Effective interactions for self-energy. I. Theory

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(Received 21 May 1986)

A systematic way of deriving effective interactions for self-energy calculations in Fermi-liquid systems is presented. The self-energy expression contains effects of density and spin fluctuations and also multiple scattering between particles. Results for arbitrarily polarized one-component Fermi-liquid systems and unpolarized two-component systems are explicitly given.

I. INTRODUCTION

In an earlier paper Vignale and Singwi[1] have constructed an effective particle-particle interaction between two quasiparticles in a degenerate Fermi liquid. This effective interaction is appropriate for studying phenomena like the superconducting transition[2] but is not appropriate for studying properties like the self-energy which is determined mainly by particle-hole interactions. In this paper we extend their approach to construct an effective interaction appropriate for self-energy calculations. The main assumption made is that the irreducible particle-hole interactions in a certain channel depend only on the momentum transfer in that particular channel.

II. THEORY

We start with the Ward identity[3] which relates the self-energy and the irreducible particle-hole interaction with zero momentum transfer in a one-component fermion system:

$$\delta\Sigma_\sigma(p)=\sum_{\sigma'}\int \tilde{I}_{p\sigma;p'\sigma'}(0)\delta G_{\sigma'}(p')\frac{d^4p'}{(2\pi)^4}\ , \tag{1}$$

where $p=(\mathbf{p},p_0)$ is the four momentum and $\sigma,\sigma'=\pm\frac{1}{2}$ is the spin index and $\tilde{I}_{p\sigma;p'\sigma'}(0)$ the irreducible particle-hole interaction with momentum transfer zero. $\tilde{I}_{p\sigma;p'\sigma'}(0)$ is the sum of all interaction diagrams which are irreducible in the particle-hole channel with momentum transfer zero. These diagrams have two incoming lines with four momenta and spins p,σ and $p'\sigma'$, respectively, and two outgoing lines with the same four momenta and spins. For example, diagram (a) in Fig. 1 belongs to $\tilde{I}_{p\sigma;p'\sigma'}(0)$, while diagram (b) should not be included. Notice that if we make the "local" approximation, i.e.,

$$\tilde{I}_{p\sigma;p'\sigma'}(0)\simeq\tilde{I}_{\sigma\sigma'}(p-p')=-V_{\rm eff}^{\sigma\sigma'}(p-p')\ , \tag{2}$$

then Eq. (1) can be solved for $\Sigma_\sigma(p)$:

$$\Sigma_\sigma(p)=-\sum_{\sigma'}\int V_{\rm eff}^{\sigma\sigma'}(q)G_{\sigma'}(p-q)\frac{d^4q}{(2\pi)^4}+\mu^*\ , \tag{3a}$$

where μ^* is a constant of integration which can be absorbed by renormalizing the Fermi energy. Thus, as long as we are not interested in the absolute values of the self-energy, we can as well write

$$\Sigma_\sigma(p)=-\sum_{\sigma'}\int V_{\rm eff}^{\sigma\sigma'}(q)G_{\sigma'}(p-q)\frac{d^4q}{(2\pi)^4}\ . \tag{3b}$$

Following Vignale and Singwi,[1] we write $\tilde{I}_{p\sigma;p'\sigma'}(0)$ as the sum of three types of diagrams: (a) those *reducible* in the particle-hole channel with momentum transfer $p-p'$, (b) those *reducible* in the particle-particle channel, and (c) those *irreducible* in all possible two-body channels.

First we consider diagrams in class (a). The sum of all diagrams in class (a) can be represented conveniently by the following equations:[1,3]

$$-\tilde{I}^{(a)}_{p\sigma;p'\sigma}(0)=\sum_{\sigma''}\int \tilde{I}^{\alpha}_{(p\sigma,p'\sigma;p+p''\sigma'',p'+p''\sigma'')}\times G_{\sigma''}(p+p'')G_{\sigma''}(p'+p'')\times\Gamma_{(p+p''\sigma'',p'+p''\sigma'';p\sigma,p'\sigma)}\frac{d^4p''}{(2\pi)^4}\ , \tag{4a}$$

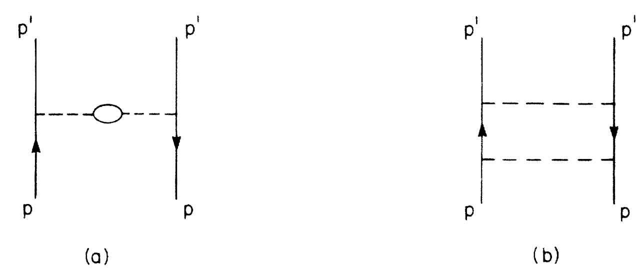

FIG. 1. An example of diagrams: (a) irreducible in the particle-hole channel with momentum transfer zero; (b) reducible in the particle-hole channel with momentum transfer zero. (a) is reducible in the particle-hole channel with momentum transfer $p-p'$, while (b) is irreducible.

$$-\tilde{I}^{(a)}_{p\sigma;p'-\sigma}(0)=\int \tilde{I}^{\alpha}_{(p\sigma;p'-\sigma;p+p''\sigma,p'+p''-\sigma)} \times G_\sigma(p+p'')G_{-\sigma}(p'+p'') \times \Gamma_{(p+p''\sigma,p'+p''-\sigma;p\sigma,p'-\sigma)}\frac{d^4p''}{(2\pi)^4}\ , \tag{4b}$$

where the $\tilde{I}^{\alpha}$'s are particle-hole interactions irreducible in the particle-hole channel with momentum transfer $p-p'$. The Γ's are full vertex functions.[3] Equations (4a) and (4b) are shown schematically in Fig. 2, where the momentum and spin labels of the incoming and outgoing lines are shown explicitly. Notice that there is no internal summation of spin index in (4b) because of spin conservation.

In general, to find Γ and evaluate (4) is a very difficult process which involves solving four-dimensional integral equations.[3] To overcome this difficulty Vignale and Singwi made a drastic approximation by assuming that the particle-hole interactions $\tilde{I}^{\alpha}$ and, correspondingly, the vertex functions Γ in (4) are functions of their momentum transfer $p-p'$ only.[1] With this approximation, Eqs. (4a) and (4b) reduce to

$$-\tilde{I}^{(a)}_{p\sigma;p'\sigma}(0)\simeq \sum_{\sigma''}\tilde{I}_{\sigma\sigma''}(p-p')\chi_{0\sigma''}(p-p')\Gamma_{\sigma''\sigma}(p-p') \tag{5a}$$

and

$$-\tilde{I}^{(a)}_{p\sigma;p'\sigma'}(0)\simeq\tilde{I}^{t}_{(p-p')}\chi^{t}_{0}(p-p')\Gamma^{t}(p-p')\ , \tag{5b}$$

where

$$\chi_{0\sigma}(q)=\int \frac{d^4p}{(2\pi)^4}G_\sigma(p)G_{-\sigma}(p+q) \tag{6a}$$

and

$$\chi^{t}_{0}(q)=\int \frac{d^4p}{(2\pi)^4}G_\sigma(p)G_{-\sigma}(p+q)\ . \tag{6b}$$

Similarly,[1,3] the four-dimensional integral equations for the Γ's reduce to algebraic equations:

$$\Gamma_{\sigma\sigma'}(q)=\tilde{I}_{\sigma\sigma'}(q)+\sum_{\sigma''}\tilde{I}_{\sigma\sigma''}(q)\chi_{0\sigma''}(q)\Gamma_{\sigma''\sigma'}(q) \tag{7a}$$

and

$$\Gamma^{t}(q)=\tilde{I}^{t}(q)+\tilde{I}^{t}(q)\chi^{t}_{0}(q)\Gamma^{t}(q)\ , \tag{7b}$$

which can be solved easily.

Diagrams in class (b) can also be represented by an equation similar to (4a) and (4b).

$$-\tilde{I}^{(b)}_{p\sigma;p'\sigma'}(0)=\int \frac{d^4p''}{(2\pi)^4}\tilde{J}_{(p\sigma,p'\sigma';p+p''\sigma,p'-p''\sigma')} \times G_\sigma(p+p'')G_{\sigma'}(p'-p'') \times \Gamma_{(p+p''\sigma,p'-p''\sigma';p\sigma,p'\sigma')}\ , \tag{8}$$

where $\tilde{J}$ is the particle-particle interaction irreducible in the particle-particle channel. Equation (8) is shown

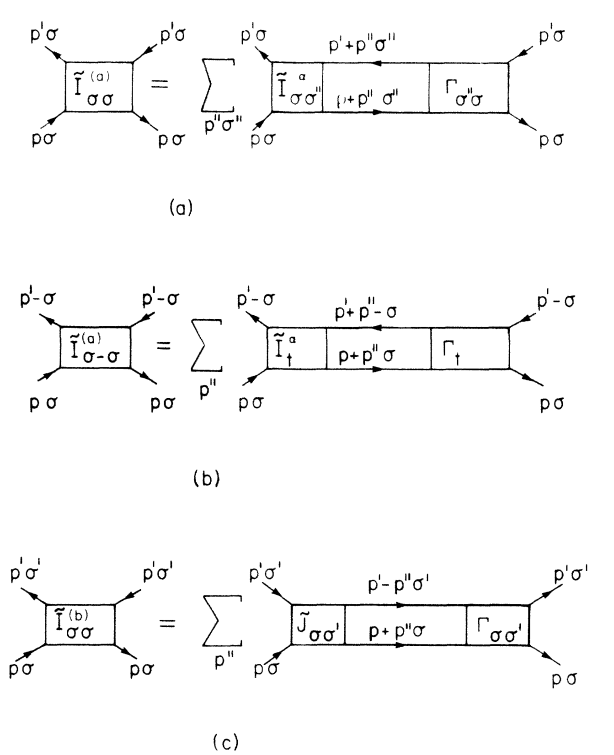

FIG. 2. Sum of all diagrams irreducible in the particle-hole channel with momentum transfer zero and simultaneously reducible in the particle-hole channel with momentum transfer $p-p'$. (a) $\sigma=\sigma'$, (b) $\sigma=-\sigma'$, (c) sum of all diagrams irreducible in the particle-hole channel with momentum transfer zero and simultaneously reducible in the particle-particle channel.

schematically in Fig. 2(c). Notice that there is an exchange contribution for $\sigma=\sigma'$.

The lowest-order contribution to class-(c) terms is the bare interaction. Higher-order diagrams in this class have complicated topological structures in general and are difficult to analyze.[1] Therefore we retain only the lowest-order term

$$\tilde{I}^{(c)}_{p\sigma;p'\sigma'}(0)\simeq V(0)-\delta_{\sigma\sigma'}V(p-p')\ , \tag{9}$$

where $V(q)$ is the bare interaction.

Notice that if we approximate $\tilde{J}$ in (8) by the bare interaction V and combine (8) with (9) we get

$$\tilde{I}^{(b)}_{p\sigma;p'\sigma}(0)+\tilde{I}^{(c)}_{p\sigma;p'\sigma}(0)\simeq -T^{\sigma\sigma}_{pp'}(p-p')+T^{\sigma\sigma}_{pp'}(0)\ , \tag{10a}$$

$$\tilde{I}^{(b)}_{p\sigma;p'-\sigma}(0)+\tilde{I}^{(c)}_{p\sigma;p'-\sigma}(0)\simeq -T^{\sigma-\sigma}_{pp'}(0)\ , \tag{10b}$$

where $T_{kk'}(q)$ is the T matrix with transfer of momentum q in the center of the mass frame of the two incoming particles with four momenta k and k', respectively.

III. EFFECTIVE INTERACTIONS

In this section we apply our results (5), (7), and (10) to set up effective interactions for a few different systems. We shall consider explicitly two different situations: (a) a paramagnetic normal Fermi liquid and (b) a spin-polarized Fermi liquid.

(a) Paramagnetic Fermi liquid. In this case, Eqs. (7a) and (7b) can be solved easily. Combining with (5a) and (5b), we get

$$-\tilde{I}^{(a)}_{p\sigma;p'\sigma}(0) \simeq [\tilde{I}^{s}(p-p')]^2 \chi_s(p-p') + [I^{a}(p-p')]^2 \chi_a(p-p') , \qquad (11a)$$

$$-\tilde{I}^{(a)}_{p\sigma;p'-\sigma}(0) \simeq [\tilde{I}^{t}(p-p')]^2 \chi_t(p-p') , \qquad (11b)$$

where

$$\tilde{I}^{s}_{(p-p')} = \tfrac{1}{2}[\tilde{I}_{\sigma\sigma}(p-p') + \tilde{I}_{\sigma-\sigma}(p-p')] , \qquad (11c)$$

$$\tilde{I}^{(a)}_{(p-p')} = \tfrac{1}{2}[\tilde{I}_{\sigma\sigma}(p-p') - \tilde{I}_{\sigma-\sigma}(p-p')] , \qquad (11d)$$

$$\chi_s(q) = \frac{\chi_0(q)}{1-\tilde{I}^{s}(q)\chi_0(q)} , \qquad (12a)$$

$$\chi_a(q) = \frac{\chi_0(q)}{1-\tilde{I}^{a}(q)\chi_0(q)} , \qquad (12b)$$

and

$$\chi_t(q) = \frac{\frac{1}{2}\chi_0(q)}{1-\tilde{I}^{t}(q)\frac{1}{2}\chi_0(q)} . \qquad (12c$$

χ_s and χ_a are the longitudinal density- and spin-response functions of the Fermi liquid, respectively. $\chi_t(q)$ is the transverse spin-response function. $\chi_0(q)$ is the usual Lindhard function. Notice that for a paramagnetic system, $G_\uparrow(p) = G_\downarrow(p) = G(p)$ and $\chi_t = \frac{1}{2}\chi_a$.[3] Therefore, (11a) and (11b) can be combined to give

$$-\tilde{I}^{(a)}_{p\sigma;p'\sigma}(0) + \tilde{I}^{(a)}_{p\sigma;p'-\sigma}(0) \simeq [\tilde{I}^{s}(p-p')]^2 \chi_s(p-p') + 3[\tilde{I}^{a}(p-p')]^2 \chi_a(p-p') . \qquad (13)$$

To determine the irreducible particle-hole interactions $\tilde{I}^{s}$ and $\tilde{I}^{a}$, Vignale and Singwi[1] suggested that they can be identified with the local-field factors[4] introduced in the case of electron liquid, through

$$I^{s}(q) \simeq V(q)[1-G^{s}(q)] , \qquad (14)$$

$$I^{a}(q) \simeq -V(q)G^{a}(q) , \qquad (15)$$

or they can be identified with the appropriate polarization potentials[5] through (12) in the case of liquid ^{3}He.

Furthermore, making the local approximation

$$T_{pp'}(q) \simeq \bar{T}(q) , \qquad (16)$$

where $\bar{T}(q)$ can be identified as some appropriate average of the T matrix, we get from combining Eqs. (1), (2), (3), (10), (13), and (16),

$$\Sigma(p) \simeq -\int V_{\text{eff}}(q) G(p-q) \frac{d^4q}{(2\pi)^4} , \qquad (17)$$

where

$$V_{\text{eff}}(q) = \bar{T}(q) + [\tilde{I}^{s}(q)]^2 \chi_s(q) + 3[\tilde{I}^{a}(q)]^2 \chi_a(q) , \qquad (18)$$

which is the same expression as the one used by Friman and Krotscheck[6] for calculating self-energy of liquid ^{3}He. [Notice that the $\bar{T}(0)$ terms from (9) only renormalize the Fermi energy and are thus not shown explicitly in (18).]

(b) Polarized Fermi liquid. Solving (7a) and (7b) for a polarized Fermi liquid, we get

$$\Gamma_{\sigma\sigma}(q) = \frac{\tilde{I}_{\sigma\sigma}(q)[1-\tilde{I}_{\sigma-\sigma}(q)\chi_{0-\sigma}(q)] + [\tilde{I}_{\sigma-\sigma}(q)]^2 \chi_{0-\sigma}(q)}{\Delta(q)} \qquad (19a)$$

$$\Gamma_{\sigma-\sigma}(q) = \frac{\tilde{I}_{\sigma-\sigma}(q)}{\Delta(q)} , \qquad (19b)$$

where

$$\Delta(q) = [1-\tilde{I}_{\sigma\sigma}(q)\chi_{0\sigma}(q)][1-\tilde{I}_{-\sigma-\sigma}(q)\chi_{0-\sigma}(q)] - [\tilde{I}_{\sigma-\sigma}(q)]^2 \chi_{0\sigma}(q)\chi_{0-\sigma}(q) \qquad (19c)$$

and

$$\Gamma_t(q) = \frac{\tilde{I}^{t}(q)}{1-\tilde{I}^{t}_{(q)}\chi^{t}_0(q)} . \qquad (19d)$$

Notice that $\chi_{0\sigma} \neq \chi_{0-\sigma} \neq \chi^t_0$ for a polarized liquid. Combining (19) with (2), (3), (5), (10), and (16), we get

$$\Sigma_\sigma(p) = -\Sigma_{\sigma'} \int V^{\sigma\sigma'}_{\text{eff}}(q) G_{\sigma'}(p-q) \frac{d^4q}{(2\pi)^4} , \qquad (20a)$$

where

$$V^{\sigma\sigma}_{\text{eff}}(q) = \bar{T}^{\sigma\sigma}_{(q)} + \{[\tilde{I}_{\sigma-\sigma}(q)]^2 \chi_{0-\sigma}(q)[1+\tilde{I}_{\sigma\sigma}(q)\chi_{0\sigma}(q)] + [\tilde{I}_{\sigma\sigma}(q)]^2 \chi_{0\sigma}(q)[1-\tilde{I}_{-\sigma-\sigma}(q)\chi_{0-\sigma}(q)]\} \frac{1}{\Delta(q)} , \qquad (20b)$$

$$V^{\sigma-\sigma}_{\text{eff}}(q) = [\tilde{I}^{t}(q)]^2 \chi^{t}(q) . \qquad (20c)$$

Notice that $\bar{T}^{\sigma\sigma}_{(q)} \neq \bar{T}^{-\sigma-\sigma}_{(q)}$ for the present case. Furthermore, there exists no simple relationship between the transverse and longitudinal spin fluctuations in a polarized system. Thus, although $\tilde{I}_{\sigma\sigma}$ and $\tilde{I}_{\sigma-\sigma}$ can be identified with the generalized local fields or polarization potentials for the polarized systems, $\tilde{I}^{t}$ has to be determined separately. Notice also that one cannot relate $V^{\sigma\sigma}_{\text{eff}}(q)$ to the density- or spin-response functions in a simple way as in (13). The reason is that in a polarized system, density and spin-fluctuations are not decoupled, which is reflected also in the complicated structure of Eqs. (19a) and (19b).

IV. GENERALIZATION TO MULTICOMPONENT SYSTEMS

The above effective interactions can be generalized to multicomponent systems in a straightforward way. For example, Eq. (1) becomes

$$\delta\Sigma_{\sigma_i}^{(i)}(p)=\sum_{j,\sigma_j}\int \widetilde{I}^{(ij)}_{p\sigma_i;p'\sigma_j}(0)\delta G^{(j)}_{\sigma_j}(p')\frac{d^4p'}{(2\pi)^4}\ , \tag{21}$$

where $i,j=1\ldots N$, N being the number of components of the system. $\widetilde{I}^{(ij)}_{p\sigma_i;p'\sigma_j}(0)$ can be analyzed in a similar way, as in Sec. II. In the following, we shall study in detail the effective interactions for a paramagnetic two-component system. One example of such a system would be electron-hole liquid discovered in some semiconductors.

For a paramagnetic system, $G^{(i)}_{\sigma}(p)=G^{(i)}_{-\sigma}(p)$, therefore, (21) can be written as

$$\delta\Sigma^{(1)}(p)=-\int V^{(11)}_{\text{eff}}(p,p')\delta G^{(1)}(p')\frac{d^4p'}{(2\pi)^4} - \int V^{(12)}_{\text{eff}}(p,p')\delta G^{(2)}(p')\frac{d^4p'}{(2\pi)^4}\ , \tag{22}$$

where (1) stands for any one of the two components and (2) stands for the other component:

$$-V^{(11)}_{\text{eff}}(p,p')=\widetilde{I}^{(11)}_{p\sigma;p'-\sigma}(0)+\widetilde{I}^{(11)}_{p\sigma;p'\sigma}(0)\ , \tag{23a}$$

$$-V^{(12)}_{\text{eff}}(p,p')=\widetilde{I}^{(12)}_{p\sigma;p'-\sigma}(0)+\widetilde{I}^{(12)}_{p\sigma;p'\sigma}(0)\ . \tag{23b}$$

As before, to evaluate $\widetilde{I}^{(ij)}(0)$ we consider three types of diagrams. The sum of all diagrams reducible in the particle-hole channel with momentum transfer $p-p'$ can be represented by the following set of equations:

$$-\widetilde{I}^{(11)(\text{a})}_{p\sigma;p'\sigma}(0)=\sum_{j,\sigma_j}\widetilde{I}^{(1j)\alpha}_{(p\sigma,p'\sigma;p+p''\sigma_j,p'+p''\sigma_j)}G^{(j)}_{\sigma_j}(p+p'')G^{(j)}_{\sigma_j}(p'+p'')\Gamma^{(j1)}_{(p+p''\sigma_j,p'+p''\sigma_j;p\sigma,p'\sigma)}\frac{d^4p''}{(2\pi)^4}\ , \tag{24a}$$

$$-\widetilde{I}^{(11)(\text{a})}_{p\sigma;p'-\sigma}(0)=\int \widetilde{I}^{(11)\alpha}_{(p\sigma,p'-\sigma;p+p''\sigma,p'+p''-\sigma)}G^{(1)}_{\sigma}(p+p'')G^{(1)}_{-\sigma}(p'+p'')\Gamma^{(11)}_{(p+p''\sigma,p'+p''-\sigma;p\sigma,p'-\sigma)}\frac{d^4p''}{(2\pi)^4}\ , \tag{24b}$$

$$-\widetilde{I}^{(12)(\text{a})}_{p\sigma;p'\sigma'}(0)=\int \widetilde{I}^{(12)\alpha}_{(p\sigma,p'\sigma';p+p''\sigma,p'+p''\sigma')}G^{(1)}_{\sigma}(p+p'')G^{(2)}_{\sigma'}(p'+p'')\Gamma^{(12)}_{(p+p''\sigma,p'+p''\sigma';p\sigma,p'\sigma')}\frac{d^4p''}{(2\pi)^4}\ , \tag{24c}$$

where the $\widetilde{I}^{(ij)\alpha}$'s and $\Gamma^{(ij)}$'s have a similar meaning as in Sec. II, except that they are generalized' for two-component systems. In the "local" approximation, we have

$$-\widetilde{I}^{(11)(\text{a})}_{p\sigma;p'\sigma}(0)\simeq\sum_{j,\sigma_j}\widetilde{I}^{(1j)}_{\sigma\sigma_j}(p-p')\chi^{(jj)}_{\sigma\sigma_j}(p-p')\times\Gamma^{(j1)}_{\sigma_j\sigma}(p-p')\ , \tag{25a}$$

$$-\widetilde{I}^{(11)(\text{a})}_{p\sigma;p'-\sigma}(0)\simeq\widetilde{I}^{(11)t}_{(p-p')}\chi^{(11)}_0(p-p')\Gamma^{(11)t}_{(p-p')}\ , \tag{25b}$$

$$-\widetilde{I}^{(12)(\text{a})}_{p\sigma;p'\sigma'}(0)\simeq\widetilde{I}^{(12)t}_{(p-p')}\chi^{(12)}_0(p-p')\Gamma^{(12)t}_{(p-p')}\ , \tag{25c}$$

where

$$\chi^{(ij)}_0(q)=\int\frac{d^4p}{(2\pi)^4}G^{(i)}(p)G^{(j)}(p+q)\ . \tag{26}$$

Notice that there is no spin dependence on the effective interaction between particles of two different components. Also, $\widetilde{I}^{(ij)}=\widetilde{I}^{(ji)}$ for exchange symmetry reasons. The corresponding approximate equations for $\Gamma^{(ij)}_{\sigma\sigma'}$ and the $\Gamma^{(ij)t}$'s are

$$\Gamma^{(ij)}_{\sigma\sigma'}(q)\simeq\widetilde{I}^{(ij)}_{\sigma\sigma'}(q)+\sum_{k,\sigma''}\widetilde{I}^{(ik)}_{\sigma\sigma''}(q)\chi^{(kk)}_0(q)\Gamma^{(kj)}_{\sigma''\sigma'}(q)\ , \tag{27a}$$

$$\Gamma^{(ij)t}(q)=\widetilde{I}^{(ij)t}(q)+\widetilde{I}^{(ij)t}(q)\chi^{(ij)}_0(q)\Gamma^{(ij)t}(q)\ , \tag{27b}$$

where $i,j,k=1,2$.

Diagrams reducible in the particle-particle channels can be represented by an equation similar to (8) except that there also exists scattering between particles of different components. Combining these diagrams with lowest-order diagrams in class (c) (diagrams irreducible in all two-body channels), as in Sec. II, we get in local approximation

$$\widetilde{I}^{(11)(\text{b})}_{p\sigma;p'\sigma}(0)+\widetilde{I}^{(11)(\text{c})}_{p\sigma;p'\sigma}(0)\simeq-\overline{T}^{\,\sigma\sigma}_{11}(p-p')+\overline{T}^{\,\sigma\sigma}_{11}(0)\ , \tag{28a}$$

$$\widetilde{I}^{(11)(\text{b})}_{p\sigma;p'-\sigma}(0)+\widetilde{I}^{(11)(\text{c})}_{p\sigma;p'-\sigma}(0)\simeq\overline{T}^{\,\sigma-\sigma}_{11}(0)\ , \tag{28b}$$

$$\widetilde{I}^{(12)(\text{b})}_{p\sigma;p'\sigma'}(0)+\widetilde{I}^{(12)(\text{c})}_{p\sigma;p'\sigma'}(0)\simeq\overline{T}_{12}(0)\ , \tag{28c}$$

where the $\overline{T}_{ij}$'s are averaged T matrices between particles of component i and component j.

Solving Eqs. (27) and combining with (22), (23), (25), and (28), we get after some algebra

$$\Sigma^{(1)}(p)=-\int V^{(11)}_{\text{eff}}(q)G^{(1)}(p-q)\frac{d^4q}{(2\pi)^4}\ , \quad -\int V^{(12)}_{\text{eff}}(q)G^{(2)}(p-q)\frac{d^4q}{(2\pi)^4}+\mu^{(1)*}\ , \tag{29}$$

where

$$V^{11}_{\text{eff}}(q)\simeq\overline{T}_{11}(q)+V^{11}_s(q)+V^{12}_s(q)+3V^{11}_a(q)\ , \tag{30a}$$

$$V^{12}_{\text{eff}}(q)\simeq\frac{[\widetilde{I}^{(12)t}(q)]^2\chi^{(12)}_0(q)}{1-\widetilde{I}^{(12)t}(q)\chi^{(12)}_0(q)}\ , \tag{30b}$$

where

$$V_s^{11}(q)=\frac{[\tilde{I}_s^{(1)}(q)]^2\chi_0^{(1)}(q)}{1-\tilde{I}_s^{(1)}(q)\chi_0^{(1)}(q)}\ , \tag{31a}$$

$$V_s^{12}(q)=\frac{[\tilde{I}^{(12)}(q)]^2\chi_0^{(2)}(q)}{[1-\tilde{I}_s^{(1)}(q)\chi_0^{(1)}(q)]\Delta(q)}\ , \tag{31b}$$

$$\Delta(q)=[1-\tilde{I}_s^{(1)}(q)\chi_0^{(1)}(q)][1-\tilde{I}_s^{(2)}(q)\chi_0^{(2)}(q)] -[\tilde{I}^{(12)}(q)]^2\chi_0^{(1)}(q)\chi_0^{(2)}(q)\ , \tag{31c}$$

and

$$V_a^{11}(q)=\frac{[\tilde{I}_a^{(1)}(q)]^2\chi_0^{(1)}(q)}{1-[\tilde{I}_a^{(1)}(q)]\chi_0^{(1)}(q)}\ . \tag{31d}$$

$\chi_0^{(i)}=2\chi_0^{(ii)}$ [see Eq. (26)] is the usual Lindhard function for the ith component and

$$\tilde{I}_s^{(i)}(q)=\tfrac{1}{2}[\tilde{I}_{\sigma\sigma}^{(ii)}(q)+\tilde{I}_{\sigma-\sigma}^{(ii)}(q)]\ , \tag{32a}$$

$$\tilde{I}_a^{(i)}(q)=\tfrac{1}{2}[\tilde{I}_{\sigma\sigma}^{(ii)}(q)-\tilde{I}_{\sigma-\sigma}^{(ii)}(q)]\ , \tag{32b}$$

for $i=1,2$.

It is easy to see the physical meaning of various terms in (30). $V_s^{11}(q)$ represents the density-fluctuation contribution in component (1) as if the other component is absent. $V_s^{12}(q)$ represents the density fluctuation coming from the coupling of the density fluctuations of both components, whereas $V_a^{11}(q)$ represents the spin-fluctuation contribution coming from component 1 which is decoupled from component 2, or vice versa. Notice that contribution from $\tilde{T}(0)$ is absorbed in μ^* as before.

The terms $\tilde{I}_{s(a)}^{(i)}$ and $\tilde{I}^{(12)}$ can again be identified with appropriate local-field factors for the two-component system,[7,1] whereas $\tilde{I}^{(12)t}$ has to be determined separately.

V. CONCLUSIONS AND COMMENTS

In this paper we have derived in a systematic way simple effective interactions suitable for self-energy study. The main uncertainty of the approach is the "local" approximation introduced which simplifies drastically the mathematics involved leading to simple, analytical expressions for the effective interactions. It is important to note that "local" approximation is not good under certain circumstances and one must be careful not to use the present results in these situations. For example, it would be inaccurate to use a local approximation for the T matrix for a system which is close to superconducting transition since a singularity arises, in this case, in the T matrix with respect to the *total* energy momentum $p+p'$ but *not* with respect to the energy-momentum transfer $p-p'$. Another similar situation would be in a system of electron-hole liquid close to liquid- (exciton) gas transition. Again, the T matrix between electron-hole pairs builds up singularity with the total energy momentum but not with the energy-momentum transfer. Under these circumstances, one has to use the more fundamental expression (1) with nonlocality properly treated.

Another more fundamental problem concerns the self-consistency of the irreducible particle-hole interactions $\tilde{I}$ between, say, the left- and right-hand sides of Eq. (4). It is easy to convince oneself that once we assume a local approximation, then self-consistency for $\tilde{I}$ is destroyed. This is the price we have paid for obtaining a simple, analytical form of effective interaction which is easy to use. One of the methods trying to remedy this problem is the induced-interaction model proposed by Babu and Brown,[8] which has been improved later by others.[9] However, drastic simplification is also needed in their approach in order to come up with expressions which are mathematically easy to handle.

In the following paper, we shall use the effective interactions derived here to study effective masses in electron and electron-hole liquids. We shall see that some interesting result is indeed found in the case of an electron-hole liquid.

ACKNOWLEDGMENTS

This work was supported in part by the Materials Research Center of Northwestern University under National Science Foundation Grant No. DMR-82-16972, and in part by the U.S. Department of Energy.

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