Formation of domains in the random-field Ising model

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Monte Carlo simulations of the random-field Ising model for dimensions $d=2$ and $d=3$ show the occurrence of fractal domains that are pinned by the random-field configuration. For $d=3$, the domain size distribution satisfies scaling properties below the critical temperature, while for $d=2$ the largest domain becomes frozen as the temperature approaches zero.

I. INTRODUCTION

Recently, there have been several theoretical papers on the evolution of domains in the random-field Ising model (RFIM),[1–6] to explain the metastability observed in neutron scattering experiments of dilute antiferromagnets in an external magnetic field.[7,8] In a continuum approximation for the domain surface, Villain[1] has shown that domains grow logarithmically in time, and similar results were obtained in a lattice model by Grinstein and Fernandez.[2] Mean-field theory has been applied to study the stability of domains[3,4] and arguments have been advanced[4] for the pinning of domain walls in a continuum interface model in a random field. In addition, Monte Carlo simulations have been carried out[6,9,10] to obtain numerically the time evolution of these domains, and to compare the results with current theory. However, the validity of basic assumptions in these theoretical models have not been tested directly in the numerical calculations. For example, since the original proposal of Imry and Ma,[11] it is generally assumed that the total random-field energy fluctuation of the domains are determined by the root-mean-square deviation of the random-field distribution. However, this neglects the fact that the domain walls evolve freely and can seek deeper random-field wells in a quenched field configuration. Furthermore, the continuum approximations assume that the domain walls can be treated as differentiable surfaces, but our finding that in some cases these walls are fractal indicates that this assumption may not always be valid.

In order to elucidate the mechanism for domain formation and pinning in the random-field Ising model, we have observed graphically the evolution of these domains and evaluated several of their properties by Monte Carlo simulations. In three dimensions, where the equilibrium critical behavior has been recently determined numerically for a value of the random field $H=1$ (in units of J, the coupling constant),[12] we have studied the domains for this value of H and for $H=1.5$ in a range of temperatures below the observed critical temperature, and in two dimensions we considered temperatures well below the critical temperature for the pure Ising model. Although many domain properties are found to be similar in two and three dimensions, the essential difference is that for $d=2$ the largest domain became frozen as the temperature was decreased, while for $d=3$ we obtained an equilibrium distribution for the domain size which satisfies a scaling form as a function of temperature. The behavior for $d=2$ is consistent with this being the lower critical dimensionality for the RFIM as has been shown now rigorously,[13] and for $d=3$ the scaling results are in agreement with the equilibrium properties observed previously above T_c.[12]

II. MONTE CARLO SIMULATIONS

A. $d=2$

The Monte Carlo simulations of the RFIM were carried out on a square lattice of linear size $L=100$ with a random field $\pm H$ at every site. Starting from a completely ordered spin configuration (down spins), the system was warmed up to a fixed temperature T, using the Metropolis algorithm, until a steady state was reached. In Fig. 1, we show the growth of domains (up spins) in a quenched random-field (RF) configuration with $H=1$, at a temperature $T=1$. Only the boundaries of the domains are shown, against the background of the RF configuration, where dots (blanks) represent the up (down) orientation of the random field. The growth of the domains indicates the following general pattern.

(i) A nucleation of the domains occurs preferentially where there is a locally large RF fluctuation in the up direction (seed).

(ii) Growth of the domains occurs by steps, such that the domain wall searches for a new configuration which maximizes in absolute value the field fluctuation just inside and outside the boundary.

(iii) After a sufficient time (Monte Carlo steps), the large domains reach a maximal size and are pinned to a fixed location determined by the RF configuration, while the thermal fluctuations are confined to the vicinity of the domain wall.

The magnetic energy is closely correlated with the surface energy; we followed the time evolution of these quantities for a growing cluster, and found that the two energies balance each other during the evolution such that the total energy remains constant, as indicated in Fig. 2. The domains shown in Fig. 1 appear to be more or less com-

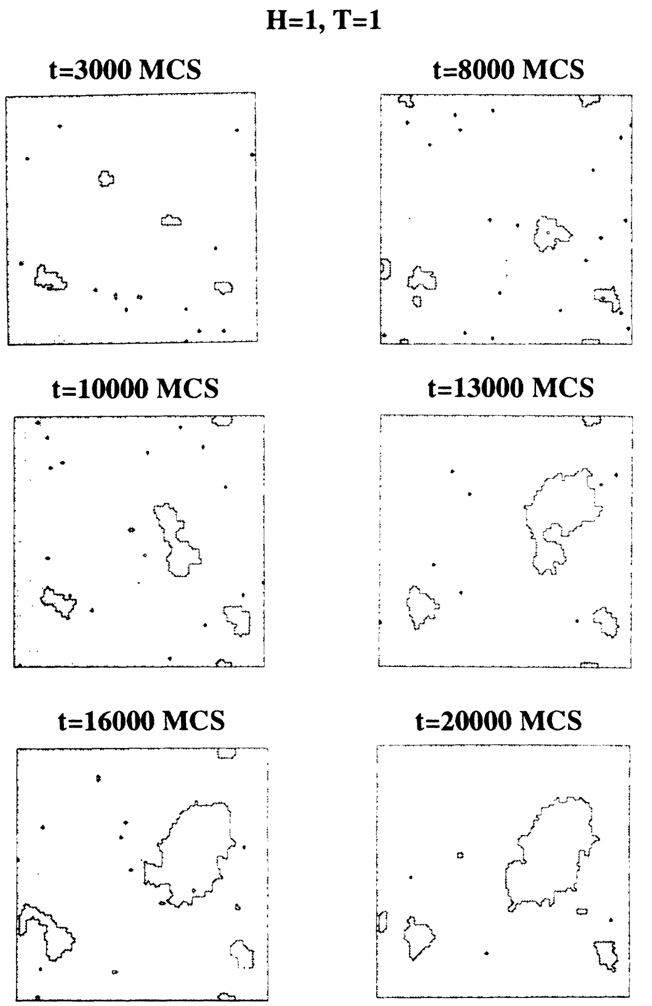

FIG. 1. Domain evolution from ordered start. The boundary of the domains of up spins is outlined against the RF configuration; a dot represents an up orientation of the random field, while the down orientation is left blank. Notice the concentration of dots along the walls inside the domain and the blank zones outside the walls. The largest cluster stabilizes at $t \approx 15\,000$ iterations.

pact at these temperatures. We assume a power law between the surface s and the number n of spins in a domain,[14]

$$s \approx n^{\sigma} , \tag{1}$$

where the surface is measured as the total number of broken bonds for a given domain. Figure 3 shows the validity of the relation, and the value of the surface exponent σ is found to be

$$\sigma \approx 0.59 \pm 0.04$$

which is close to the lower limit $1-1/d=\frac{1}{2}$ for a completely compact domain. Similar measurements[15] in the pure case ($H=0$) for $T>2$ lead to a larger value $\sigma \approx 0.7$; the difference is explained by the fact that the temperature here is sufficiently low to reduce the thermal fluctua-

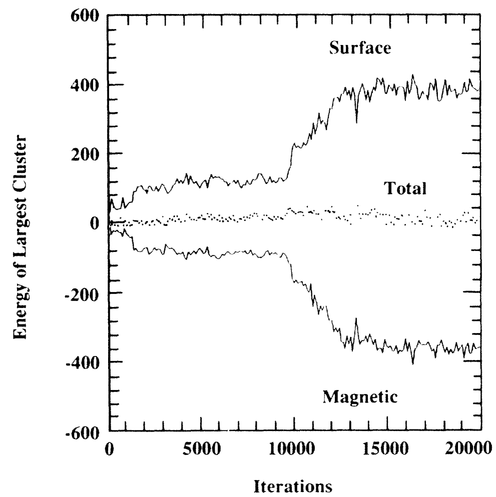

FIG. 2. Time evolution of the energy of a large domain. Magnetic and surface energies are strongly anticorrelated, leaving the total energy undisturbed.

tions considerably, so that there are no "internal" surfaces (holes) in the domains.

The total RF fluctuations in a domain have generally been considered to be typically a rms deviation, and therefore to be proportional to the square root of the number of spins n in the domain. We assume here a more general power law,

$$h(n) \approx n^{\zeta} , \tag{2}$$

where ζ is an undetermined exponent. We verify Eq. (2) from our data shown in Fig. 4 which gives

$$\zeta \approx 0.66 \pm 0.04 ,$$

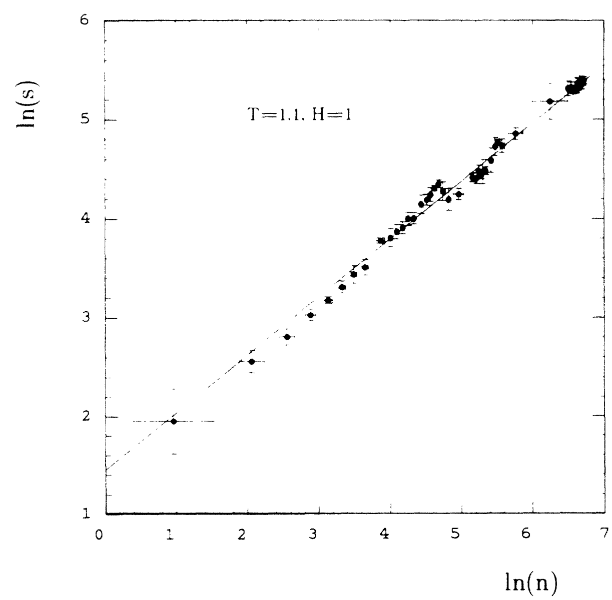

FIG. 3. Logarithmic plot of surface versus size. The power-law relation $s \approx n_{\sigma}$ holds, giving $\sigma \approx 0.6$.

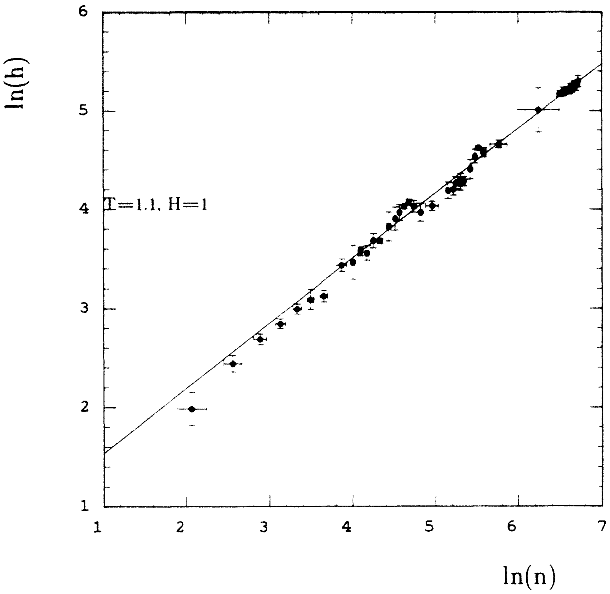

FIG. 4. Logarithmic plot of field fluctuation versus size. The power-law relation $h \approx n_\zeta$ holds and gives $\zeta \approx 0.66$.

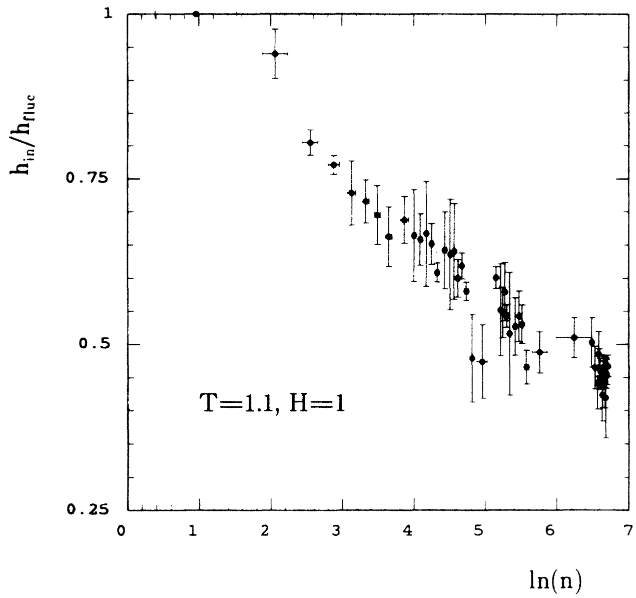

FIG. 6. Ratio h^{in}/h^{fluc}, where h^{fluc} is the total field fluctuation in a domain. The ratio is always $\gtrsim 50\%$ even for very large sizes, which indicates that most of the magnetic energy is contained in the boundary layer.

a significantly higher value than the expected one of $\zeta = \frac{1}{2}$. The domain adjusts itself with the RF configuration to minimize its total energy; a careful observation of the domains (as in Fig. 1) reveals that the boundary is actually located between layers (≈ 1 lattice spacing deep) of large and opposite RF fluctuations. We measure the local fluctuations in these layers "inside" and "outside" the domain, h_{in} and h_{out}; Fig. 5 shows that these fluctuations are proportional to the perimeter of the domain, instead of its square root as assumed in current theories.[1,2] We emphasizes that Figs. 3 through 6 show the data for clusters evolving towards stable sizes (as seen in Fig. 1).

We find also that the mechanism for stability and pinning of the domains is quite different from the formulation of Bruinsma and Aeppli.[5] In particular, components of the domain wall are pinned at fixed locations determined by the RF configuration, independent of the initial spin state of the lattice (ordered or disordered), which maximizes the magnitude of the RF magnetic energy of the boundary layer of the domain. This energy is an important contribution to the total RF magnetic energy, as seen in Fig. 6, which shows the ratio of the RF energy in the inside layer over the total RF energy of a domain for all sizes. Finally, Fig. 7 shows that these domains appear to be stable configurations at $T=0$: As we cool the lat-

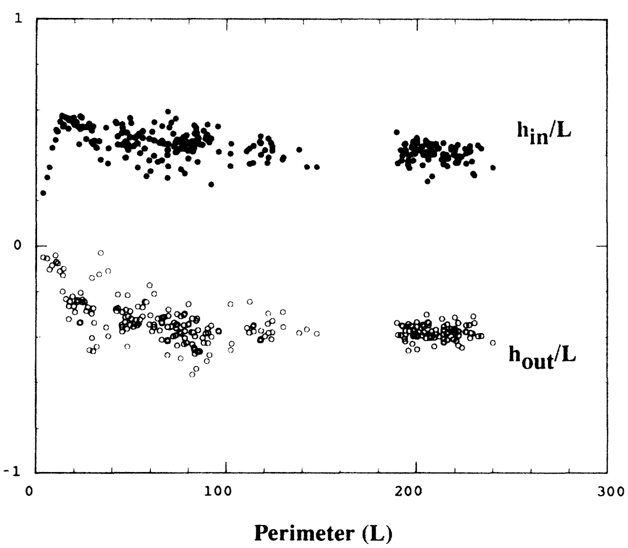

FIG. 5. Field fluctuations in layers inside and outside the domain boundary, h^{in} and h^{out}. The y coordinate is the ratio h/L, with L the perimeter length, and the x ordinate is the perimeter length L. Neglecting the scatter, the constant and symmetric values of the ratios show that $h^{in,out} \approx L$.

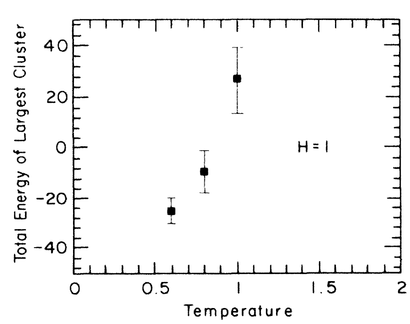

FIG. 7. Total energy of an isolated large domain, produced from an ordered start in the presence of a random field H, and later cooled ($T=1 \to 0.8 \to 0.6$) with the same field. The total energy becomes negative at $T \approx 0.9$.

tice in the presence of a random field, the total energy of the large domains becomes negative for sufficiently low temperatures, consistent with the proof that the ground state of the $d=2$ RFIM is disordered.

B. $d=3$

The $d=3$ simulations of the RFIM were performed on a cubic lattice of size $L=32$. As in the $d=2$ case, domain of up-spins were seen to nucleate, grow, and remain fixed on the lattice for a given RF configuration. This behavior must be contrasted with runs performed for the pure Ising model[15] where similar domains occur, but were not localized. The pinning of domains is therefore an essential characteristic of both the $d=2$ and 3 RFIM. The spontaneous magnetization as a function of the temperature is shown in Fig. 8 for $H=1$, 1.5, and includes for comparison the pure Ising model ($H=0$). It is evident that in the presence of a RF the critical temperature decreases and that the magnetization curve is steeper and shows a discontinuity at the phase transition, in accordance with the results of Ref. 12. The strong-field case ($H=1.5$) does not have a smooth behavior even below the transition, which may be due to lack of equilibrium. For $H=1$, the system was observed to equilibriate faster, and we obtained the effective magnetic exponent $\beta \approx 0.15 \pm 0.02$ from a power-law fit to the data below the discontinuity. This result is consistent, within error bars, with the β of the pure $d=2$ Ising model, and with the corresponding $d=2$ effective exponents obtained in a previous simulation[12] as well as in experiments.[8]

In contrast with the $d=2$ RFIM, the distribution of domain sizes satisfies a scaling form as a function of the reduced temperature $t=(T_c-T/T_c)$, for T below an apparent percolation threshold at a temperature T_p lower than the critical temperature T_c. For $T>T_p$ this scaling breaks down and only a single large domain becomes important near T^c. This can be seen in Fig. 9 where we separated the concentration of up spins into two parts, one due to the sum of all domains of "finite" size c^{fin}, and the other to the isolated contribution of the percolating ("infinite") domain, c^{∞}. In particular, Fig. 9 shows that the discontinuity of the magnetization seen in Fig. 8 corresponds to the discontinuity of the size of the percolation domain, c^{∞}, as a function of the temperature.

The scaling analysis of the size distribution for $T<T^P$ is similar to that of the pure Ising model,[14,15] which takes the asymptotic form

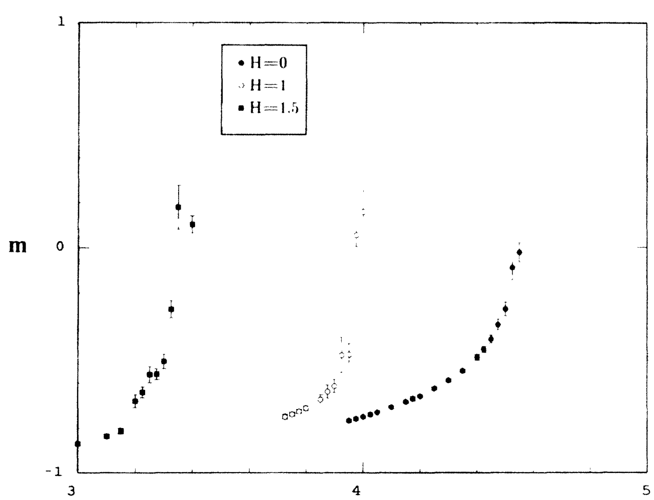

FIG. 8. Magnetization of $d=3$ system showing a discontinuity in the presence of a RF and the shift of the transition temperature towards lower temperatures. By comparison, the smooth behavior of the pure system ($H=0$) is shown on the same plot. The data at $H=1.5$ may not be completely equilibriated for values close to the transition.

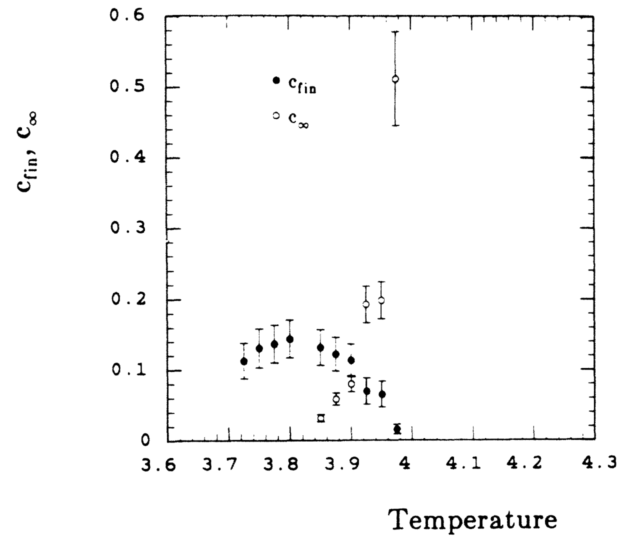

FIG. 9. Concentration of up spin obtained from finite-size domains (c_{fin}) and from the largest percolating domain (c_∞). The discontinuity in c_∞ at the transition reflects the discontinuity for the magnetization.

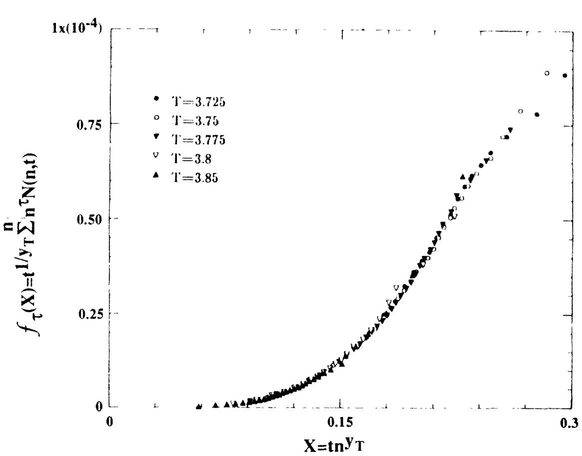

FIG. 10. Integrated scaling function $f_\tau(x)$, obtained from the τ^- movement of the size distribution. This function depends only on the scaled variable $x=tn^y$, and therefore is universal in temperature. The data for temperatures below the percolation threshold is consistent with the scaling assumption.

$$N(n,t)\approx n^{-\tau}N(tn^{y}) , \tag{3}$$

where τ and y are two characteristic exponents of the distribution, related to the usual thermodynamic arguments, and $N(x)$ is an analytic function of its argument. We evaluate a partial summation to

$$\sum^{n} n^{\tau}N(n,t)\approx \int dn\, N(tn^{y}) \approx t^{-1/y}f^{\tau}(X) , \tag{4}$$

where we have defined the integrated scaling function f^{τ} by

$$f^{\tau}(X)=\int^{X} dx\, x^{(1-y)/yN(x)} \tag{5}$$

and $X=tn_{y}$.

A plot of this scaling function for several temperatures is a test of the scaling assumption of the size distribution, Eq. (3). The result is shown in Fig. 10 for $H=1$, at temperatures below T^{P} but still within the scaling region, as determined by the magnetization, and for $T^{c}=3.98$, $\tau=2.06$, and $y=0.26$. However, since the scaling regime is not close to the critical temperature, similar results are obtained if we assume that the size distribution scales with respect to the percolation threshold.

The domains for $d=3$ have an apparent fractal surface, in contrast with the $d=2$ case, for which the cluster shapes were seen to be more compact. The logarithmic plot of the surface-to-size relation, Eq. (1), is shown in Fig. 11 for the two values of the RF and for a choice of three temperatures. We have shifted the data vertically by one unit for each temperature and a linear fit to the data gives

$$\sigma\approx 0.84\pm 0.04$$

which is close to the result in the pure Ising model[15]

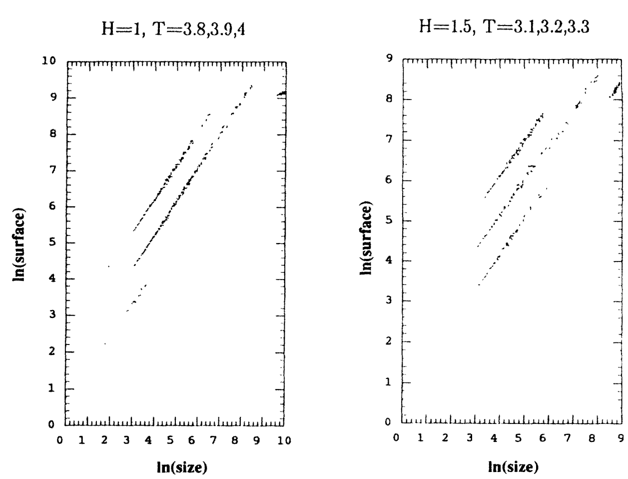

FIG. 11. Surface-size relation for $d=3$. The log-log plot shows the validity of the power law for several temperatures and fields. The vertical coordinate is shifted by one for each temperature to indicate the constancy of the slope with different temperatures. The lowest temperature is on top of the scale, and the correct scale is for the medium one. The slope obtained gives $\sigma\approx 0.85$.

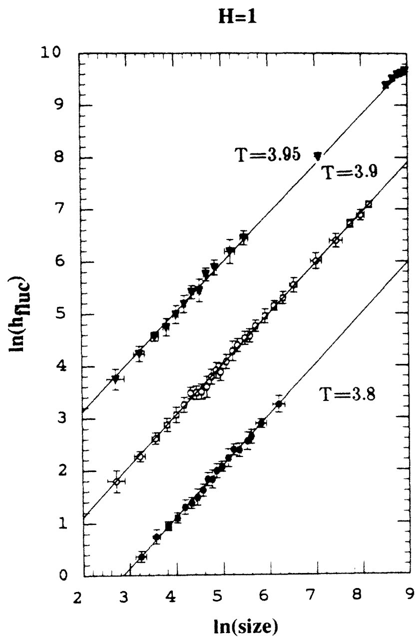

FIG. 12. Field fluctuation versus size plot. The log-log plot shows the validity of the power-law relation, and we have again shifted the data vertically by one unit for each temperature. The slope obtained gives $\zeta\approx 0.96$.

where the exponent $\sigma\approx 0.88$. The data for the RF fluctuations inside the domains shown in Fig. 12, indicates that the power-law dependence, Eq. (2), is again valid for all temperatures; we find for the exponents ζ the value

$$\zeta\approx 0.96\pm 0.04 .$$

However, it should be remarked that a small curvature exists for both of these plots, which leads to an effective variation of the exponents as the size of the domain grows: σ tends to increase while ζ decreases slightly for very large domains.

We expect again that most of the RF fluctuation of the domain is contained in the surface, and we can check this by plotting the ratio of the RF magnetic energy fluctuations $h^{\text{in}}/h^{\text{fluc}}$, as we did for the $d=2$ case. Figure 13

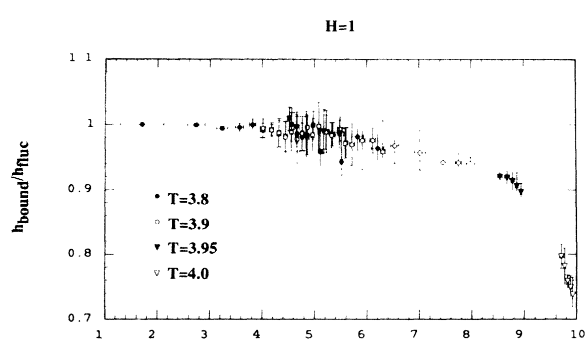

FIG. 13. Ratio $h_{\text{in}}/h_{\text{fluc}}$ for $d=3$. The ratio is always very close to one, which indicates that almost all of the magnetic energy is contained in the boundary layer.

shows that even for very large sizes this ratio is very close to one. Therefore the picture of the domains near t^c is appreciably different from the one assumed in current theoretical models;[1,2,11] instead of a compact object for which the RF magnetic energy is a bulk effect, we find that the domain is a fractal object with almost all of the RF energy contained in the boundary region.

III. CONCLUSIONS

We have found that the mechanism for domain formation in the RFIM has some properties which are quite different from those usually assumed in current theoretical models. In particular, the RF magnetic energy of a domain exceeds considerably the value calculated from a typical rms RF fluctuation. In three dimensions the surface of the domains is fractal and accounts for most of the RF magnetic energy fluctuation, while in two dimensions the domains are more compact, and are stable as the temperature approaches zero. It should be pointed out that, as in most Monte Carlo calculations, we have used values of the RF of the same order as the spin-spin coupling, while the effective experimental RF is much smaller. However, in view of the agreement between the Monte Carlo evaluation of the critical exponents[12] and the experimental values[8] we are encouraged to believe that the domain properties discussed in this paper are also relevant to smaller values of the RF.

ACKNOWLEDGMENTS

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[1]J. Villain, Phys. Rev. Lett. **52**, 1543 (1984), in *Scaling Phenomena in Disordered Systems,* edited by R. Pynn and A. T. Skjeltorp (Plenum, New York, 1986).

[2]G. Grinstein and J. R. Fernandez, Phys. Rev. B **29**, 6389 (1984).

[3]H. Yoshisawa and D. P. Belanger, Phys. Rev. B **30**, 5220 (1984).

[4]C. Ro, G. S. Grest, C. M. Soukoulis, and K. Levin, Phys. Rev. B **31**, 1682 (1985).

[5]R. Bruinsma and G. Aeppli, Phys. Rev. Lett. **52**, 1547 (1984).

[6]D. Stauffer, C. Hartztein, K. Binder, and A. Aharony, Phys. Condens. Matter **55**, 325 (1984).

[7]R. J. Birgenau, R. G. Colley, G. Shirane, and Y. Yoshisawa, J. Stat. Phys. **34**, 817 (1984); Phys. Rev. Lett. **54**, 2147 (1985).

[8]D. P. Belanger, A. R. King, and V. Jaccarino, Phys. Rev. Lett. **54**, 577 (1985); Solid State Commun. **54**, 79 (1985); Phys. Rev. B **32**, 503 (1985); **31**, 4538 (1985); **28**, 2522 (1985).

[9]M. Grant and J. D. Gunton, Phys. Rev. B **29**, 1521 (1984); B **29**, 6266 (1984); E. T. Gawlinski, K. Kaski, M. Grant, and J. D. Gunton, Phys. Rev. Lett. **53**, 2266 (1984).

[10]E. Pytte and J. F. Fernandez, J. Appl. Phys. **57**, 3274 (1985); Phys. Rev. B **31**, 616 (1985).

[11]Y. Imry and S. Ma, Phys. Rev. Lett. **35**, 1977 (1975).

[12]A. P. Young and M. Nauenberg, Phys. Rev. Lett. **54**, 2429 (1985).

[13]D. S. Fisher, J. Frolich, and T. Spencer, J. Stat. Phys. **34**, 863 (1984); J. Z. Imbrie, Phys. Rev. Lett. **53**, 1747 (1984).

[14]M. E. Fisher, Physics (N.Y.) **3**, 255 (1967); K. Binder, Ann. Phys. **98**, 390 (1976).

[15]J. L. Cambier and M. Nauenberg Phys. Rev. B **34**, 8071 (1986).

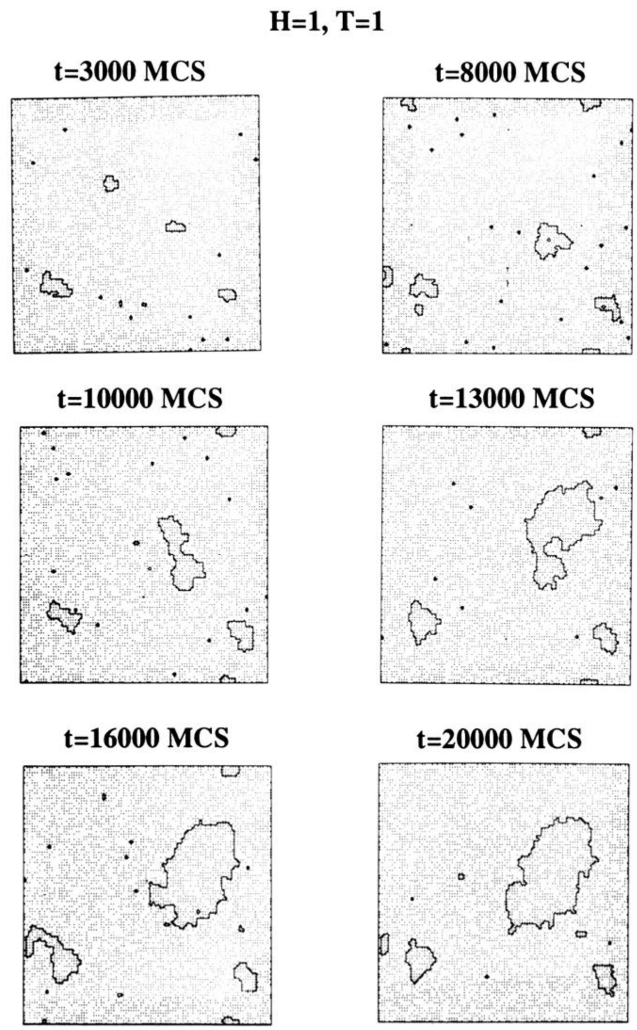

FIG. 1. Domain evolution from ordered start. The boundary of the domains of up spins is outlined against the RF configuration; a dot represents an up orientation of the random field, while the down orientation is left blank. Notice the concentration of dots along the walls inside the domain and the blank zones outside the walls. The largest cluster stabilizes at $t \approx 15\,000$ iterations.