Plasmon band structure in a lateral multiwire semiconductor superlattice

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The collective modes of a lateral multiwire superlattice made from two-dimensional strips of alternating equilibrium electron densities are investigated in the semiclassical hydrodynamic approximation. Numerical results for the dispersion relation of the lowest three plasmon bands are given explicitly and the optical absorption spectra (appropriate for far-infrared spectroscopic experiments) for the system are obtained. Both bulk and edge plasmons are considered in the theory.

There has been a lot of recent interest[1–17] in the collective properties of low-dimensional electron gases as occurring, for example, in inversion layers and multilayer semiconductor superlattices. Plasmon dispersion has been studied both theoretically[1–11] and experimentally[12–17] in a single two-dimensional layer as well as in the periodic multilayer system. Theoretical attention in the very recent literature has focused on surface plasmons in superlattices,[3,4] edge plasmons in two-dimensional layers,[5,6] and on one-dimensional plasmons in quantum wire structures.[10] In this paper we consider the interesting system of a periodic lateral superlattice consisting of finite two-dimensional strips of alternating electron density (we refer to such systems as multiwire superlattices). Such a modulated two-dimensional electron system, where the electron density changes periodically along one direction, has recently been fabricated[15,18] in silicon inversion-layer structures. In addition to calculating plasmon dispersion in these structures, we will also report calculated results on far-infrared absorption spectra.

Our model for the lateral multiware superlattice assumes a purely two-dimensional electron later in the x-y plane, which has a periodic density modulation in the y direction (with the density being uniform along the x axis). The equilibrium electron density for the system is, therefore, given by

$$n_0(\mathbf{r})=n_0(x,y)\delta(z)\ , \tag{1}$$

$$n_0(x,y)=\begin{cases} n_A & \text{for } ld<y<a+ld\ ,\\ n_B & \text{for } a+ld<y<(l+1)d\ ,\end{cases} \tag{2}$$

with $l=0, \pm 1, \pm 2$, etc. The model of Eqs. (1) and (2) describes a system containing alternate metal strips (of zero thickness in the z direction) of widths a and b $(=d-a)$ in the y direction (and infinitely long in the x direction) with alternating two-dimensional electron densities n_A and n_B, respectively. Thus the system has complete translational invariance in the x direction and has periodic symmetry with period $d=a+b$ along the superlattice direction y. Our approximation of neglecting the thickness of the layer is consistent with the experimental systems (e.g., modulated inversion layer), where the average thickness of the confining wave function in the z direction is much smaller than the typical length scales (e.g., d,a,b) in the x-y plane. We treat the system as a strict two-dimensional system in the x-y plane.

Before doing any calculation, one can guess that there will be multiple branches of plasmon modes due to the periodic density modulation in the y direction. In addition, the bulk (two-dimensional) plasmons and the edge plasmons of various strips will interact to form plasmon bands (with the bandwidths determined by the Coulomb interaction between different strips). In principle, there will be infinitely many branches of plasmon bands, even though the spectral weight of each branch will diminish as one goes up in energy. In this paper we will consider the low-lying plasmon bands using a semiclassical hydrodynamical approach.[19] In an earlier paper[10] we had considered the extreme quantum case where only the lowest plasmon band (which has quasi-one-dimensional plasmon character) was investigated. Currently fabricated experimental systems[15,18] cannot yet reach the extreme one-dimensional quantum limit and results reported in this paper are more appropriate for the currently existing experimental systems.

Hydrodynamic theory for calculating plasmon dispersion in an electron gas is fairly standard[19] and we omit the formal mathematical details. After linearizing the equation of continuity, the Euler's equation and the Poisson's equation, we get the following for the self-consistent density fluctuation $n_1(y)$ of the system

$$\left[\omega^2-\omega_p^2(y)-\beta^2(y)\left[q^2-\frac{\partial^2}{\partial y^2}\right]\right]n_1(y) + \left[\frac{\partial^2}{\partial y^2}\beta^2(y)\right]\frac{\partial n_1}{\partial y}(y)+E_y(y)\frac{\partial}{\partial y}n_1(y)=0\ , \tag{3}$$

where β is the electronic compressibility (which is also a measure of nonlocal effects in the plasmon dispersion of the electron gas). In Eq. (3) the density fluctuation is a function only of y since we assume translational invariance in the x direction and take

$$n_1(\mathbf{r},t)=[n_1(y)e^{iqx-i\omega t}]\delta(z)\ ,$$

with q as the conserved wave vector in the x direction. The two-dimensional frequency ω_p^2 in Eq. (3) depends on whether one is located in strip A or B and is given approximately by

$$\omega_p^2=\begin{cases}\omega_A^2 & \text{with } y\in(ld,a+ld)\ ,\\ \omega_B^2 & \text{with } y\in(a+ld,ld+d)\ ,\end{cases} \tag{4}$$

and

$$\omega_{A,B}^2=\left[\frac{2\pi n_{A,B}e^2}{\kappa m^*}\right](q^2+k_y^2)^{1/2}\ , \tag{5}$$

where $\kappa_y \approx m_A\pi/a$ or $m_B\pi/b$ are the allowed wave vectors in the y direction (with m_A,m_B integers) and m^*, κ are the effective mass and the static dielectric constant, respectively. The self-consistent electric field $E_y(y)$ entering Eq. (3) is obtained by solving Poisson's equation:

$$E_y(y)=-2eq\int dy'\mathrm{sgn}(y-y')K_1(q\,|y-y'|\,)n_1(y')\ , \tag{6}$$

where $K_1(x)$ is the modified Bessel function. When Eqs. (4)—(6) are combined with Eq. (3) one gets a nontrivial integro-differential equation as the condition for the self-consistent plasma oscillation of the system. Instead of working with the modified Bessel-function kernel of Eq. (6), we use the relationship $(\partial/\partial x)K_0(x)=-K_1(x)$ and the approximate kernel suggested[5] by Fetter by replacing $K_0(x)$ with $(2\pi)2^{-3/2}\exp(-2^{1/2}x)$.

The standard technique[8,19] for obtaining the plasma dispersion relation is to apply boundary conditions involving the continuity of charge density and current density at the boundary between alternating wires. We use the following ansatz[8] for the density fluctuations with frequency ω in the range $\omega_B>\omega>\omega_A$:

$$n_1(y)=\begin{cases}Ae^{ip_Ay}+Be^{-ip_Ay} & \text{for } y\in(0,a)\ ,\\ Ce^{-p_By}+De^{p_B(y-d)} & \text{for } y\in(a,b)\ ,\end{cases} \tag{7}$$

with

$$p_A^2=\beta_A^{-2}(\omega^2-\omega_A^2)-q^2$$

and (8)

$$p_B^{-2}=\beta_B^{-2}(\omega_B^2-\omega^2)+q^2\ ,$$

and the density fluctuation at arbitrary y is obtained by using the Bloch theorem associated with the periodicity in the y direction:

$$n_1(y)=n(y+ld)e^{ikld}\ . \tag{9}$$

Concentrating on the plasma oscillations in the range $\omega_B>\omega>\omega_A$ we can now get a set of four equations for the unknown amplitudes A, B, C, and D by applying Bloch's condition and the boundary conditions (continuity of charge and current densities). This leads to the secular equation

$$\begin{vmatrix} e^{ip_Aa} & e^{-ip_Aa} & -e^{-p_Ba} & -e^{p_B(a-d)} \\ e^{ikd} & e^{ikd} & -e^{-p_Bd} & -1 \\ (n_B-n_A)E_A(a) & (n_B-n_A)E_B(a) & (n_B-n_A)E_C(a) & (n_B-n_A)E_D(a)e^{-p_Bd} \\ -i\beta_A^2p_Ae^{ip_Aa} & +i\beta_A^2p_Ae^{ip_Aa} & -\beta_B^2p_Be^{-p_Ba} & +\beta_B^2p_Be^{p_B(a-d)} \\ (n_B-n_A)E_A(d) & (n_B-n_A)E_B(d) & (n_B-n_A)E_C(d) & (n_B-n_A)E_D(d)e^{-p_Bd} \\ -i\beta_A^2p_Ae^{ikd} & +i\beta_A^2p_Ae^{ikd} & -\beta_B^2p_Be^{-p_Bd} & +\beta_B^2p_B \end{vmatrix}=0\ . \tag{10}$$

The coefficients E_A, E_B, E_C, and E_D depend on q, p_A, p_B, ω_A, ω_B, k, and, of course, a and d. We do not show them here for the sake of brevity. The wave vector k is the plasmon-band wave vector arising from the y periodicity of the system as defined in Eq. (9).

We have solved the above secular equation [Eq. (10)] numerically to obtain the low-lying plasma bands of the system. The results are shown in Fig. 1. The lowest three bands ($m=0,1,2$) are shown as a function of qa, the wave vector along the x direction. The band edges are at $kd=0$ or π, where k is the wave vector along the superlattice direction. Parameters chosen for our numerical calculation are $n_A/n_B=0.3$, $b/a=2$, $\beta_A^2=\beta_B^2=0.1\omega_0^2a^2$, where $\omega_0=(2\pi e^2n_A/\kappa m^*a)^{1/2}$ is taken as the natural unit of plasma energy in the problem. The bands are classified by the integer m $(=m_A=m_B)$ which defines the "local" plasmon modes (the higher-multipole[19] branches) in the individual wires. Thus our plasmon band calculation is a linear-combination-of-atomic-orbitals-(LCAO-) type band calculation, in contrast to the work[9] of Krasheninnikov and Chaplik who used a nearly-free-electron-type perturbative method to obtain the plasmon band gaps.

We would also like to point out that our approximation of replacing the exact kernel $K_1(x)$ in Eq. (6) with the approximate exponential kernel is a nonessential quantitative approximation made only to facilitate our numerical work. This does not in any way affect our qualitative results. We should emphasize that, unlike in Ref. 5, we do not solve the problem of a *single* finite strip, but using translational invariance, directly solve the problem of an *infinite* superlattice so that the difficulties associated with fringing fields (in the nonlocal integral equation), as discussed in Ref. 5, do not arise in our method. One additional approximation we make is the neglect of interband mixing terms ("band hybridization") so that our plasmon bands are allowed to cross. If interband interaction terms are retained in the calculation, small gaps will open up in Fig. 1 at wave numbers where two bands cross. Our results will remain unaffected for other wave numbers.

Our results for the calculated long-wavelength optical-absorption spectrum for the system are shown in Fig. 2. For optical absorption we use the following formula:[20]

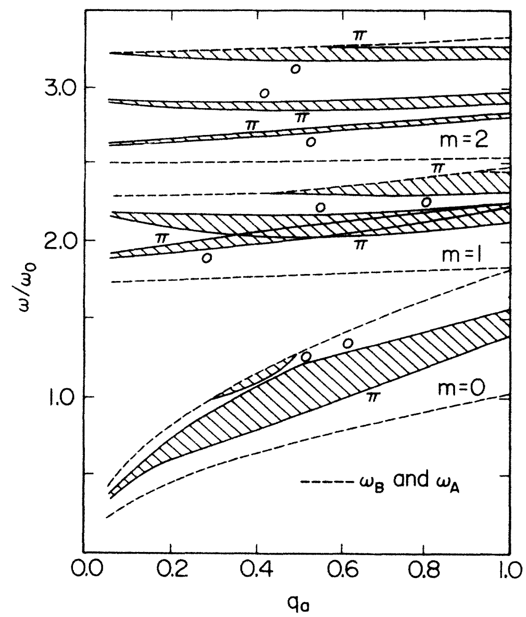

FIG. 1. Shows the calculated plasmon dispersion in a lateral multiwire superlattice as a function of the wave vector q along the strip. Three sets of plasmon bands ($m=0,1,2$) are shown with the band edges being at $k_y d=0$ and π where $d=a+b$ is the superlattice period and a,b are the widths of the alternating strips ($\omega_0, \omega_A, \omega_B$ are defined in the text).

$$P=\frac{1}{2}\,\mathrm{Re}\sigma_{\mathrm{eff}}(\omega)\,|\,E_x(q\approx 0, z\approx 0)\,|^2 ,$$

where σ_{eff} is the effective high-frequency conductivity of the system and E_x is the electric field. We choose $qa=0.1$ for the actual numerical work shown in Fig. 1. The optical-absorption spectrum clearly shows structure due to higher plasma modes, with the structure being very weak for $m>2$.

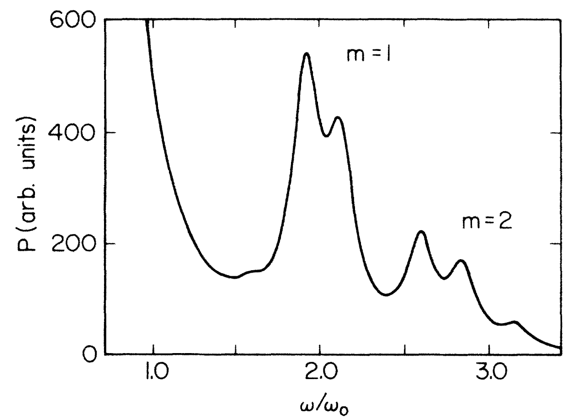

FIG. 2. Shows the long wavelength optical absorption by the lateral superlattice as a function of the frequency. Various peaks due to plasmon effects are identified.

In conclusion, we have calculated the plasmon band structure and the far-infrared optical-absorption spectrum of a lateral two-dimensional multiwire superlattice of alternating electron density n_A and n_B (per unit area) confined in alternating strips of widths a and b, respectively. We use a semiclassical hydrodynamic approach assuming that quantum confinement effects are not important (which is a correct assumption for the currently existing experimental systems). Our work is relevant to the plasmon dispersion in a two-dimensional electron gas with spatially modulated charge density.

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