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Lattice and continuum percolation transport exponents: Experiments in two-dimensions

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The conductivity and the elasticity exponents were measured in a system which can model lattice or continuum percolation in two dimensions. The conductivity exponent is the same in both cases, but the elastic exponent is larger in the continuum percolation. All the results are in very good agreement with the analysis of Halperin, Feng, and Sen.

In this Rapid Communication, I present experimental results pertaining to the transport properties (electrical conductivity and elasticity) for the cases of lattice and continuum percolation in two dimensions (2D). Recently, Halperin, Feng, and Sen[1] calculated that there are significant differences between the lattice- and continuum-percolation exponents. They considered the transport properties near the percolation threshold of a random-void model, where spherical holes are randomly distributed in the medium. In 2D (which is the case of this Rapid Communication) they found that the conductivity exponent t is the same as in lattice percolation. But the elasticity exponent T is much larger in continuum percolation. It is known[2] that in a lattice $T_l \simeq 3.5$, and Halperin *et al.*[1] found that in a continuum $T_c - T_l = \frac{3}{2}$. In this paper results of experiments are presented which show excellent agreement with the analysis of Halperin *et al.*[1]

The system, already used[3] to determine the exponent T, consists of a metallic sheet in which holes are punched, and the strain under the application of a known stress is measured. This system has two important advantages: (a) One can measure simultaneously the elastic constants and the electrical conductivity, and (b) one can model either lattice or a continuum percolation by varying only the method of punching the holes. In the first case (lattice) the holes are punched randomly on the sites of a square lattice, with the hole diameter slightly larger than the lattice constant. In the second case (continuum) the holes are punched randomly on the sample. In this manner, it is possible to have a continuous distribution of the channel width[1] δ with a finite limit for its probability distribution $P(\delta)$ for $\delta \rightarrow 0+$, as necessary to observe a change in the exponent T.

The experiments were performed in the following manner.

(a) The samples are 20×21-cm^2 sheets of copper (thickness 0.2 mm) and the diameter of the holes is 1.1 cm. On the same sample the conductivity and the elastic constants were measured as a function of ϕ. In lattice percolation, ϕ is the hole number, and in continuum percolation it is the removed surface. In this latter case, ϕ is determined by weighing the metal pieces removed by the punching.

(b) The mechanical measurements were made in the two different configurations shown in Fig. 1. Configuration a is the tensile stress geometry used previously;[3] configuration b measures the shear response. A schematic view of the experimental setup of this configuration is given in Fig. 2. The problem in configuration b is to keep the sample planar under a shear stress. For this purpose, two identical sheets were used, as shown in Figs. 1 and 2. The two samples are fixed to the basis plate by their opposite ends, while their other ends are clamped on a mobile

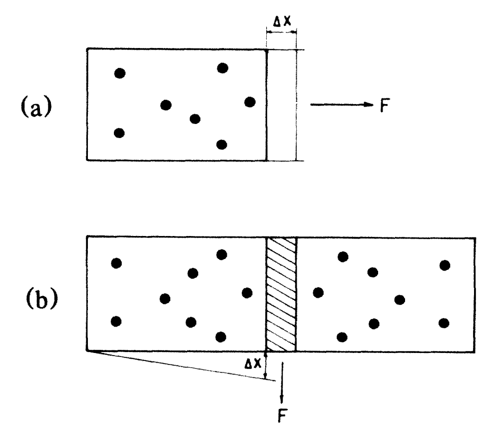

FIG. 1. Configurations of the sample in the elasticity measurements. In b two identical samples are used, with the holes at the same positions. ΔX is displacement of the plate and F is the applied force.

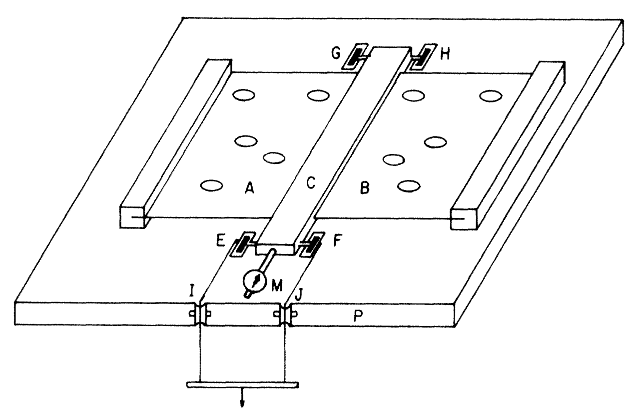

FIG. 2. Schematic view of the device for measuring the elastic constant in the configuration of Fig. 1(b). A, B are samples. P is the basis plate. C is the mobile beam. M is the micrometer. E, F, G, H, I, and J are the locations of ball bearings.

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beam. The displacement of the beam under application of a known weight is measured by a micrometer. These two configurations involve different combinations of the elastic constants.[4] It will be shown that the two configurations give the same exponent T (as also occurs for the lattice geometry[5]) showing that the two elastic constants (C_{11} and C_{44} or k and μ) vanish at the elastic threshold with the same exponent.

(c) The threshold ϕ_c was determined by fitting the conductivity data by the expression $\sigma \sim (\phi_c - \phi)^t$, since it is possible to perform measurements of σ much nearer to ϕ_c than can be done for the elastic constants. It is found that $\phi_c(\text{lattice}) \simeq 0.60$ and $\phi_c(\text{continuum}) \simeq 0.64$.

(d) It is important to go as near ϕ_c as possible. In the present case, there are two limitations. First, the finite size of the sample limits ϕ to values for which the correlation length is smaller than the sheet size. This takes place for $|\phi - \phi_c| \gtrsim 0.05$. Further, the stress that one has to apply in order to be always in the linear regime (see Ref. 3 for details) decreases with $|\phi - \phi_c|$. It may happen that the residual friction is of the same order of magnitude as the applied stress and the measurement is no longer possible. Care has been taken in order to diminish the friction by the use of good-quality ball bearings.

For the conductivity σ, the same exponent $t = 1.1 \pm 0.2$ is found in both kinds of percolation, as predicted by Halperin *et al.*[1]

The results of the elastic measurements are shown in Fig. 3 where the lattice and continuum elastic constants in the configurations a and b are plotted. Since the units of $C_{a,b}$ are arbitrary, the curves were displaced to separate them. The behavior of the two kinds of percolation is clearly seen. For the lattice, the exponent T_l is found equal to 3.5 (as in Refs. 3 and 5). It is clear that the exponent T_c of the continuum percolation is larger than T_l. From these curves C_b and C_a, we can estimate $T_c = 5 \pm 0.5$. The importance of the results is that T_c is found to be larger than T_l, with a difference $T_c - T_l$ between 1 and 2. The ratio of the absolute values of the elastic constants in the two kinds of percolation is of order of 2; $C_{a,b}$ being larger for the lattice case, at the same value of $\phi_c - \phi$.

Since Halperin *et al.*[1] related the elastic properties to the distribution $P(\delta)$ of the channel width, and its behavior for $\delta \rightarrow 0+$ the distribution $P_1(\delta)$ of the distance between two holes was determined. For small values of δ, the two distributions are identical. A hole A is chosen (Fig. 4) and the distances from its side to all its "neighbors" are measured. A hole is considered to be a neighbor of the hole A if it is possible to draw a straight line (from center to center) between the two holes without crossing another hole. In Fig. 4, the lines joining the hole A to all its neighbors are shown. It is clear also that two overlapping holes are not taken into account, since there is no metal part between them. This definition is different from that of Halperin *et al.*,[1] but my distribution is much simpler to determine experimentally.

Photographs of the sample were taken at different stages of the measurements. From these photographs, the distributions were determined. In Fig. 5 the distributions $P_1(\delta)$ of lattice percolation for $\phi = 0.4$ ($\phi_c - \phi = 0.2$) and for $\phi = 0.55$ ($\phi_c - \phi = 0.05$) are shown. In spite of the relatively important change in ($\phi_c - \phi$), there are only minor changes in $P_1(\delta)$. One observes gaps in $P_1(\delta)$, reflecting the discrete nature of the hole positioning. The smallest channel width is 3.2 mm (between 2 and 4 mm) and for $\delta \rightarrow 0+$, $P_1(\delta) = P(\delta) = 0$. In Figs. 6(a) and 6(b), the distributions of continuum percolation are drawn for $\phi = 0.425$ ($\phi_c - \phi = 0.225$) and for 0.59 ($\phi_c - \phi = 0.05$). For $\phi \rightarrow 0+$, the distribution goes to a finite value and it is different for the two values of $|\phi - \phi_c|$. At large values of δ, $P_1(\delta)$ does not have the same behavior for the two kinds of percolation. $P_1(\delta)$ of the lattice percolation is different from zero for relatively large δ. This results from the definition of a neighbor and from a canal effect typical of a lattice.

To conclude, the 2D conductivity and elasticity ex-

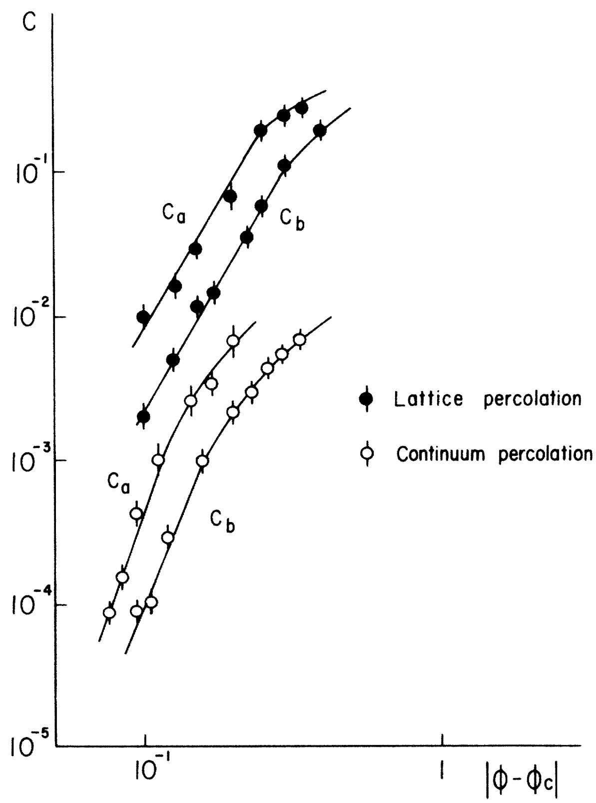

FIG. 3. Elastic constants vs ($\phi_c - \phi$). a corresponds to Fig. 1(a) and b corresponds to Fig. 1(b).

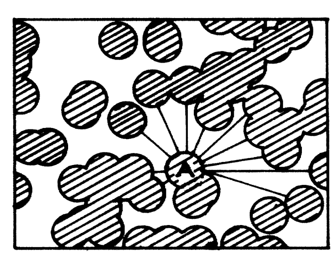

FIG. 4. One hole (A) and all its "neighboring" holes. From A to a neighbor, it is possible to draw a line which does not cross any other hole.

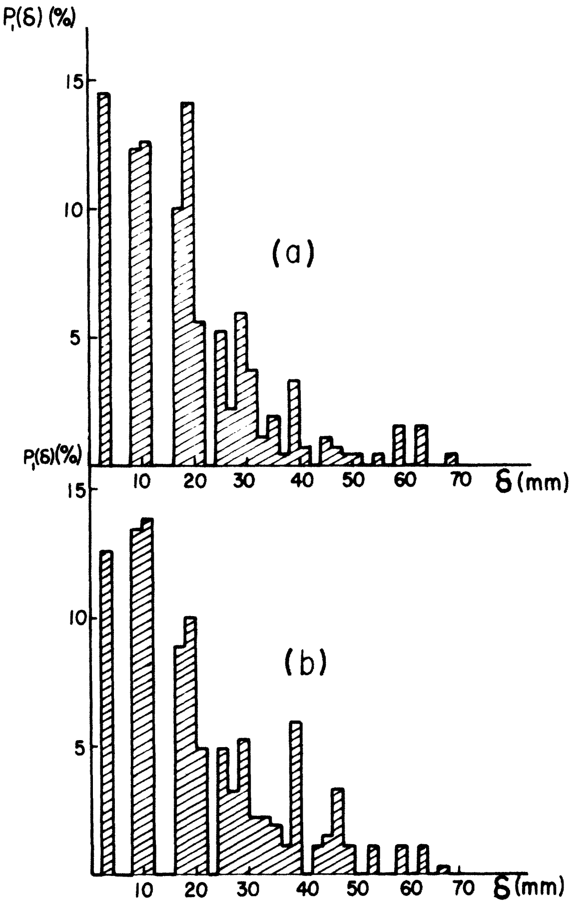

FIG. 5. Distribution $P_1(\delta)$ in the lattice percolation for (a) $\phi_c-\phi=0.2$ and (b) $\phi_c-\phi=0.05$.

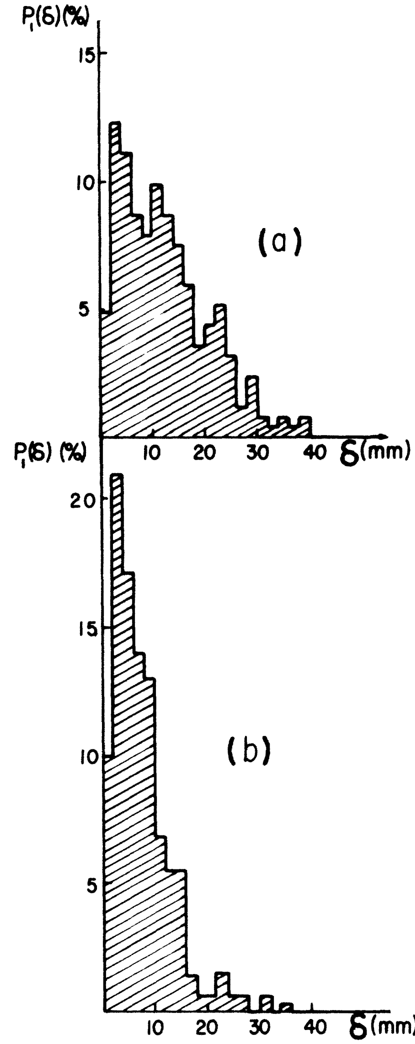

FIG. 6. Distribution $P_1(\delta)$ in the continuum percolation for (a) $\phi_c-\phi=0.225$ and (b) $\phi_c-\phi=0.05$.

ponents in lattice and continuum percolation were determined. The conductivity exponent is the same, but the elastic exponent T_c for continuum percolation is larger (~ 5) than the lattice-percolation exponent (~ 3.5). At the same time, the distributions $P_1(\delta)$ of the distances between holes were determined, showing the difference between the two kinds of percolations. All these results are in very good agreement with the results of Halperin *et al.*[1]

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[1]B. I. Halperin, S. Feng, and P. N. Sen, Phys. Rev. Lett. **54**, 2391 (1985).

[2]S. Feng, P. N. Sen, B. I. Halperin, and C. J. Lobb, Phys. Rev. B **30**, 5386 (1984); Y. Kantor and I. Webman, Phys. Rev. Lett. **52**, 1981 (1984); D. J. Bergman, Phys. Rev. B **31**, 1696 (1985).

[3]L. Benguigui, Phys. Rev. Lett. **53**, 2028 (1984).

[4]In the first geometry [Fig. 1(a)] the measured elastic constant is neither E (valid for a very long sample) nor C_{11} (valid for a very short sample), but a combination of them ($C_a^{-1}=\alpha C_{11}^{-1}+\beta E^{-1}$, with α and β depending on the sample size). By the same kind of argument, one can easily see that in the second geometry [Fig. 1(b)] a combination of E and μ is measured. This point is discussed in D. J. Bergman and L. Benguigui (unpublished).

[5]L. Benguigui, in *Physics of Finely Divided Matter,* proceedings of the Winter School Les Houche, France, 1985, edited by N. Boccara and M. Daoud, Springer Proceedings in Physics, Vol. 5 (Springer-Verlag, Berlin, 1985), p. 188. In the two configurations, the same value of T is found, showing that the two elastic constants (C_{11} and C_{44} or k and μ) go to zero when $\phi\rightarrow\phi_c$ with the same exponent.