,\omega)$ occurs at $\Omega_{q\neq 0}>\Omega_{q=0}$. For the small disorder chosen (cf. Table I), we still have $\Omega_q-\frac{1}{2}\tau_p>\omega_0$, see Fig. 2. Therefore the breakdown of screening is fully reflected in the plasmon peak, as it would be in a self-consistent calculation, and our lowest-order theory is valid. With in-

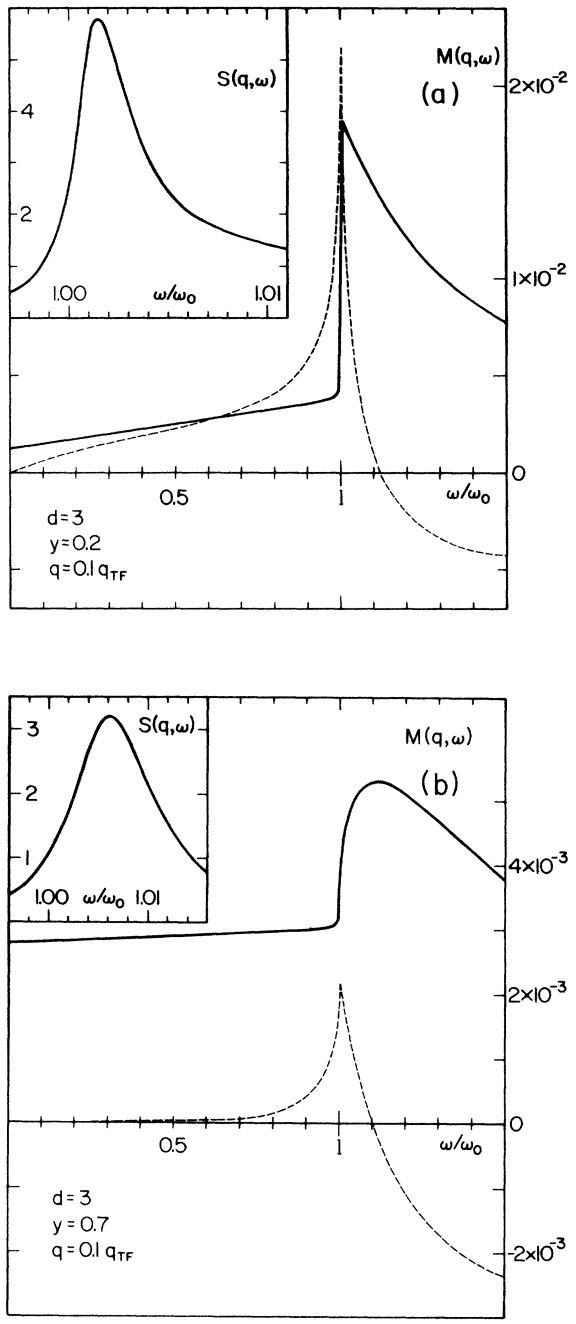

FIG. 2. Imaginary part (solid line), and real part (dashed line) of current relaxation kernel $M(q,\omega)$ versus frequency ω, for $d=3$, and (a) $y=0.2$; (b) $y=0.7$. The inset shows the plasmon peak in the dynamical structure factor $S(q,\omega)$. M is given in units of $4\varepsilon_F$, S is given in arbitrary units.

TABLE I. values of disorder parameter Z^2n_i/n used for the results shown in Figs. 1–3. The numbers are obtained by scaling Z^2n_i/n with y (y^2) for $d=2$ (3). According to the criterion given in Ref. 10, these values are sufficient to always be far from the metal-insulator transition.

y	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$10^2Z^2n_i/n$ ($d=2$)	3	6	9	12	15	18	21	24	27	30
$10^4Z^2n_i/n$ ($d=3$)		8	18	32	50	72	98	128	162	200

creasing density, screening becomes better and better, and ImM becomes less and less frequency dependent for $\omega < \omega_0$. Consequently, τ_p/τ_t increases. Also, the rise of ImM at ω_0 is less dramatic for larger y, cf. the prefactor of the nonanalyticity in Eq. (2.14). Therefore also the non-Lorentzian shape of $S(q,\omega)$ with its anomalously large width gets less and less pronounced, Fig. 2(b), and τ_p/τ_t approaches unity at larger y.

For 2D systems, the picture is somewhat different. Since the nonanalyticity in $M(q,\omega)$ due to the bare plasma pole occurs at $\omega=0$, Eq. (2.15), M shows no particular structure at the plasma frequency. However, as in $d=3$ the overall frequency dependence of M is more pronounced for small densities, hence τ_p/τ_t increases with increasing y. Contrary to $d=3$, however, the result at small densities depends rather strongly on the wave number chosen. The reason for this is the fact that in $d=2$ the plasma frequency goes to zero with vanishing wave number. At small densities, where q_c and the corresponding frequency ω_c are larger in the natural Fermi liquid units ($2k_F$ and $4\varepsilon_F$), different choices of q result in very different plasma energies. It is clear that for small q, and therefore small Ω_q, the difference between $M(q,\Omega_q)$ and $M(0,i0)$ will be smaller than for larger q. Consequently, τ_p/τ_t will be larger for smaller q. We have chosen the disorder as shown in Table I. Since z_i enters only via the vertex function V_2, Eq. (2.12b), its effects on $M(q,\Omega_q)$ and $M(0,i0)$ are not very different, and we have found τ_p/τ_t to be only weakly dependent on z_i. For simplicity, therefore, we put $z_i=0$. For these parameters, Fig. 3 shows τ_p/τ_t for $q=0.1\ q_{\rm TF}$, and $q=0.2k_F$, respectively. Since $q_{\rm TF}$ in $d=2$ is independent of the density, the first choice results at low densities in a larger $\Omega_q/4\varepsilon_F$, hence $M(q,\Omega_q)$ is substantially larger than its zero-frequency value, and τ_p/τ_t is strongly suppressed. For the second choice of q, this effect is much less pronounced, and τ_p/τ_t is of order unity except for the very lowest densities. This conclusion should remain valid even in real 2D structures because form-factor effects are small in the long-wavelength limit.

In conclusion we have studied the plasmon lifetime compared to the transport relaxation time for 2D and 3D electron liquids by means of the memory-function technique. For 3D systems at metallic densities, we have found $\tau_p \simeq \tau_t$, but τ_p/τ_t is considerably smaller at semiconductor densities. The plasma peak in the dynamical structure factor has an anomalous shape. This has been shown to be due to the breakdown of dynamical screening at the plasma frequency. At densities typical for bulk semiconductors, the screening shows a relatively strong frequency dependence, and τ_p/τ_t may be as small as 0.2.

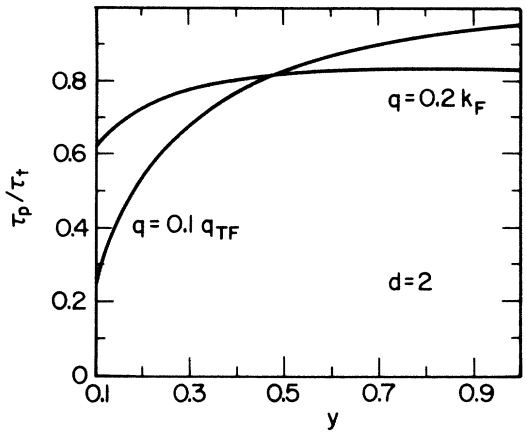

FIG. 3. Ratio of plasmon and transport lifetime, τ_p/τ_t, versus electron density parameter y, for $d=2$. See text for further explanation.

For 2D systems, τ_p/τ_t also increases monotonously with increasing density, and its value at low densities depends strongly on the wave number. For large wave numbers, i.e., large plasma frequencies, and low density τ_p/τ_t may be as small as 0.2, while for smaller plasma frequency one has always $\tau_p \simeq \tau_t$. In sharp contrast to the ratio of τ_t and the single-particle lifetime τ_s, which in Ref. 6 has been found to be strongly dependent on the distance parameter z_i of 2D systems, τ_p/τ_t does not significantly depend on z_i.

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