

Quantitative prediction of permeability in porous rock

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We show that percolation concepts lead to the definition of a characteristic length for the permeability in random porous media. Application of the model to sandstone and carbonate rocks yields quantitative agreement between theory and experiment with no adjustable parameters.

The transport properties of fluids in porous media are of broad interest in science and technology. They belong to the general class of transport in random media including electronic transport in amorphous conductors. Transport in porous media is central to technological problems in porous electrochemical electrodes, filters, and gels and to establishing the producibility of petroleum reservoirs. There have been numerous attempts over the last sixty years to establish a relationship between the permeability of a random porous material and other characteristic material properties.¹ Early work¹ expressed the permeability, which has units of length squared, as a function of hydraulic radius (the pore volume divided by the pore surface area) and such macroscopic properties as the total porosity. The hydraulic radius has been used as the characteristic length up to the present time.^{2,3} Recent work attempts to justify the Carmen-Kozeny equation on the basis of network or effective-medium models.^{3,4} These attempts fail to give satisfactory results for two reasons. First, they attempt to relate the transport properties to the macroscopic geometric properties. Second, they do not recognize the fundamental physical significance and sensitivity of the length scale in the permeability prediction.

We propose and present experimental evidence supporting the following relationship for the permeability of rocks saturated with a single liquid phase:

$$k = cl_c^2(\sigma/\sigma_0) . \quad (1)$$

The permeability k is defined by the Darcy relation: $\mathbf{v} = (k/\eta)\nabla p$, where \mathbf{v} is the velocity of fluid flow through the rock, η is the fluid viscosity, and ∇p is the pressure gradient imposed across the rock sample. Here l_c is some characteristic length of the pore space, σ is the conductivity of the rock saturated with a brine solution of conductivity σ_0 , and c is a constant on the order of $\frac{1}{226}$. The conductivity term reflects the connectedness of the pore space, whereas the length term sets the scale for the permeability. We show that Eq. (1) follows from the percolation arguments of Ambegaokar, Halperin, and Langer (AHL),⁵ which lead to a definition of the characteristic length l_c . The length l_c is a unique transport length scale and dominates the magnitude of the permeability. New contributions of this work include recognizing the importance of this length to the physics of permeability and directly measuring l_c from mercury injection experiments.⁶ Our results verify the *quantitative* accuracy of the AHL approximation—in agreement with recent computer

simulations⁷—and support the conjecture that mercury injection has a percolation geometry.⁸

It is well established⁹ that pore spaces of many reservoir rocks are characterized by a broad distribution of pore sizes. A broad distribution of pore sizes suggests that transport through the pore space must be understood in terms of a broad distribution of local conductances. For each point \mathbf{r} in the pore space, we assign a value $\lambda(\mathbf{r})$, where $\lambda(\mathbf{r})$ represents the diameter of the largest sphere that contains the point \mathbf{r} and is fully contained in the pore space. We next group the points into continuous patches of pore space with common value λ . We assume a patch where assigned value λ is also roughly of linear dimension λ [we can relax this constraint without invalidating our argument, but it is important that no long-range correlations in the function $\lambda(\mathbf{r})$ persist].

AHL⁵ suggest that transport in a random system with a broad distribution of conductances is dominated by those conductances with magnitudes greater than some characteristic value g_c . The characteristic conductance g_c represents the largest conductance, such that the set of conductances $\{g \mid g > g_c\}$ still forms an infinite, connected cluster. Hence, transport in such a system reduces to a percolation problem with threshold value g_c . Kirkpatrick¹⁰ and Shante¹¹ have carried these ideas further. They assign all local conductances with values $g \geq g_c$ the value g_c , and set all conductances with values $g < g_c$ to zero; they arrive at a trial solution for the sample conductance of the form

$$g = \bar{k}g_c[p(g_c) - p_c]^t , \quad (2)$$

which can be maximized with respect to the variable g_c . Here, $p(g_c)$ denotes the probability that a given conductance is greater than or equal to g_c , and \bar{k} is an appropriate constant. In three dimensions, the percolation exponent $t = 1.9$.¹²

We interpret transport within rock pore spaces in terms of these percolation ideas. Since the local hydraulic conductance is a function of the length l , the threshold conductance g_c defines a characteristic length l_c , which is the length that appears in Eq. (1). Moreover, the length that marks the percolation threshold in the hydraulic conductance problem also defines the threshold in the electrical conductance case. We can, therefore, study Eq. (2) as a function of the length parameter l :

$$g(l) = \phi g_c(l)[p(l) - p_c]^t . \quad (2')$$

The porosity ϕ appears here to ensure a proper normalization of the fluid or the electric charge density. As we de-

crease l past the threshold value l_c , the function $g_c(l)$ (which we take equal to $\bar{c}l^3$ for the fluid flow problem and equal to $c'l$ for the analogous electrical problem) also decreases, but the power-law expression $[p(l) - p_c]^t$ will increase as more and more of the pore space is included in the largest cluster that spans the sample. For appropriate choices of the function $p(l)$, the conductance $g(l)$ assumes a maximum value for some $l_{\max} \leq l_c$. Since Eq. (2') should always yield at least a lower bound to the true rock conductance, we take l_{\max} to be the "best" choice to use in the trial solution [Eq. (2')]. We observe that, in general, l_{\max}^h for the hydraulic conductance will be different from l_{\max}^e for the electrical conductance problem, since the conduction pathways have different weights for the two cases.

We can derive some general results concerning the relationship between permeability and conductivity without specifying the function $p(l)$. These relationships hold as long as $p(l)$ allows for a maximum in the conductance and the maximum occurs for $l_{\max} \leq l_c$. If these conditions are satisfied, we find to first order in Δl_e and Δl_h

$$l_{\max}^e = l_c - \Delta l_e = l_c \{1 - t/[1 + t + l_c t p''(l_c)/p'(l_c)]\} \quad (3)$$

$$l_{\max}^h = l_c - \Delta l_h = l_c \{1 - t/[3 + t + l_c t p''(l_c)/p'(l_c)]\} .$$

For very broad pore size distributions [such that $l_c t p''(l_c)/p'(l_c) \ll 1$] Eqs. (3) reduce to

$$l_{\max}^e = l_c [1 - t/(1 + t)] = 0.34 l_c \quad (4)$$

$$l_{\max}^h = l_c [1 - t/(3 + t)] = 0.61 l_c .$$

Using the above results for broad distributions of conductances, we establish a relationship between the electrical conductivity

$$\sigma = \alpha \phi [p(l_{\max}^e) - p_c]^t$$

and the permeability

$$k = \beta \phi (l_{\max}^h)^2 [p(l_{\max}^h) - p_c]^t .$$

To first order in Δl_e or in Δl_h ,

$$[p(l_{\max}^{e,h}) - p_c] = -\Delta l_{e,h} p'(l_c) .$$

To interpret the constants α and β , we assume that locally the rock conductivity is simply σ_0 , the conductivity of the brine solution, and that the local pore geometry is cylindrical. These assumptions imply that $\alpha = \sigma_0$ and $\beta = \frac{1}{32}$.¹³ Combining the above relationships, we obtain $k = \frac{1}{226} l_c^2 \sigma / \sigma_0$, which is of the same form as Eq. (1).

To determine the characteristic length l_c of actual rock samples, we performed mercury injection¹⁴ experiments on a set of 50 rocks that range in permeability from 50 microdarcies to 5 darcies. Mercury, which is assumed to be an ideal nonwetting fluid, is forced into the evacuated pore space under quasistatic conditions. For each externally applied pressure, the diameter of the mercury-pore space interface is determined by the Washburn equation:¹⁴ $p_c = -4\gamma \cos\theta/d$, where p_c is the capillary pressure, which is the difference in pressures on the two sides of the meniscus, γ is the surface tension ($=485$ dyn/cm), θ is the contact angle (130°), and d is the local diameter of the pore space. (The experimental uncertainties in the surface

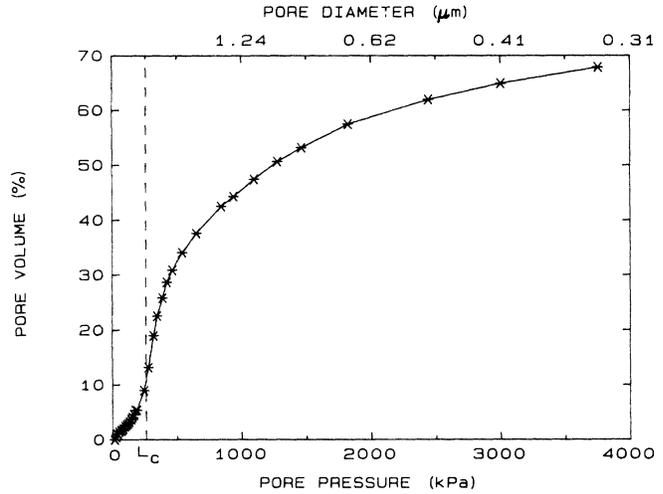


FIG. 1. Typical mercury injection curve taken on a quartz cemented sandstone from eastern Utah. The characteristic length is labeled l_c . The pore diameters are calculated from the pore pressures using the Washburn equation.

tension and contact angle yield errors that are small compared to the expected uncertainty in measuring the permeability.) The Washburn equation assumes a local cylindrical geometry.

Figure 1 shows a typical mercury injection curve where the volume of mercury intruded (normalized by the total pore volume) is plotted versus the applied pressure. Several authors⁸ have interpreted the rapid rise in the curve to occur when the intruded mercury initially forms a connected cluster that spans the sample. The initial portion of the intrusion curve with positive curvature is associated with surface defects and a broad variation in sample thickness found in the rock chips used in the experiment. We take the inflection point¹⁵ of the rapidly rising portion of the curve to mark the threshold pressure p_t for the formation of the "infinite" cluster. From the Washburn

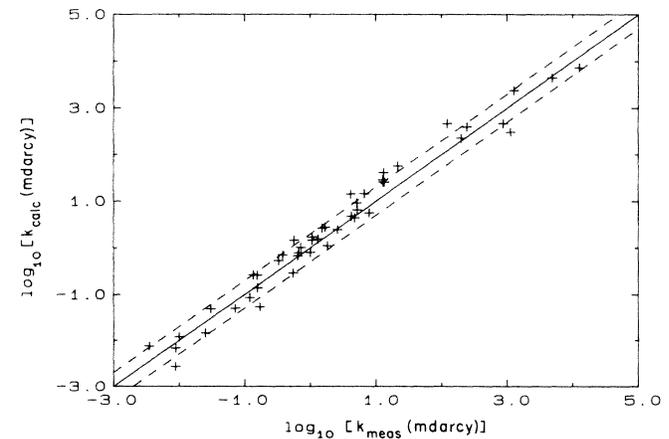


FIG. 2. Calculated permeability k_{calc} vs measured permeability k_{meas} for various sandstones and carbonates. The dashed lines indicate a factor of 2 deviation. Note that the unit of permeability is the millidarcy (md) $= 10^{-11}$ cm².

equation, we conclude that the pore widths l included in the infinite cluster satisfy the relation $l \geq -4\gamma\cos\theta/p_i$; moreover, it is not possible to form an infinite cluster such that the included pore widths are strictly greater than $-4\cos\theta/p_i$. This expression then defines the characteristic length l_c .

Figure 2 presents the gas permeabilities extrapolated to infinite pressure¹⁶ of sandstone samples and a few porous carbonate rocks versus the permeabilities calculated from Eq. (2). The gas permeabilities extrapolated to infinite hydrostatic pressure are customarily assumed to represent the geometrically determined absolute permeabilities. The conductivities were measured at 20 kHz (these measurements agree well with dc values) on rock samples saturated with 10000 and 100000 ppm NaCl brine. The permeabilities, conductivities, and l_c 's were all measured on the same piece of rock sample. The experimental errors are dominated by the measurements of l_c and k . The cumulative errors are estimated to be plus or minus a factor of 2. The absolute values of the permeabilities of the least permeable samples are systematically high because of the difficulty in perfectly sealing rock samples within the measurement rig. The line drawn in Fig. 2 is the prediction of Eq. (1). Agreement between the measured and predicted

permeabilities is within experimental error.

The length scale l_c determined from the inflection point on a capillary pressure curve, is accurate to $\pm 15\%$ of its value. The inflection point is a unique, well-defined experimental point and corresponds to the formation of electrical continuity across the sample. In contrast, the choice of a length in the neighborhood of l_c would alter the value of the constant $\frac{1}{226}$ and would not be directly relevant to the critical path for percolation.

In conclusion, we cite several phenomenological expressions in the geophysics literature relating permeability and parameters derived from mercury intrusion data. A relationship established by Swanson¹⁷ comes close quantitatively to our Eq. (1). Swanson's derivation, however, is very different from ours. The excellent predictability he obtains reflects his choice of a characteristic length close to l_c or l_{\max} . There are also numerous expressions for k that use the porosity raised to some large power.^{18,19} Large exponents are required to give the observed range of k values in the absence of an l_c measurement. In our experiment l_c varies by three orders of magnitude while σ/σ_0 varies by a factor of 20. Expressions involving large exponents on porosity are reasonable only when ϕ tracks l_c .

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