Josephson current in low-dimensional proximity systems and the field effect

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The system S_α-M_β-S_γ (S_α and S_γ are superconductors, M_β is a semiconductor, a semimetal, or a normal metal) is studied. Particular attention is paid to the case when M_β contains a two-dimensional electron gas (e.g., an inversion layer). A one-dimensional (1D) case is also considered. The Josephson current is evaluated and the main factors determining the field effect are studied. A special diagrammatic method allowing one to calculate the thermodynamic Green's function and, consequently, the Josephson current, has been developed. The current depends strongly on the electron concentration which leads to a noticeable field effect. The dependence of j_m on other factors, such as temperature, mobility, effect mass, etc. is also studied. The field effect appears to be stronger for low-dimensional systems. An analysis of the experimental data obtained recently for the Nb-InAs-Nb system is carried out.

I. INTRODUCTION

Systems of the type S_α-M_β-S_γ (S_α and S_γ are superconductors, M_β is a normal metal, a semiconductor, or a semimetal) have attracted a lot of interest. The flow of the Josephson current in such systems is characterized by peculiar features and is promising from the point of view of applications. In this connection, we would like to point out a paper[1] in which the field effect has been used to bring about a noticeable change in the amplitude of the Josephson current flowing through the inversion layer in the system Nb-InAs-Nb. It is important to note that the inversion layer contains a two-dimensional electron gas. In this connection it is of interest to carry out a theoretical analysis of tunneling system with low-dimensional coupling.

The Josephson current in S-M-S systems has been studied by several authors.[2–6] They have described the case of the usual three-dimensional M subsystem and calculated the corresponding current.

In this paper we focus our attention on the case when M is a two-dimensional electron gas, although the general method developed here can be applied to the 3D case. A major example of such a 2D system is an inversion layer. Another example is a size-quantizing film. We shall consider also the 1D case. It will be shown that a transition to low-dimensional systems is crucial for the intensity of the field effect.

Because of the presence of degenerate electron gas, the M_β part of the system differs in a striking way from a usual tunnel barrier: the proximity effect plays a key role. The length of M_β (see Fig. 1) exceeds the coherence length ξ_N and therefore it is necessary to take into account the space dependence of the pair amplitude. Our approach is based on the method of thermodynamic Green's functions and the corresponding diagrammatic technique. This method allows one to describe the phenomenon of interest in any temperature region.

Our main interest is in the nature of the field effect and in the analysis of the main factors which determine the change of the Josephson current due to an external electric field. Low-dimensional systems appear to be very promising for the field effect.

The structure of the paper is as follows. Section II addresses the problem of obtaining the main equations. The Josephson current will be evaluated in Sec. III. We will discuss also the nature of the field effect and its dependence on various parameters. Section IV contains an analysis of the experimental data and a general discussion.

II. MAIN EQUATIONS

Consider the system S_α-M_β-S_γ (Fig. 1) where S_α and S_γ are superconductors and M_β contains two-dimensional degenerate electron gas. Suppose that only the lowest subband of the transverse motion is filled [e.g., for InAs this assumption is valid up to the surface-carrier concentration $N_{sf}^{\beta} \sim 10^{12}$ cm^{-2} (Refs. 1, 7 and 8)]. Note that the thickness $d \sim 10^2$ Å (see, e.g., Ref. 8, and for size—quantizing films[9]). The case of several filled subbands can be considered in a similar way and will be discussed elsewhere.

The flow of a nondissipative current in the system of interest is not the usual Josephson tunneling through a barrier. The length L of the normal film is large [up to $\sim 5\times 10^3$ Å (Refs. 1 and 7)], moreover M_β is not an insulator and contains its own electron system which participates in the current-flow state. This state is caused by the proximity effect. A complex state is induced in the M_β

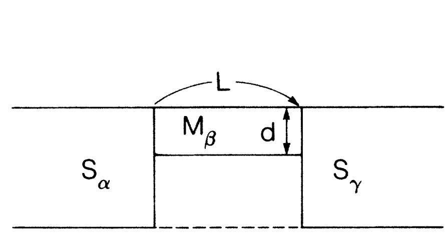


FIG. 1. S_α-M_β-S_γ system.

system; it is determined by two superconductors with different phases.

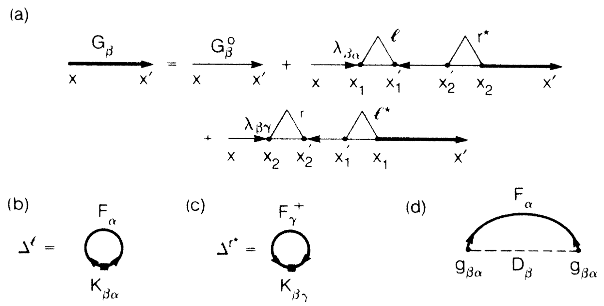
Our subsequent consideration will be based on the method of thermodynamic Green's functions (see, e.g., Ref. 10). As is known, this method allows one to evaluate the temperature dependences of various quantities. Another important advantage of this method is the possibility to use the diagrammatic technique in order to calculate the corresponding Green's function. We consider the thermodynamic electronic Green's functions G^s_α, G_β, and G^s_γ (the index s means that the α and γ metals are superconductors). In this connection we can introduce the complete sets $\{\psi_\alpha\}$, $\{\psi_\beta\}$, and $\{\psi_\gamma\}$. For example, the functions $\{\psi_\alpha\}$ are the eigenfunctions of the Hamiltonian H_α describing the state of an isolated α metal in the absence of the proximity effect. The sets $\{\psi_\beta\}$ and $\{\psi_\gamma\}$ are similar. The Green's function $G^0_\beta(\mathbf{r},\mathbf{r}',\omega_n)$ corresponds to an isolated β system and can be written in the form

$$G^0_\beta(\mathbf{r},\mathbf{r}',\omega_n)=\sum_k \psi^*_{\beta k}(\mathbf{r})\psi_{\beta k}(\mathbf{r}')[i\omega_n-\xi_k]^{-1}\ , \qquad (1)$$

where $\omega_n=(2n+1)\pi T$, $\xi_k=\epsilon_\beta(k)-\epsilon_F$; in the effective-mass approximation, $\epsilon_\beta(k)=k^2/2m^*_\beta$.

The proximity effect is due to peculiar features at the boundary region which separates α and β (γ and β) systems. Namely, Cooper pairs can penetrate from the superconductor to the normal crystal γ. In addition, the β electrons can make transitions into the α and γ superconductors with formation of Cooper pairs. These processes can be described by the special pairing self-energy parts Δ^l and Δ^r (the indices l and r correspond to the S_α-M_β and S_γ-M_β contacts, respectively).

We can write the following diagrammatic equation for the Green's function G_β:



(2)

where (a) is the diagrammatic equation for the thermodynamic Green's function; Δ^l describes a $\beta\rightarrow\alpha$ transition with the formation of a Cooper pair, $\lambda_{\beta\alpha}$ is the corresponding vertex; (b) and (c) are the self-energy parts Δ^l and Δ^{r*} in the local approximation, F^α and F^+_γ are the anomalous Green's functions; (d) is the self-energy part due to the electron-phonon coupling. In the analytic form,

$$G_\beta(\mathbf{r},\mathbf{r}',\omega_n)=G^0_\beta(\mathbf{r},\mathbf{r}',\omega_n)+\int G^0_\beta(\mathbf{r},\mathbf{r}_1,\omega_n)\Delta^l(\mathbf{r}_1,\mathbf{r}'_1;\omega_n)G^0_\beta(\mathbf{r}'_2,\mathbf{r}'_1;-\omega_n)\Delta^{r*}(\mathbf{r}'_2,\mathbf{r}_2,\omega_n)G_\beta(\mathbf{r}_2,\mathbf{r}',\omega_n)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}'_1d\mathbf{r}'_2$$
$$+\int G^0_\beta(\mathbf{r},\mathbf{r}_2,\omega_n)\Delta^r(\mathbf{r}_2,\mathbf{r}'_2;\omega_n)G^0_\beta(\mathbf{r}'_1,\mathbf{r}'_2;-\omega_n)\Delta^{l*}(\mathbf{r}'_1,\mathbf{r}_1,\omega_n)G_\beta(\mathbf{r}_1,\mathbf{r}',\omega_n)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}'_1d\mathbf{r}'_2\ . \qquad (2')$$

Here

$$\mathbf{r}=\{x,y,z\},\quad \mathbf{r}'=\{x',y',z'\},\quad \mathbf{r}_1=\{0,y_1,z_1\}\ ,$$
$$\mathbf{r}'_1=\{0,y'_1,z'_1\},\quad \mathbf{r}_2=\{L,y_2,z_2\},\quad \mathbf{r}'_2=\{L,y'_1z'_2\}\ .$$

We made a Fourier transformation with respect to the imaginary time τ; $\omega_n=(2n+1)\pi T$.

We would like to stress that the diagrams and the corresponding equation (2') are given in the coordinate representation. The self-energy parts Δ^l and Δ^r are introduced in this representation; they describe the electronic transitions $\alpha\leftarrow\beta$ and $\gamma\leftarrow\beta$ with formation of Cooper pairs occurring in the boundary regions.

The factor

$$\int G^0_\beta(\mathbf{r}'_2,\mathbf{r}'_1;-\omega_n)\Delta^{r*}(\mathbf{r}'_2,\mathbf{r}_2,\omega_n)G_\beta(\mathbf{r}_2,\mathbf{r}',\omega_n)d\mathbf{r}'_1,d\mathbf{r}'_2 \qquad (2'')$$

represents the anomalous Green's function $F^+_\beta(\mathbf{r}'_1,\mathbf{r}',\omega_n)$ which differs from zero because of the proximity effect. Note also that the right-hand side of Eq. (2) contains as a factor the exact Green's function G_β, thus it is not assumed that the self-energy parts are proportional to any small parameter.

One can see directly from Eq. (2) that the self-energy parts Δ^l and Δ^r are the amplitudes of formation of Cooper pairs, or, which is the same, the amplitudes of electron-hole transitions. Such processes are known as Andreev's reflection[11] (see also Ref. 12). The functions Δ^l and Δ^r describe this phenomenon.

The general equation (2) can be simplified for our system. First of all, the system is uniform along the y direction. Hence $G_\beta(\mathbf{r},\mathbf{r}',\omega_n)$ depends on $y-y'$, and we can make a Fourier transformation with respect to this variable. Moreover, the self-energy parts Δ^l and Δ^r can be taken in the local approximation. As a consequence of locality and uniformity, these functions do not depend on y.

We then obtain

$$G_\beta(x,x';p_y;z,z';\omega_n)=G^0_\beta(x,x';p_y;z,z';\omega_n)$$
$$+\int G^0_\beta(x,0;p_y;z,z_1;\omega_n)\Delta^l(\omega_n)G^0_\beta(L,0;-p_y;z_2;z_1;-\omega_n)\Delta^{r*}(\omega_n)G_\beta(L,x';p_y;z_2;z';\omega_n)dz_1dz_2$$
$$+\int G^0_\beta(x,L;p_y;z,z_2;\omega_n)\Delta^r(\omega_n)G^0_\beta(0,L;-p_y;z_1,z_2;-\omega_n)\Delta^{l*}(\omega_n)G_\beta(0,x';p_y;z_1,z';\omega_n)dz_1dz_2\ . \qquad (3)$$

Note that if the β system were a 3D electron gas, then one would make a Fourier transformation with respect to $\rho=\{y,z\}$. In our case the β system is size quantizing.

The Green's functions G^0_β and G_β can be written in the diagonal representation (see Ref. 13), with respect to the eigenfunctions $\chi_k(z)$ describing the transverse motion of the electrons, i.e.,

$$G^0_\beta(x,x';p_y;z,z';\omega_n)=\sum_k G^0_{\beta k}(x,x';p_y;\omega_n)\chi^*_k(z)\chi_k(z') .$$

As a result we obtain

$$G_{\beta k}(x,x';p_y;\omega_n)=G^0_{\beta k}(x,x';p_y;\omega_n)+G^0_{\beta k}(x,0;p_y;\omega_n)\Delta^l(\omega_n)G^0_{\beta k}(L,0;-p_y;-\omega_n)\Delta^{r*}(\omega_n)G_{\beta k}(L,x';p_y;\omega_n)$$
$$+G^0_{\beta k}(x,L;p_y;\omega_n)\Delta^r(\omega_n)G^0_{\beta k}(0,L;-p_y;-\omega_n)\Delta^{l*}(\omega_n)G_{\beta k}(0,x';p_y;\omega_n) . \quad (4)$$

Approximately, the projection p_z can be used as a transverse quantum number. Different subbands correspond to its different quantized values $p_z=\pi k/d$ (d is the scale of the transverse motion, e.g., the thickness of the inversion layer).

We focus on the case when only the lowest subband is filled. For example, for InAs this is valid up to $N\sim10^{12}$ cm^{-2}. Hence, we can put $k=1$ (the index k will be omitted below) and finally we obtain

$$G_\beta(x,x';p_y;\omega_n)=G^0_\beta(x,x';p_y;\omega_n)+G^0_\beta(x,0;p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(L;0;-p_y;-\omega_n)\Delta^{r*}(\omega_n)G_\beta(L,x';p_y;\omega_n)$$
$$+G^0_\beta(x,L;p_y;\omega_n)\Delta^r(\omega_n)G^0_\beta(0,L;-p_y;-\omega_n)\Delta^{l*}(\omega_n)G_\beta(0,x';p_y;\omega_n) . \quad (5)$$

The evaluation of the Green's function G^0_β can include the contribution of ordinary scattering [see Fig. (2) part (a)]. An estimate of these diagrams is given in the Appendix. It turns out that this contribution is small in the region of small p_y (this region makes the major contribution to the Josephson current, see below). The term describing multiple successive contributions of the pairing self-energy parts near the boundary [see Fig. (2) part (b)] also appears to be small (see the Appendix).

Hence, the Green's function G_β satisfies Eq. (5). This equation can be solved (see the Appendix), and we arrive at the expression:

$$G_\beta(x,x';p_y;\omega_n)=G^0_\beta(x,x';p_y;\omega_n)+[G^0_\beta(x,0;p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(L,0;-p_y;-\omega_n)\Delta^{r*}(\omega_n)f$$
$$+G^0_\beta(x,L;p_y;\omega_n)\Delta^r(\omega_n)G^0_\beta(0,L;-p_y;-\omega_n)\Delta^{l*}(\omega_n)g]R^{-1} , \quad (6)$$

where

$$f=a_1(1-c)-ac_1,\ g=a(1-b_1)+a_1b,\ R=(1-c)(1-b_1)-c_1b,\ a=G^0_\beta(0,x';p_y;\omega_n) ,$$
$$a_1=G^0_\beta(L,x';p_y;\omega_n),\ b=G^0_\beta(0,0';p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(L,0;-p_y;-\omega_n)\Delta^{r*}(\omega_n) ,$$
$$b_1=G^0_\beta(L,0;p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(0,L;p_y;\omega_n)\Delta^{r*}(\omega_n),\ c=G^0_\beta(0,L;p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(0,L;-p_y;-\omega_n)\Delta^{r*}(\omega_n) , \quad (7)$$
$$c_1=G^0_\beta(L,L;p_y;\omega_n)\Delta^r(\omega_n)G^0_\beta(0,L;-p_y;-\omega_n)\Delta^{l*}(\omega_n) .$$

If $L\gg\xi_N$ [this is the main case we consider; $\xi_N=(P_F/2\pi m^*_\beta T)$, we put $\hbar=1$], then the Green's function $G^0_\beta(0,L;p_y;\omega_n)$ contains the exponential factor $\exp(-L/\xi_n)$, see below, Eq. (15). Hence, in the lowest-order approximation with respect to this small exponential factor, $f\simeq a_1$, $g\simeq a$, $R\simeq1$, and we can write

$$G_\beta(x,x';p_y;\omega_n)=G^0_\beta(x,x';p_y;\omega_n)+G^0_\beta(x,0;p_y;\omega_n)\Delta^l(\omega_n)G^0_\beta(L,0;-p_y;-\omega_n)\Delta^{r*}(\omega_n)G^0_\beta(L,x';p_y;\omega_n)$$
$$+G^0_\beta(x,L;p_y;\omega_n)\Delta^{l*}(\omega_n)G^0_\beta(0,L;-p_y;-\omega_n)\Delta^r(\omega_n)G^0_\beta(0,x';p_y,\omega_n) . \quad (8)$$

III. JOSEPHSON CURRENT, FIELD EFFECT

A. Supercurrent

Let us turn to the calculation of the Josephson current. Consider the general expression

$$\mathbf{j}(\mathbf{r})=\frac{ie}{m^*}T\sum_{\omega_n}[(\nabla_{\mathbf{r}}-\nabla_{\mathbf{r}'})G(\mathbf{r},\mathbf{r}';\omega_n)|_{\mathbf{r}'=\mathbf{r}} .$$

Thanks to the equation of continuity, $\mathrm{div}\mathbf{j}=0$, one can evaluate the current at any point x ($j_x=$const, see, e.g., Ref. 14). The expression describing the current can be written in the form

$$j_x=\frac{ie}{m^*}T\sum_{\omega_n}\int dp_y(\nabla_x-\nabla_{x'})G_\beta(x,x';p_y,\omega_n)|_{x=x'=L/2} . \quad (9)$$

We have selected $x=L/2$.

Based on Eq. (8), one can obtain, after simple manipulations, the following expression:

$$j_x=\frac{2e}{m^*}\mathrm{Im}\sum_{\omega_n}\int dp_y\Delta^l(\omega_n)\Delta^{r*}(\omega_n)$$
$$\times G^0_\beta(L,0;-p_y;-\omega_n)S_\beta(L/2,p_y,\omega_n) , \quad (10)$$

where

$$S^0_\beta(x,p_y,\omega_n)=G^0_\beta(x,0;p_y;\omega_n)\frac{\partial}{\partial x}G^0_\beta(L,x;p_y;\omega_n)-G^0_\beta(L,x;p_y;\omega_n)\frac{\partial}{\partial x}G^0_\beta(x,0;p_y;\omega_n)\ . \quad (10')$$

Consider the self-energy parts Δ^l and Δ^{r*}. The corresponding diagrammatic equations are presented by Eq. (2) part (b); (we can write similar equations for Δ^{l*} and Δ^r). The functions F_α and F^+_γ are the total anomalous Green's functions of the superconductors α and γ respectively; $K_{\beta\alpha}$ and $K_{\beta\gamma}$ are the effective vertices describing the proximity effect. For example, $K_{\beta\alpha}$ describes the electronic transition $\beta\rightarrow\alpha$ with the formation of a Cooper pair. Equation (2) part (c) corresponds to the passage of a pair from the γ superconductor to the β system. It is important to note that the dependence of j_x on different parameters is determined mainly not by the vertices, but by other factors (see below).

The vertices $K_{\beta\alpha}$ and $K_{\beta\gamma}$ are determined by a number of mechanisms (strictly speaking, $K_{\beta\alpha}=\sum_i K^i_{\beta\alpha}$, the index i denoting the various mechanisms). First of all, we should take tunneling into consideration. This mechanism has been considered by Aslamazov *et al.*[2] In this case $K_{\beta\alpha}$ can be expressed in terms of the tunneling matrix element.[15] Note that the use of the diagrammatic technique allows us to consider large values of the penetration coefficient.

Another possibility is connected with specific features of the electron-phonon interaction in the boundary region. This interaction may result in an electronic transition $\beta\rightarrow\alpha$, described by the Hamiltonian

$$\hat{H}_{\alpha\beta}=\int g(\mathbf{r})\psi^\dagger_\alpha(\mathbf{r})\psi_\beta(\mathbf{r})\phi(\mathbf{r})d\mathbf{r}$$

$\psi^\dagger$ and ψ are the electron operators, ϕ is the phonon operator, $g(\mathbf{r})$ differs from zero only in the boundary region. The corresponding self-energy part Δ^l is shown in Eq. (2) part (d). The main contribution to pairing comes from the short-wave part of the phonon spectrum. As a result, the phonon Green's function D can be taken in the δ function approximation (see Ref. 16), that is, $D_{\omega_n}(\mathbf{r},\mathbf{r}')=D_{\omega_n}\delta(\mathbf{r}-\mathbf{r}')$ and this leads to the local approximation [Eq. 2(b)] with $K_{\alpha\beta}=g^2_{\alpha\beta}D_{\omega_n-\omega_{n'}}$ (in the weak coupling approximation, $D\sim 1$, see Ref. 17). It is possible that the low-frequency part of the phonon spectrum also contributes to the proximity effect. In this case it is necessary to go beyond the local approximation. This question will be considered in detail elsewhere.

The self-energy parts Δ^r and Δ^l are directly related to the anomalous Green's functions F_α and F_γ [see Eq. (2) parts (b) and (c)]. Each one of them is characterized by its own phase, so that $F^\alpha=|F_\alpha|e^{i\theta_\alpha}$, $F_{\gamma*}=|F_\gamma|e^{-i\theta_\gamma}$. Based on these expressions and Eq. (10) we obtain an expression for the Josephson current $j_x=j_m\sin(\theta_\alpha-\theta_\gamma)$, where

$$j_m=\frac{\pi e}{m^*_\beta}s_{\beta\alpha}s_{\beta\gamma}T\sum_{\omega_n}\frac{\Delta_\alpha\Delta_\gamma}{(\omega_n^2+\Delta_\alpha^2)^{1/2}(\omega_n^2+\Delta_\gamma^2)^{1/2}}I(L,\omega_n)\ . \quad (11)$$

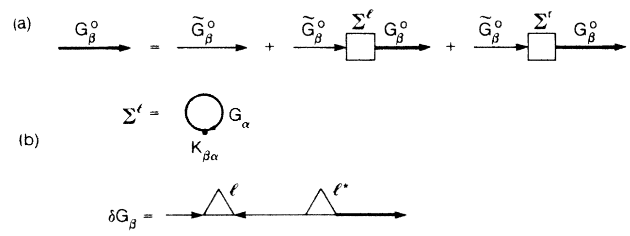


FIG. 2. (a) $\beta\alpha$ and $\beta\gamma$ scattering; (b) successive pairing near the boundary.

Here

$$I(L,\omega_n)=\int_0^\infty dp_y G^0_\beta(L,0;p_y;-\omega_n)S(L/2;p_y,\omega_n)\ . \quad (12)$$

We took into account the fact that the integral in Eq. (11) is an even function of p_y. We used expressions for Δ^l and Δ^r which can be obtained from Eq. (2) (see also the Appendix), e.g.,

$$|\Delta^l(\omega_n)|=K_{\beta\alpha}|F_\alpha(0)|=s_{\beta\alpha}\int d\xi\frac{\Delta_\alpha}{\omega_n^2+\xi^2+\Delta_\alpha^2}=\frac{\pi}{2}s_{\beta\alpha}\Delta_\alpha/(\omega_n^2+\Delta_\alpha^2)^{1/2}\ . \quad (13)$$

Δ_α is the absolute value of the order parameter in the α superconductor, $s_{\beta\alpha}=K_{\beta\alpha}\nu_\alpha$, $s_{\beta\alpha}=K_{\beta\gamma}\nu_\gamma$, and $\nu_{\alpha(\gamma)}$ is the density of states. Note that Δ_α is the order parameter in the presence of the proximity system and, strictly speaking, is not equal to Δ^0_α, where Δ^0_α is the order parameter of an isolated α superconductor. Its values can be evaluated from the self-consistent equation for Δ_α. The deviation of Δ_α from Δ^0_α depends on the value of $K_{\alpha\beta}$. A similar expression can be obtained for $|\Delta^r|$.

If we study a junction S_α-M_β-S_α with two identical superconductors, then Eq. (11) can be written in the form

$$j_m=\tilde{\gamma}T\sum_{\omega_n}\frac{\Delta_\alpha^2}{\omega_n^2+\Delta_\alpha^2}I(L,\omega_n)\ , \quad (14)$$

where

$$\tilde{\gamma}=\frac{\pi e}{m^*_\beta}s^2_{\beta\alpha}\ ;$$

$I(L,\omega_n)$ is defined by Eq. (12).

We see that the problem of evaluating the Josephson current is reduced to the calculation of the quantity $I(L,\omega_n)$ and the subsequent evaluation of the sum in Eq. (11). As usual, we should distinguish two major cases, namely the "clean" and "dirty" limits for the β system.

B. "Clean" β system: Dependence on N

The function $I(L,\omega_n)$ [see Eq. (12)] contains the Green's function of an isolated β system (in the absence of the proximity effect). The length L is assumed large ($\sim 2-5\times 10^3$ Å); it exceeds noticeably the atomic scale and even the size of the Cooper pair ξ_N. Under such conditions, we can classify states as in bulk material (see Ref. 14). In some sense, our system is described by the model in which each subsystem α, β, and γ is replaced by a "sheet" of 3D space (the β system is restricted in the z direction), and these sheets are connected along the lines $x=0$ ($\alpha\beta$ coupling) and $x=L$ ($\gamma\beta$ coupling). This means that plane waves can be taken as eigenfunctions describing the electron states of the β system. The dispersion relation will be taken in the effective mass approximation. Then the Green's function G_β^0 is equal to (see, e.g., Refs. 14 and 18)

$$G_\beta^0=\lambda_{\omega_n}^{-1}m^*\exp(i\lambda_{\omega_n}|x-x'|)\ , \qquad (15)$$

where

$$\lambda_{\omega_n}=[2m^*(i\omega_n-\xi_\beta)]^{1/2};\ \ \xi_\beta=p_y^2/2m^*-\epsilon_{F\perp}\ . \qquad (15')$$

$\epsilon_{F\perp}$ is the Fermi energy corresponding to the lowest transverse level $\epsilon_\perp$; $\epsilon_{F\perp}=\epsilon_F-\epsilon_\perp$.

Substituting (15) into Eq. (12), and choosing the branches of λ_{ω_n} which correspond to damping of the Josephson current, we arrive, after some manipulations, at the expression

$$j_m=\tilde{\gamma}p_F(m_\beta^*)^3T\sum_{\omega_n>0}\frac{\Delta_\alpha\Delta_\gamma}{(\omega_n^2+\Delta_\alpha^2)^{1/2}(\omega_n^2+\Delta_\gamma^2)^{1/2}}\times\int_0^1 d\zeta u^{-2}\exp[-2uL\sin(\eta/2)]\ , \qquad (16)$$

where $\zeta=p_y/p_F$, $p_F=(2m_\beta^*\epsilon_{F\perp})^{1/2}$ is the two-dimensional Fermi momentum, $u=\tilde{u}(2m)^{1/2}$, $\tilde{u}=(\omega_n^2+\xi_\beta^2)^{1/4}=[\omega_n^2+\epsilon_{F\perp}^2(1-\zeta^2)^2]$, and $\eta=\arctan(\omega_n/|\xi_\beta|)=\arctan[\omega_n/\epsilon_F(1-\zeta^2)]$. At first, we consider the case when the parameter $L/\xi_N^c>1$, where $\xi_N^c=p_F/(2\pi Tm^*)$ is the coherence length.[19] We will see that this case is the most promising from the point of view of the field effect. An analysis of the integral in Eq. (16) shows that the main contribution comes from the region of small ζ (qualitatively, this means that the current is due mainly to electrons moving almost perpendicular to the boundary). The integral can be easily evaluated by the method of steepest descent and we obtain [for simplicity we restrict ourselves to the case of two identical superconductors S, see Eq. (17); generalization to systems containing different S_α and S_γ is straightforward]

$$j_m=\tilde{\gamma}\left[\frac{\pi p_F}{m_\beta^*L}\right]^{1/2}p_F^{-1}(m_\beta^*)^3T\times\sum_{\omega_n>0}\frac{\Delta_\alpha^2}{(\omega_n^2+\Delta_\alpha^2)\omega_n^{1/2}}\exp\left[-\frac{2\omega_n m_\beta^*}{p_F}L\right]\ . \qquad (17)$$

Because of the condition $2\pi Tm_\beta^*L/p_F>1$ (see above), we can keep the first term only (other terms contain an additional exponential smallness). We then obtain

$$j_m=A\exp\{-2\pi Tm_\beta^*L/p_F\}\ , \qquad (18)$$

$$A=\tilde{\gamma}(p_F/m_\beta^*LT)^{1/2}p_F^{-1}(m_\beta^*)^3\Delta_\alpha^2[(\pi T)^2+\Delta_\alpha^2]^{-1}\ . \qquad (18')$$

For example, if $L\sim5\times10^{-5}$ cm and $V_F=p_F/m_\beta^*\approx7\times10^7$ cm/s, then expression (18) is valid in the region $T\gtrsim3$ K.

One can see from Eq. (18) that the Josephson current in the system of interest depends on many factors. If we are interested in the field effect, the most important factor is the dependence of j_m on the electron concentration n, because applied voltage affects mainly this quantity.

For a 2D system, $p_F=(2\pi N_{sf})^{1/2}$, $N_{sf}=nd$, and hence

$$j_m=A\exp[-\rho/(N_{sf})^{1/2}]\ , \qquad (19)$$

where $\rho=(2\pi)^{1/2}Tm_\beta^*L$.

The exponential dependence of j_m on the electron concentration can lead to noticeable effects. The sharpness of this dependence is determined by other parameters, namely by L, m^*, and T. We can see from Eq. (19) that an increase in their values results in an increase of this sharpness, that is, in an increase of the field effect (although the absolute value of j_m is getting smaller). A more detailed discussion is given in Sec. IV.

Consider now the case when $2\pi TLm^*/p_F\ll1$ (e.g., the low-temperature region, although the smallness of this ratio can be caused by other factors as well). Assume that $\tilde{\gamma}$ is small; then we can use Eq. (8) despite the inequality $L\ll\xi_N$.

We can pass from summation to integration ($2\pi T\sum_{\omega_n}\rightarrow\int d\omega$, see, e.g., Ref. 10) and we obtain

$$j_m\propto\Delta_\alpha[\mathrm{ci}(b)\sin b-\mathrm{si}(b)\cos b]\ , \qquad (20)$$

where ci(x) and si(x) are the cosine and sine integrals and $b=2L\Delta_\alpha/V_F$. If $b\gg1$, we use the asymptotic values of ci and si (see, e.g., Ref. 20), and we obtain $j_m\propto p_F/L$. It is interesting to note that in this case j_m depends on L not exponentially, but according to a power law. The presence of the degenerate β electron system makes the picture different from the usual Josephson tunneling through an insulator.

The field effect does not manifest itself strongly in the low-temperature region. The effect is much stronger at intermediate temperatures and in the region $T\sim T_c$ [see Eq. (18)].

C. "Dirty" case: Dependence on the concentration

We have considered a "clean" β system ($\xi_N\ll l$). This case is realistic even for strongly doped materials (see below, Sec. IV). In this section we consider the opposite ("dirty") case.

We use the method developed in Ref. 2; it is applicable to a 2D system. Based on Eq. (11), we can obtain

$$j_m=\tilde{\gamma}T\sum_{\omega_n}\frac{\Delta_\alpha\Delta_\gamma}{(\omega_n^2+\Delta_\alpha^2)^{1/2}(\omega_n^2+\Delta_\gamma^2)^{1/2}}W(0,L;2i|\omega_n|)\ . \qquad (21)$$

Here

$$W(x_1,x_2,2i|\omega_n|)=\overline{\int dp_y G_\beta^0(x_1,x_2;-p_y;-\omega_n)G_\beta^0(x_2,x_i;p_y;\omega_n)}\ .$$

The bar denotes averaging over the positions of the impurities. The function W satisfies the diffusion equation[21] and can be written in the form[2]

$$W(x_1,x_2,2i\,|\omega_n|)=\frac{1}{L}\sum_{k=-}\frac{\cos(\pi k x_1/L)\cos(\pi k x_2/L)}{2|\omega_n|+\frac{1}{2}(\pi k/L)^2V_F^2\tau_{tr}}\,, \tag{22}$$

τ_{tr} is the transport time between collisions. Based on Eqs. (21) and (22), we obtain

$$j_m=\tilde{\gamma}\pi T\sum_{k=-}(-1)^k\sum_{\omega_n>0}\frac{\Delta_\alpha\Delta_\gamma}{(\omega_n^2+\Delta_\alpha^2)^{1/2}(\omega_n^2+\Delta_\gamma^2)^{1/2}}\times\frac{1}{2|\omega_n|+\frac{1}{2}(\pi k/L)^2V_F^2\tau_{tr}}\,. \tag{23}$$

If $T>T^*$ [an expression for T^* will be obtained below, see Eq. (26)], we keep only the term with $n=0$. Then we arrive at the following expression (for simplicity we consider the case $\Delta_\alpha=\Delta_\gamma$):

$$j_m=\tilde{\gamma}(m^3/4p_F)(\pi\tau T)^{-1/2}\exp(-L/\xi_N^D)\,, \tag{24}$$

where $\xi_N^D=(D/2\pi T)^{1/2}=(\xi_N^c l/2)^{1/2}$, $D=V_F^2\tau/2$ is the diffusion constant, ξ_N^c is the coherence length for the "clean" system (see above), and

$$l=\mu V_F m_\beta^*/e \tag{25}$$

is the mean free path (μ is the mobility). A dependence similar to (24) has been obtained in Ref. 3 on the basis of the Ginzburg-Landau theory. The effect of localization has been studied in Ref. 22. It has been shown that this effect also results in a decrease of the coherence length.

One can neglect the terms $n\geq 1$ [see Eq. (23)] if $(2\pi T/D)^{1/2}L\gg 1$; then these terms contain an additional exponential smallness. Hence Eq. (24) is valid if $T>T^*$, where

$$T^*=D/2\pi L^2\,, \tag{26}$$

or $T^*=V_F l/4\pi L^2$. For example, if $\mu=5\times 10^3$ cm^2/V s, $l\simeq 3\times 10^2$ Å, $V_F=7\times 10^7$ cm/s, $L=4\times 10^{-5}$ cm, we obtain $T^*\approx 0.1$. K. A similar analysis can be used in the 3D case studied experimentally in Ref. 23.

We are interested mainly in the dependence of j_m on the electron concentration because this dependence is related to the field effect. We can see from Eq. (24) that the major contribution to the dependence of j_m on the electron concentration $N\equiv N_{sf}$ (see above) as in the "clean" case [see Eq. (18)], comes from the exponential factor

$$j_m\propto\exp\{-r/[N\mu(N)]^{1/2}\}\,, \tag{27}$$

or

$$j_m\propto\exp(-\rho_d/N^{1/2}),\quad \rho_d=\rho(\hbar/\pi\tau T)^{1/2}\,, \tag{28}$$

where $r=(2m^*Te)^{1/2}L$, ρ is the quantity introduced earlier [see Eq. (19)]. In addition to the direct dependence of j_m on N, the field effect is affected also by the N dependence of μ.

D. 1D case

In this paper we are concerned mainly with the situation when the coupling between two superconductors is provided by a two-dimensional electron gas. However, from the point of view of the field effect, it is interesting to consider the Josephson current in the junction S_α-M_β-S_γ with the β system being a 1D gas. It can be, for example, an inversion layer limited in two directions. M_β can be, for instance, a film of InAs, characterized by size quantization in the y direction due to the small thickness L_y; quantization along the z axis, like in the usual case, is due to the presence of an inversion layer. The electric field is perpendicular to the plane. Here we consider the "clean" case only. A more detailed analysis will be given elsewhere.

Evaluation of the current is analogous to the 2D case [see above, Eq. (16)]; one should not integrate over p_y. As a result, we obtain

$$j_m=\gamma_L(m_\beta^*)^3p_F^{-2}T\sum_{\omega_n>0}\Delta_\alpha^2(\omega_n^2+\Delta_\alpha^2)^{-1}\times\exp(-2\omega_n m_\beta^* L/p_F)\,. \tag{29}$$

We consider the case of two identical superconductors. If $2\pi TLm_\beta^*/p_F>1$, then

$$j_m=\gamma_L(m_\beta^*)^3p_F^{-2}T\Delta_\alpha^2[(\pi T)^2+\Delta_\alpha^2]^{-1}\times\exp(-2\pi Tm_\beta^*L/p_F)\,. \tag{30}$$

Let us analyze the dependence of j_m on the electron concentration. In the 1D case, $p_F=2\pi N_L$, where N_L is the linear concentration:

$$j_m\propto\exp(-\tilde{\rho}/N_L)\,, \tag{31}$$

where $\tilde{\rho}=\rho/2\pi$ [see, Eq. (19)]. The dependence of j_m on the concentration becomes, generally speaking, stronger than in the 2D case where $j_m\sim\exp(-r/N_S^{1/2})$ [see Eq. (19)]. Of course, the sharpness of $j_m(N)$ depends on the value of the quantity ρ (see Sec. IV), but one can expect a strong field effect in the 1D case.

IV. DISCUSSION

We are studying the current-flow state in the system S_α-M_β-S_γ. This state is described by Eqs. (19), (24), (27), and (31). In this section we are going to discuss in detail the field effect and various factors contributing to it.

A voltage applied to the system affects mainly the electron concentration. The exponential dependence of j_m on the electron concentration results in a noticeable change of the current. Of course, quantitatively, it is necessary to take into account the pre-exponential factor, but the main contribution comes from the exponential factor. The latter is equal to $\exp(-\rho/N^{1/2})$ in the "clean" case, and to $\exp\{-r/[N\mu(N)]^{1/2}\}$ in the "dirty" case, where $\rho=(2\pi)^{1/2}m_\beta^*TL$ and $r=(2em^*T)^{1/2}L$. Note that both factors ρ and r increase with increasing m_β^*, T, and L. It is important to stress that an increase of the parameters m_β^*, T, and L leads in both cases to a decrease in the absolute value of the Josephson current, but at the same time it leads to an increase of the field effect, that is, the dependence $j_m(N)$ is getting sharper. Indeed, if for example, $r=2.2\times10^9$ $\mathrm{V^{-1/2}\ cm^{-1}s^{-1}}$, then $B=j_m(N_2)/j_m(N_1)=2$, if $N_1=5\times10^{11}$ $\mathrm{cm^{-2}}$, $N_2=10^{12}$ $\mathrm{cm^{-2}}$, $\mu(N_1)=4\times10^3$ $\mathrm{cm^2/V\,s}$, $\mu(N_2)=6.5\times10^2$ cm/V s. If we change some of the parameters m^*_β, T, or L so that $B=4\times10^9$ $\mathrm{V^{-1}cm^{-1}s^{-1}}$, then the ratio B is getting bigger and becomes approximately equal to 6. Hence increasing the effective mass m^*_β, temperature, and the length L is favorable to the field effect. Quantitatively, we obtain different values of j_m in the "clean" and "dirty" cases, but the nature of the dependence $j_m(m^*_\beta,T,L)$ is similar. Speaking of the dependence $\mu(N)$, one should note that this dependence can be nonmonotonic (see, e.g., Refs. 1 and 7). One can see directly from Eq. (27) that the field effect is stronger if the electron concentration corresponds to the region where $\mu(N)$ is an increasing function of N. For example, for InAs, $\mu(N)$ is an increasing function of N up to $N\sim10^{12}$ $\mathrm{cm^{-2}}$.[1,7] The subsequent decrease of μ in the density $N>10^{12}$ $\mathrm{cm^{-2}}$ is due to the filling of the next subband.[7] In this case each subband is characterized by its own coherence length, and usually $\xi_{N1}\gg\xi_{N2}$. The main contribution comes from the lowest subband, but the mobility decreases sharply because of the appearance of a new relaxation channel: intersubband scattering. This case should be considered separately; this will be done in detail elsewhere.

Equation (19) is valid in the "clean" case (if the condition $L>\xi^c_N$ is satisfied). As was noted above, this case can be relevant even for heavily doped semiconductors. The current is described by Eq. (19) or Eq. (24), depending on the ratio $l/\xi^c_N=(2\pi/e)\mu m^*_\beta T$ [see Eq. (25)]. One should use Eq. (19) if $(2\pi/e)\mu m^*_\beta T\gg1$. For example, this condition is satisfied if $\mu=5.5\times10^3$ $\mathrm{cm^2/V\,s}$, $m^*=0.05m_e$, $T\simeq12$ K.

Let us consider the system studied in Ref. 1. Assume that the electron concentration changes from $N_1=5\times10^{11}$ $\mathrm{cm^{-2}}$ to $N_2=10^{12}$ $\mathrm{cm^{-2}}$. Assume that $T=2$ K, $L=3\times10^{-5}$ cm, and $m_\beta=0.025m_e$. All these values are realistic for the inversion layer of InAs.[1] The mobility $\mu(N)$ changes from $\mu(N_1)=3.3\times10^3$ $\mathrm{cm^2/V\,s}$ to $\mu(N_2)=6\times10^3$ $\mathrm{cm^2/V\,s}$. In this case, the ratio $l/\xi^c_N=0.16$ and hence the current is described by Eq. (24) (dirty case). The dependence $j_m(N)$ is given by the factor $j_m\propto(\mu N)^{-1/2}\exp[-(\mu N)^{-12/}r]$, where $r=(2m^*_\beta Te)^{1/2}L$. In our case, $r\simeq1.3\times10^8$ $\mathrm{V^{-1/2}s^{-1/2}}$. With the use of these values of r, μ_i, and N_i $(i=1,2)$, we obtain $j_m(N_2)/j_m(N_1)\simeq2.3$. This is in good agreement with the experimental data.[1]

Hence, the exponential dependence of j_m on N results in a noticeable change of the Josephson current. As was noted above, an increase in temperature results in an increase of the field effect. For example, if $T=4$ K and the other parameters are the same, then $j_m(N_2)/j_m(N_1)\simeq4.2$. The values of T are restricted by the critical temperature of the proximity system and the latter is related to T_c of the superconductors. From this point of view, it would be interesting to study the field effect in systems containing NbN, because $T_{c\,|\,\mathrm{NbN}}>T_{c\,|\,\mathrm{Nb}}$. Of course, an increase in T leads to a decrease in the value of j_m, but the ratio $j_m(N_2)/j_m(N_1)$ is getting larger.

Consider another numerical example. Assume that $T=12$ K, $L=2\times10^{-5}$ cm, $m^*_\beta=0.05m_e$, and $\mu=5.5\times10^3$ $\mathrm{cm^2/V\,s}$. Then $l/\xi^c_N\simeq2$. In this case Eq. (19) provides a more accurate description of the current. Then $j_m\propto N^{-1/2}\exp(-\rho N^{-1/2})$, where $\rho=(2\pi)^{1/2}Tm^*L$. If $N_1=5\times10^{11}$ $\mathrm{cm^{-2}}$ and $N_2=10^{12}$ $\mathrm{cm^{-2}}$, we obtain $j_m(N_2)/j_m(N_1)\simeq3.6$.

In Sec. II we considered also the case when the β system is a 1D electron gas. As was noted [see Eq. (31)], the dependence $p_F\propto N_L$ leads to a possibility to observe the strong dependence of j_m on N_L. For example, consider the situation when $T=5$ K, $L=3\times10^{-5}$ cm, $m^*=2.5\times10^{-29}$ g, $N_{L1}=10^5$ $\mathrm{cm^{-1}}$, and $N_{L2}=2\times10^5$ $\mathrm{cm^{-1}}$. According to Eq. (30), $j_m\propto N_L^{-2}\exp(-Tm^*L/N_L)$. Then $j_m(L_2)/j_m(L_1)\simeq2.7$. If $N_{L1}=7.5\times10^4$ $\mathrm{cm^{-1}}$ and $N_{L2}=1.5\times10^5$ $\mathrm{cm^{-1}}$ then $j_m(L_2)/j_m(L_1)\simeq5.9$.

Thus, the field effect is affected by a number of parameters (m^*_β, L, T, μ, N). This variety of parameters allows to change the current in the desired direction.

V. SUMMARY

In this paper the state of the proximity system S_α-M_β-S_γ has been studied (S_α and S_γ are superconductors, M_β is a normal metal, semimetal, or a semiconductor). We have focused on the current-flow state of the system for the case when M_β is a 2D degenerate electron gas (inversion layer, size-quantizing film). The main results are as follows.

(i) Based on the method of thermodynamic Green's functions, a general method of describing the proximity system has been developed. This nonuniform system is described by a diagrammatic technique in the coordinate representation [see Eqs. (2), (2′), and (5)]. The nonuniformity is due to the presence of different materials and the coordinate dependence of the pair amplitude.

(ii) The Josephson current in the system has been evaluated [see Eqs. (10), (18), and (24)]. Its behavior depends strongly on the temperature [see Eqs. (18) and (24)], thickness L, the mobility, etc. The "clean" and "dirty" cases are analyzed.

(iii) The field effect is caused mainly by the dependence of the current on the electron concentration [see Eqs. (19) and (27)]. The exponential dependence of j_m on N leads to a noticeable effect. The magnitude of this effect can be affected by the values of various parameters.

(iv) The effect of electric field on the Josephson current has been observed experimentally in Ref. 1. A comparison with the experimental data is carried out (see Sec. IV).

(v) The field effect in the case when M_β is a 1D electron gas is considered [see Eqs. (30) and (31)].

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APPENDIX

Let us note the following points.

(1) The Green's function $G_\beta(\mathbf{r},\mathbf{r}',\omega_n)$ satisfies the following equation:

$$G_\beta(\mathbf{r},\mathbf{r}';\omega_n)=G_B^0(\mathbf{r},\mathbf{r}';\omega_n)+\int G_\beta^0(\mathbf{r},\mathbf{r}_i';\omega_n)\Delta(\mathbf{r}_i,\mathbf{r}_K;\omega_n)F_\beta^+(\mathbf{r}';\mathbf{r}_K;\omega_n)d\mathbf{r}_i d\mathbf{r}_K \ , \quad (A1)$$

where $\Delta(\mathbf{r}_i,\mathbf{r}_k;\omega_n)$ is the self-energy part describing pairing. Equation (A1) is a general one and is valid even in the case of usual electron-phonon coupling. In our case of a proximity system, $\Delta(\mathbf{r}_i,\mathbf{r}';\omega_n)$ differs noticeably from zero only in the boundary region, which leads to the equation

$$G_\beta(\mathbf{r},\mathbf{r}';\omega_n)=G_\beta^0(\mathbf{r},\mathbf{r}';\omega_n)+\int G_\beta^0(\mathbf{r},\mathbf{r}_i^0,\omega_n)\Delta^l(\mathbf{r}_i^0;\mathbf{r}_K^0;\omega_n)F_\beta^+(\mathbf{r}';\mathbf{r}_K^0,\omega_n)d\mathbf{r}_i^0 d\mathbf{r}_K^0$$
$$+\int G_\beta^0(\mathbf{r},\mathbf{r}_i^0;\omega_n)\Delta^r[(\mathbf{r}_i')^0;(\mathbf{r}_K')^0;\omega_n)]F_\beta^+[\mathbf{r}',(\mathbf{r}_K')^0;\omega_n]d(\mathbf{r}_i')^0 d(\mathbf{r}_K')^0 \ . \quad (A2)$$

Here $\mathbf{r}_i^0=\{0,y_i,z_i\}$, $\mathbf{r}_K^0=\{0,y_K,z_K\}$, $(\mathbf{r}_i')^0=\{L,y_i,z_i'\}$, $(\mathbf{r}_K')^0=\{L,y_K',z_K'\}$, and the quantities Δ and Δ^r are defined by the relation

$$\int \Delta(\mathbf{r}_i,\mathbf{r}_K,\omega_n)dx_i dx_K$$
$$=\Delta^l[\mathbf{r}_i^0;(\mathbf{r}_K')^0,\omega_n]+\Delta^r[(\mathbf{r}_i')^0;(\mathbf{r}_K')^0;\omega_n] \ . \quad (A3)$$

In the local approximation, $\Delta(\mathbf{r}_i,\mathbf{r}_K,\omega_n)=\Delta(\mathbf{r}_i;\omega_n)\delta(\mathbf{r}_i-\mathbf{r}_K)$, and we obtain

$$\int \Delta(\mathbf{r}_i,\omega_n)dx_i=\Delta^l(\mathbf{r}_i^0;\omega_n)+\Delta^r[(\mathbf{r}_i')^0;\omega_n] \ . \quad (A4)$$

Δ^l and Δ^r are related to the values of the self-energy parts in the regions near the α and γ superconductors, respectively.

The Josephson effect is described by those terms in the Green's function G_β which contain both quantities Δ^l and Δ^r. Separating the corresponding set of diagrams [see Eq. (2) and Eq. (2″)], we arrive at Eq. (2′).

(2) In order to solve Eq. (5), we put $x=0$, and then $x=L$. As a result, we obtain a system of linear equations, which can be easily solved. We find

$$G(0,x';p_y;\omega_n)=[a(1-b_1)+a_1 b]R^{-1} \ , \quad (A5)$$

$$G(L,x';p_y;\omega_n)=[a_1(1-c)+ac_1]R^{-1} \ , \quad (A6)$$

where $R=(1-b_1)(1-c)-c_1 b$; the quantities a, a_1, b, b_1, c, and c_1 are defined by Eq. (7). For the symmetric S_α-M_β-S_α system, $b=c_1$ and $b_1=c$. Note that the quantities b, b_1, c, and c_1 contain an additional exponential smallness relative to a and a_1. For example, b contains the Green's function $G_\beta^0(L,0;-p_y;-\omega_n)$ [see Eq. (15)]; one can see that $G_\beta^0(L,0;-p_y;-\omega_n)\sim e^{-p_F L}\ll 1$ (the main contribution comes from the region where $|\lambda_{\omega_n}|\sim p_F$).

(3) Let us write down the equation for the Green's function $G(\mathbf{r},\mathbf{r}',\omega_n)$ (in the absence of pairing)

$$G_\beta^0(\mathbf{r},\mathbf{r}_1,\omega_n)=\tilde{G}_\beta^0(\mathbf{r},\mathbf{r}',\omega_n)+\int \tilde{G}_\beta^0(\mathbf{r},\mathbf{r}_1,\omega_n)\Sigma(\mathbf{r}_1,\omega_n)G_\beta^0(\mathbf{r}_1,\mathbf{r}',\omega_n)d\mathbf{r}_1 \ , \quad (A7)$$

where $\mathbf{r}=\{x,y,z\}$, $\mathbf{r}'=\{x',y',z'\}$, and $\mathbf{r}_1=\{x_1,y_1,z_1\}$. If we consider a 3D electron gas, then Eq. (A7) can be written in the form

$$G_\beta^0(x,x';\rho-\rho';\omega_n)=\tilde{G}_\beta^0(x,x';\rho-\rho';\omega_n)+\int \tilde{G}_\beta^0(x,x_1;\rho-\rho_1,\omega_n)\Sigma(x_1,\rho_1,\omega_n)G_\beta^0(x_1,x';\rho_1-\rho';\omega_n)dx_1 d\rho_1 \ . \quad (A8)$$

Equation (A8) is a general equation valid for any scattering mechanism, including the usual electron-phonon scattering. As is known, the electron-phonon scattering leads to renormalization of the Fermi velocity and the electron-phonon coupling (see, e.g., Ref. 10). In our case, the scattering is connected with $\beta\to\alpha$ and $\beta\to\gamma$ transitions [see Fig. 2(a)]. This scattering is described by the self-energy part $\Sigma(x_1,\rho_1)$ which is a sharp function differing from zero only in the boundary region. Hence we obtain

$$G_\beta^0(x,x';\rho-\rho';\omega_n)=\int \tilde{G}_\beta^0(x,0;\rho-\rho_1;\omega_n)\Sigma^l(\rho_1,\omega_n)G_\beta^0(0,x';\rho_1-\rho';\omega_n)d\rho_1$$
$$+\int \tilde{G}_\beta^0(x,L;\rho-\rho_1;\omega_n)\Sigma^r(\rho_1,\omega_n)G_\beta^0(L,x';\rho_1-\rho';\omega_n)d\rho_1 \ . \quad (A9)$$

Here the quantities Σ^l and Σ^r are defined by the relation

$$\int \Sigma(x_1;\rho_1;\omega_n)dx_1=\Sigma^l(\rho_1;\omega_n)+\Sigma^r(\rho_1;\omega_n) \ . \quad (A10)$$

The integrand in (A10) differs from zero in the regions near $x_1=0$ and $x_1=L$. Σ^l and Σ^r describe scattering near the left and right boundaries, respectively. By analogy with Δ^l and Δ^r (see above), Σ^l and Σ^r do not depend on ρ_1; Fourier transforming in ρ, we obtain

$$G_\beta(x,x';\mathbf{p};\omega_n)=\tilde{G}_\beta^0(x,x';\mathbf{p};\omega_n)$$
$$+\tilde{G}_\beta^0(x,0;\mathbf{p};\omega_n)\Sigma^l(\omega_n)G_\beta(0,x';\mathbf{p};\omega_n)$$
$$+\tilde{G}_\beta^0(x,L;\mathbf{p};\omega_n)\Sigma^r(\omega_n)G_\beta(L,x';\mathbf{p};\omega_n) \ . \quad (A11)$$

This equation can be solved by analogy with Eq. (5). We shall not write down the complete solution. It is easy to see that the presence of the self-energy parts Σ^l and Σ^r leads to renormalization of the Green's function. The value of the renormalization factor depends on p_y and T. In the region of small p_y [this region makes the major contribution to the Josephson current, see Eq. (16)] and either at $T \sim T_c$ or in the intermediate temperature region, this factor is small: $\tilde{\gamma}(T/\epsilon_F)p_f a \ll 1$ [$\tilde{\gamma}$ is defined by Eq. (14), the length a corresponds to the atomic scale, so that $p_f a \sim 1$].

We shall also estimate the contribution of the diagrams in Fig. 2(b), that is, terms of the type $G(x,0;p_y;\omega_n)\Delta G(0,0;p_y;\omega_n)\Delta$. One can see that these terms are small $\sim(T_c\,|\lambda_\omega|\,)p_F a \sim (T_c/\epsilon_F)p_F a \ll 1$, and these diagrams can be neglected.

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