Thickness variations and the Corbino effect

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We examined the effect of thickness variations in Corbino samples of very pure aluminum at 4.2 K in magnetic fields up to 7 T. Only thickness variations that affect the axial symmetry of the sample were found to perturb the Corbino current pattern and to lower the magnetoresistance. A simple model accounts for our data and also shows that the linear magnetoresistance, observed for simple metals in earlier experiments in the Corbino geometry, may have been caused by unintentional variations in thickness.

I. INTRODUCTION

A Corbino disk[1] is a circular conductor with one current contact in the center of the disk and the other current contact around the outer perimeter. Both current contacts are equipotential surfaces, also when a magnetic field is applied. The voltage between two points $\mathbf{r}_1$ and $\mathbf{r}_2$ is measured for static magnetic field strengths $\mathbf{B}=B\hat{\mathbf{z}}$ perpendicular to the disk. Because of the axial symmetry all equipotential surfaces are circular. When, with increasing magnetic field, the radial component of the current density J_r is kept constant, the tangential component J_ϕ will grow linearly with the applied magnetic field. The voltage V measured between $\mathbf{r}_1$ and $\mathbf{r}_2$ consists of a field-independent part, due to J_r, and a part increasing quadratically with the magnetic field, which can be considered as the Hall voltage due to J_ϕ,

$$\frac{V(\mathbf{r}_1,\mathbf{r}_2)}{I}=\frac{\ln(r_1/r_2)}{2\pi d}\rho(1+\beta^2)\ . \qquad (1)$$

Here d is the thickness of the conductor, $\beta=\omega_c\tau=R_H B/\rho$ the so-called Hall angle, R_H the Hall constant, and ρ the resistivity and generally, both R_H and ρ are magnetic field dependent.

The Corbino effect has been exploited to measure the lattice thermal conductivity in pure In,[2,3] Al,[4] and K.[5] In these experiments the large electronic component of the thermal conductivity is suppressed by the magnetic field, and the lattice component is consequently found by extrapolating the high-field values.

For a correct description of the electronic (thermal) conductivity one modifies (the thermal analogue of) Eq. (1) in two ways.

(i) The Corbino method is most useful for high magnetic fields, $\beta \gg 1$. At these fields the simple metals show a saturation of R_H and ρ, and one can substitute the saturation values for R_H and ρ in Eq. (1).

(ii) For a correct interpretation of the results one had to take the linear magnetoresistance (LMR) into account.[2] The LMR is well known from experiments on simple metals in the Hall bar geometry, but the origin of this effect is not fully understood. It is incorporated phenomenologically by multiplying the xx and yy components of the resistivity tensor by $1+\alpha\beta$, where α is a dimensionless measure of the LMR, the so-called Kohler slope. In the Corbino geometry one can account for the effect of the LMR by dividing the right-hand side of Eq. (1) by $1+\alpha\beta$, provided that $\beta \gg 1,\alpha$.

The LMR has been incorporated in all cited experiments, yielding otherwise confirmed results for the lattice thermal conductivity.[2–5] It has also been found in the results on our unperturbed Corbino sample. For LMR caused by intrinsic sources, incorporation in the resistivity tensor has some physical relevance. In this paper we will show, however, that noncircular thickness variations in a Corbino sample modify the current pattern and thus cause LMR. A description with a Corbino current pattern and a resistivity tensor with a Kohler slope might be phenomenologically satisfactory for such samples, but it clearly does not treat the underlying physics correctly.

Recently a theory for the LMR of samples in the Hall bar geometry has been proposed by Bruls *et al.*[6,7] They attribute the LMR to variations in the sample thickness along the magnetic field direction, extending perpendicularly to the direction of the current. Such thickness variations cause a magnetic-field-dependent current distribution, and yield an additional magnetic-field-dependent dissipation. The resulting magnetoresistance increases linearly with the applied field for high fields. The magnetic-field-dependent current distribution results from variations of the Hall voltage along the current direction. These variations may be caused by varying sample thickness along the current direction, but a varying Hall constant or a varying magnetic field will give rise to the same effects.

In semiconductors, inhomogeneities cause variations of the Hall constant and additional magnetoresistance. Because of the difficulty of making samples with well defined variations in the concentration of charge carriers, experimental verification of theories on this effect only yields qualitative agreement.[8] In view of the equivalence (with respect to the transport problem) of variations in the Hall constant and thickness variations, these theories are better tested in metals (e.g., aluminum) with well-defined thickness variations, than in semiconductors with poorly defined variations of the Hall constant.

The effect of inhomogeneities in a Corbino sample was discussed earlier by Beer.[9] He suggested that the absence of a Hall field in a Corbino disk will diminish the effect of inhomogeneities on the magnetoresistance. He calculated the effect of axial-symmetric nonuniformities, and concluded that they do not effect the quadratic magnetoresistance in a Corbino disk.[9] This conclusion is, however, a consequence of the symmetry of the inhomogeneity considered. In practice, inhomogeneities do not possess this symmetry, and we will show that noncircular thickness variations will cause severe distortions of the Corbino effect.

II. EXPERIMENTAL DETAILS

The samples were cut, using spark erosion, from high purity polycrystalline Al plates (residual resistivity ratio 26 000 after annealing) obtained from the Vereinigte Aluminium Werke.[10] We made circular disks of 55-mm diameter. Spark erosion was also used for forming grooves and projections. Holes of 0.8-mm diameter were drilled through the sample to attach the voltage leads and the central current contact. Then, the samples were annealed for one hour at 500 °C in air, and the contact leads were soldered with special aluminum solder. For the outer current contact an indium ring was pressed tightly to the aluminum disk. On the ring eight equally spaced current leads were soldered, and these were attached, each in series with a 10-Ω load resistor, to one terminal of the current source, while the other terminal was used for the inner contact.

Six voltage contacts were made, three on 6 mm and three on 18 mm distance from the central contact. The inner and outer contacts were positioned in pairs on lines running through the center of the disk. Measurements were done on the following four samples.

(i) Sample *A*, a flat disk of 1-mm thickness.

(ii) Sample *B*, a 1 mm thick disk with a circular groove of 0.15-mm depth, 8-mm inner radius, and 10-mm outer radius.

(iii) Sample *C* is made from sample *A*, but a straight groove was machined through the center of the disk extending over the whole sample and then the sample was annealed again. The depth of the groove is 0.30 mm, its width 2 mm.

(iv) Sample *D*, a 1.5-mm disk with a straight projection through the center of the disk over the whole sample. The height of the projection is 0.5 mm, its width 2 mm.

In samples *C* and *D* the axial symmetry is broken, and the angle of the lines of the voltage probes with the center line of the groove or projection is important. The first pair of contacts was placed on the center line of the groove or projection, the second pair at an angle of 23°, and the third pair at an angle of −138° with respect to the first pair of contacts. (The sample is sketched in the inset of Fig. 3.) The pairs of contacts will be referred to as pair 1, 2, and 3, respectively.

All measurements were performed in a 7-T superconducting magnet at 4.2 K.

III. DATA AND ANALYSIS

We first report the data for our samples *A* and *B*. In both samples the magnetoresistance is quadratic in the applied field with an additional linear magnetoresistance. This is shown in Fig. 1, where we plotted GB^2 versus the magnetic field B, with the conductance $G = I/V(\mathbf{r}_1,\mathbf{r}_2)$. For fields above 1 T, we obtain straight lines with slope $G\alpha/\mu$ and offset G/μ^2, where $\mu = R_H/\rho$. For fields below 1 T the magnetoresistance is not quadratic.

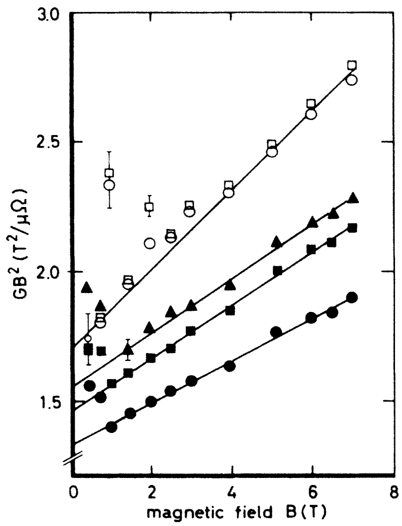

FIG. 1. GB^2 plotted against the magnetic field B, where G is the conductance for the axial-symmetric samples. Sample *A* (open symbols) is flat; sample *B* (solid symbols) has a circular groove between the voltage contacts. The different symbols refer to measurements across different pairs of voltage contacts. Error bars are drawn for fields below 2 T, for higher fields they become negligible on the scale of this graph.

In both samples the different pairs of voltage contacts on each sample showed nearly the same resistance. The small differences can be explained by unintentional differences in the geometric factor of Eq. (1). All measurements were performed for both field directions normal to the disk, and the differences in the results for the two different directions were negligible.

From the slope and the offset in Fig. 1 we find a Kohler slope for sample *A*, $\alpha = (2.7 \pm 0.3) \times 10^{-3}$ and a saturation resistivity $\rho = (3.2 \pm 0.3) \times 10^{-12}$ Ω m. For sample *B*, $\alpha = (1.8 \pm 0.3) \times 10^{-3}$ and $\rho = (2.8 \pm 0.3) \times 10^{-12}$ Ω m.

Let us next consider the samples *C* and *D*. Figure 2 shows the effect of a straight groove in a Corbino disk: The upper curve is measured on sample *A*, the lower curve on sample *C*, across the third pair of voltage contacts and it is obvious that the Corbino effect is frustrated by the presence of the groove. Figure 3 shows in more detail the voltage, divided by the measuring current (usually 1 A), measured across the different pairs of voltage contacts on sample *C* for the two directions of the magnetic field normal to the disk. These data have been collected without altering the sample's conditions.

For sample *D* the order of magnitude of the effect, the dependence on contact position and on the direction of the magnetic field agreed with the measurements on sample *C*. On sample *D*, we also did some measurements with a displaced central current contact, using the inner voltage

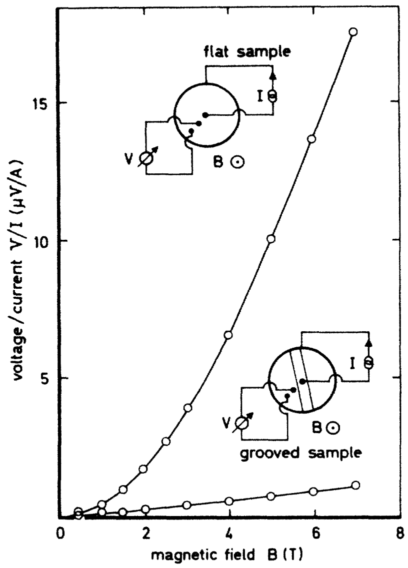

FIG. 2. Voltage divided by the current for samples *A* and *C*. The inset shows the geometry of current and voltage contacts used. The upper curve gives the results for a flat disk (sample *A*), the lower curve for this same disk after a groove had been machined (sample *C*).

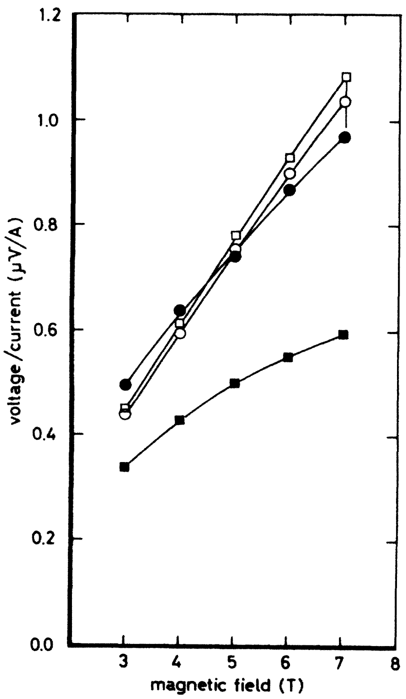

FIG. 4. Voltage divided by the current across the third pair of contacts on sample *D*, a disk with a projection, measured with the normal central (circles) and with a displaced central current contact (squares). The solid and open symbols refer to the two directions of the magnetic field as in Fig. 3.

probe of the second pair of contacts as current contact. The potential differences across the first pair of contacts were reduced for both field directions. Displacing the central current contact reduced the voltages across the third pair of contacts for only one direction of the magnetic field, as is shown in Fig. 4.

The results for samples *C* and *D* depended on the force that pressed the indium ring against the aluminum disk. Pressing the ring more tightly to the disk raised the voltage across the third pair of contacts, with reproducible maximum values. These maximum values have been used in this paper. The minimum registered voltage was, irrespective of the applied field, about 70% of this value. The voltage across the other pairs of contacts varied less systematically, but they were all of the same order of magnitude as the data in Fig. 3. For samples *A* and *B* such effects were not observed.

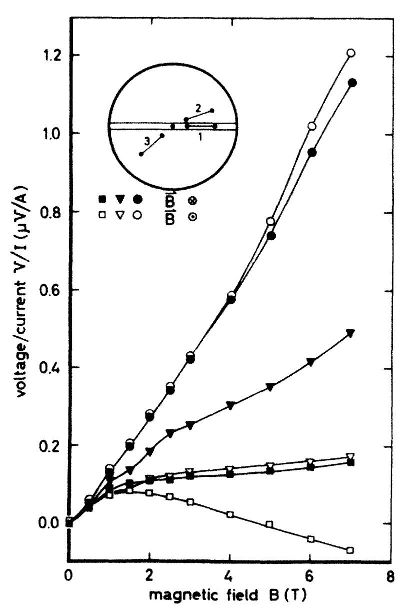

FIG. 3. Voltage divided by the current for sample *C*, a grooved disk, measured across the pairs of contacts 1 (squares), 2 (triangles), and 3 (circles) for the two different directions of the magnetic field (see inset).

The dependence of the results on the magnetic field direction and the contact position is proof that the samples *C* and *D* did not exhibit a Corbino-like current pattern. In practical situations often one single pair of contacts is available for voltage measurements and one can only check the dependence on the magnetic field direction.[2–5] If both directions yield nearly equal results, one generally assumes that the Corbino formulas are valid and the deviations from the ordinary quadratic magnetoresistance are interpreted as an intrinsic (linear) magnetoresistance. In samples *C* and *D* the voltage measured across the third pair of contacts could be erroneously interpreted as a proper Corbino result. It is interesting to analyze these data, the way we analyzed the results for samples *A* and *B* in Fig. 1. This is done in Fig. 5. For both samples we observe an LMR deviation in the Corbino magnetoresistance, similar to Fig. 1. Ignoring our knowledge of the severe deviations from the proper Corbino current pattern, we can determine values for Kohler slope α and

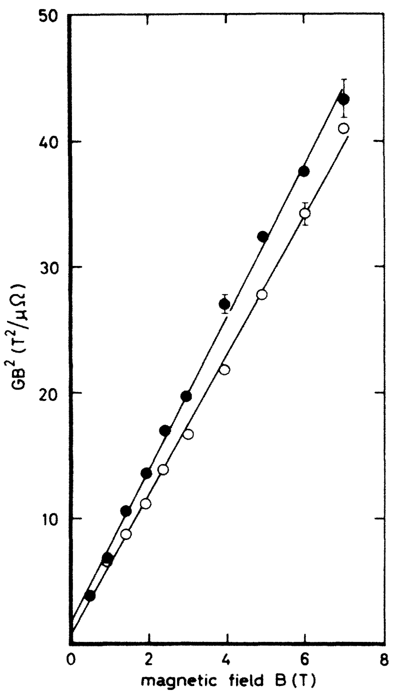

FIG. 5. GB^2 plotted against magnetic field B, where G is the current divided by the voltage, measured across the third pair of contacts on sample C (solid circles) and sample D (open circles).

saturation resistivity ρ. We find, for sample C, $\alpha=(0.11\pm0.02)$ and $\rho=(3.4\pm0.7)\times10^{-12}$ Ω m; and, for sample D, $\alpha=(0.08\pm0.02)$ and $\rho=(1.3\pm0.5)\times10^{-12}$ Ω m.

IV. DISCUSSION

This section is divided into four parts. In part A we discuss the data on the samples with axial symmetry. In part B we present a model for grooves in a Corbino disk and we show that it describes our data on the samples with noncircular thickness variations reasonably well. In part C we discuss the data on these samples in more detail. In part D we answer the question, why symmetry is important in the Corbino geometry.

A. Samples with axial symmetry

The results on sample A are representative Corbino data: We find both the strong quadratic component as predicted by Eq. (1) and an LMR deviation of the same order of magnitude as reported in Refs. 2–5. The circular groove in sample B does not damage the Corbino effect, and it has no significant effect on the Kohler slope. In both samples the axial symmetry of the current pattern is reflected in the small difference between the voltages registered across the different pairs of contacts.

B. Model for grooved samples

We performed measurements on samples with thickness variations of 30%. An exact solution of the transport problem in this case is too difficult. An interesting limiting case that can be treated mathematically, is the Corbino disk with a groove totally cut through, i.e., a segment of a circle with current contacts on the inner and outer arcs.

The current distribution in the segment is then found by a conformal mapping of the current lines in a rectangular plate with current contacts covering two opposite sides, onto this segment. The current pattern in the rectangular plate shows a magnetic-field-dependent compression in the vicinity of the current contacts.[11] In most customary four-probe configurations this effect is unobservable; but in the Corbino segment the current lines are distorted over the whole sample, as is shown in Fig. 6. It is easily shown that the voltage V_s between inner and outer current contact is given by

$$V_s=I_s\rho\frac{\ln(r_o/r_i)}{\pi d_s}\left[1+\frac{\pi}{\ln(r_o/r_i)}\beta\right], \tag{2}$$

where d_s is the thickness of the semicircular disk, I_s the current in the semicircle, and r_i and r_o the radii of the inner and outer current contacts.

A grooved sample can now be considered as a parallel circuit of a Corbino disk and circle segments. At the lowest fields the magnetoresistance shows a Corbino-like behavior; with increasing field the segments gain importance, and the linear term of Eq. (2) will result in an additional, eventually dominant LMR. Let us consider a surface of a Corbino disk, divided into n equal circle segments by narrow grooves with relative depth f. Magnetoresistance measurements with the potential leads on the current contacts, yield an LMR with Kohler slope,

$$\alpha=f\frac{n}{2\pi}\ln(r_0/r_i)\,. \tag{3}$$

In most experiments $f \ll 1$ and the factor following f is typically of order unity. This means that the grooved disk shows a Corbino-like behavior for low fields, but with increasing field the segments become more important and the LMR is dominant for $\beta\sqrt{f}>1$. The reported Kohler slopes in Refs. 2–5 are of order 10^{-2}, and the measurements were performed in the high-field limit $\beta\gg1$, so unintentional grooves in the surfaces may have been the cause of the LMR in these experiments.

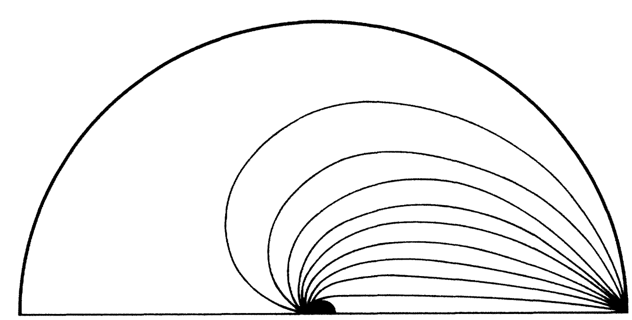

FIG. 6. Current pattern for a semicircular conductor with short-circuiting current contacts on inner and outer arc in a high magnetic field.

The potential leads were not positioned on the current contacts in our experiment. Taking this into account, our model yields for the third pair of contacts in sample *C*, $\alpha=0.12\pm0.02$, in agreement with the Kohler slope found in Fig. 5.

The deviations in the Corbino effect for sample *D*, the Corbino disk with the straight projection, resemble those found in the grooved sample. Sample *D* can therefore be considered as a Corbino disk with a parallel circuit of circle segments as well. The absolute value of the effect indicates that the circle segments in sample *D* are not the projections themselves, but rather the semicircles in the surface that are separated by the projection. We believe that the above given model for grooves is relevant to other perturbations in the axial symmetry as well. Our measurements indicate that these perturbations in samples *C* and *D* are approximately of the same magnitude, in accordance with the equivalence in relative thickness variations in both samples.

C. Samples with noncircular thickness variations

For our very simple model, only partial agreement with the data can be expected. But it will be shown next, that part of the observed dependence of the voltages on the direction of the magnetic field, on the position of the central current contact, and on the pressure of the contact ring to the disk, can be explained by this model.

Reversing the magnetic field moves the constrictions in the current pattern in Fig. 6 to the opposite corners of the circle segments. So the voltages across pairs of contacts that are not on the bisector of the circle segment, are not symmetric under reversal of the magnetic field. The third pair of potential leads in Fig. 3 is closest to this bisector, and the observed voltages are described well by our model; for the voltages across the two pairs of leads in the vicinity of the thickness variations the model is too simple.

Displacing the current contact will disturb the equivalence of the current patterns in the two semicircles and this asymmetry depends on the magnetic field direction. In the semicircle containing the displaced current contact, the path from the constriction on the inner contact to the constriction on the outer contact is shorter than the path for the central current contact. This reduction is appreciable for only one direction of the magnetic field. In that case, the voltage across the third pair of contacts (in the other half of the sample) will be appreciably reduced, in accordance with the observations in Fig. 4. The reduction of the voltages across the first pair of contacts located on the thickness variation is understood too: The (Hall) voltage registered will be smaller because, for either field direction, the current traversing the projection between the potential leads will be reduced for a displaced inner current contact.

In the experiment, the resistivities in the indium contact ring and along the periphery of the aluminum disk are small compared to the interface resistance between ring and disk. So our model, in which the outer current contact is an equipotential surface, is approximated best if the interface resistance between the ring and the disk is the same anywhere along the perimeter. Local fluctuations in the interface resistance are inevitable, but they are unimportant if their extent is small compared to the perimeter of the disk. This range is smallest for uniform pressure of the ring to the disk. Indium is easily deformed, so the most uniform contact is obtained when the ring is tightly pressed to the disk.

The current pattern in the samples *C* and *D* is rather sensitive to the uniformity of the outer current contact, because of the current constrictions discussed in Fig. 6. Especially the voltages measured across the potential leads near the thickness variations will be sensitive to changes in the uniformity of the outer current contact, i.e., on the pressure of the ring to the disk. In the axial-symmetric samples *A* and *B* fluctuations in the interface resistance are less prominent, because they perturb the current pattern over only small radial distances, due to the Corbino effect. The fact that the positions of the potential leads do not coincide with the current contacts in our experiment, suppresses these contact effects in samples *A* and *B* more than in samples *C* and *D*.

But in principle, nonuniformities in the outer contact will disturb the axial symmetry of the current pattern too, and will cause similar effects as the deliberate symmetry breaking by the thickness variations.

D. Importance of symmetry in the Corbino geometry

This experiment has taught us the following on the equipotential surfaces and the axial symmetry. It is essential for the Corbino effect that all zero-field equipotential surfaces remain equipotential surfaces when a magnetic field is applied. In a uniform conducting slab (i.e., homogeneous and of constant thickness) this condition is fulfilled, if the edges are equipotential surfaces with and without an applied transverse magnetic field. It is easily shown,[12] that for such a conductor the resistance associated with the heat dissipated is proportional to $1+\beta^2$.

There are two ways of making the edges to equipotential surfaces. First they might be short circuited by pressing a material onto it with better conductivity. In this method, the shape of the conducting slab is immaterial, as long as it is a uniform slab. In metal physics this method is not reliable, because the interface resistances between the short-circuiting medium and the slab are often larger than the resistances in the slab itself. Secondly the shape of the sample may be chosen so, that the edges are equipotential surfaces because of the symmetry, as in the Corbino disk. In metal physics this strategy is to be preferred. If the interface resistance between the current contacts and the slab is large compared to the resistance in the slab, the inner and outer perimeters are equipotential surfaces even if the interface resistance fluctuates along the perimeter. Then the magnetoresistance will obey Eq. (1), as in samples *A* and *B*. But if the axial symmetry is broken by nonuniformities, the Corbino effect breaks down, even if the inner and outer perimeters are equipotential surfaces as discussed in part B of this section. Then, the actual current pattern is also more sensitive to local differences of the interface resistance between the current contacts and the edge of the sample.

The above analysis shows that one should be cautious when using an imperfect Corbino geometry. For example, in the Corbino-like geometry that was first exploited by Störmer *et al.*[13] for a direct measurement of the electrical conductivity σ_{xx} of the two-dimensional electron gas, the angular symmetry of the sample is destroyed by its contacts. In the quantized regime, where the edges are equipotential lines due to the zero-resistance state of the electron system, the conditions for the Corbino effect are fulfilled. But out of the quantized regime the equipotential lines will be affected by an applied magnetic field, because of the perturbed angular symmetry. Therefore the proper Corbino behavior will not appear and the results cannot be interpreted as the true σ_{xx} of the electron system. An anomalous magnetic field dependence of the results that was observed near the quantized regime[13] may be explained partly by the imperfection of the Corbino effect.

V. CONCLUSION

The experiments on thickness variations in Corbino samples reported here lead to the following conclusions. A circular groove does not affect the Corbino current pattern. Both a straight groove and a straight projection will disturb the Corbino current pattern. The voltage measured across contacts some distance away from a straight thickness variation will then increase linearly with the applied magnetic field.

We have developed a model that considers the sample with the straight groove as a parallel circuit of a flat Corbino sample with a quadratic magnetoresistance, and of two semicircular segments with a linear magnetoresistance that becomes dominant for high fields. This model gives a quantitatively correct prediction of the voltage measured some distance from the groove. It gives a qualitatively correct description of the dependence of the effect on the position of the central current contact, on the magnetic field direction, and on the uniformity of the outer current contact.

The similarity of the results for the groove and the projection indicates that the loss of the axial symmetry in the sample is essential for the observed effects.

The quantitative agreement between the model presented and the measured data leads to the conclusion that unintentional thickness variations can account for the LMR reported in the literature on Corbino experiments.

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