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Elastic and plastic properties of the flux-line lattice in type-II superconductors

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The present status is reviewed for the elastic and plastic properties of the flux-line lattice in type-II superconductors, which crucially enter the modern theory of collective pinning. In contrast to atomic lattices the flux-line lattice exhibits pronounced elastic nonlocality and the screw dislocation (oriented perpendicular to the flux lines) is strongly anisotropic and can move freely since there is no Peierls potential along the flux lines. General expressions for the interaction between straight or arbitrarily curved flux lines are presented which may be used to compute equilibrium arrangements of flux lines.

I. INTRODUCTION

The theory of weak collective pinning[1] of the flux-line lattice (FLL) in type-II superconductors has been confirmed recently by experiments on thin amorphous films in a perpendicular magnetic field B_a. Both two-dimensional pinning of straight parallel FL's[2] and an abrupt transition to the three-dimensional pinning of a plastically deformed FLL[3] were observed. This transition reflects itself in a sharp jump of the critical current j_c by a factor of ≈ 10, followed by a steep increase of j_c by a further factor of ≈ 10 immediately above the corresponding transition field B_{c0}. The jump is caused by a sudden proliferation of FLL defects (probably screw dislocations nucleating at the film surface) which drastically reduces the longitudinal correlation length of the FLL and the correlated volume $V_c \sim j_c^{-1/2}$.

This jump proves, I believe clearly and for the first time, that for three-dimensional pinning plastic deformation of the FLL is essential even in the weak-pinning limit (amorphous materials have a very small j_c). This statement applies at least to materials with large Ginzburg-Landau (GL) parameter κ ($\kappa \approx 60$ in amorphous superconductors) and possibly also for clean Nb which has $\kappa \approx 1/\sqrt{2}$ and thus a very small shear stiffness of the FLL (see below). Evidence for the nonapplicability of the original collective-pinning theory to three-dimensional pinning was provided by experiments which yielded much larger values for j_c than could be explained by a mere elastic deformation of the FLL as assumed in Ref. 1.

A quantitative explanation of this jump and of three-dimensional pinning, therefore, requires detailed knowledge of the elastic and plastic properties of the FLL. We give here a short review of the present status of the theory and compile useful formulas (partly unpublished) from which further studies of linear and nonlinear elastic and plastic behavior of the FLL should start.

II. ELASTIC MODULI OF THE FLUX-LINE LATTICE

The flux lines in a type-II superconductor may be displaced from their ideal positions (a hexagonal lattice of parallel FL's) by various forces:[4] by pinning forces exerted by inhomogeneities or structural defects of the material; by surface currents caused (a) by the boundary conditions imposed on the magnetic field (by image FL's), (b) by an applied field B_a (Meissner screening currents), or (c) by an applied current (which, before the FL's are allowed to shift flows only in a surface layer); by temperature gradients; and by gradients in material parameters. For small strains the change in the energy is proportional to the square of the strains or displacements. This "linear elastic energy" is completely determined by three elastic moduli: The tilt modulus $c_{44}(k)$, the modulus for uniaxial compression $c_{11}(k)$, and the shear modulus c_{66}. The latter is approximately independent of k, the wave vector of the (periodic) displacement field. Within the same "isotropic" approximation in which c_{66} becomes independent of k, c_{44} and c_{11} depend only on the modulus $k = |\mathbf{k}|$.

The dependence of the tilt and compressional energy on k reflects the "elastic nonlocality" of the FLL:[5] For deformations with $k > 1/\lambda'$ the FLL is softer than for homogeneous strain. The characteristic length of this nonlocality is the range of the FL-FL interaction, equal to the effective magnetic penetration depth $\lambda' = \lambda/(1-b)^{1/2}$, where $b = B/B_{c2}$ is the reduced field, B (the FL density times the quantum of flux ϕ_0) is the internal field, B_{c2} the upper critical field of the superconductor, $\lambda = \kappa\xi$, and $\xi = (\phi_0/2\pi B_{c2})^{1/2}$ is the GL coherence length.

The moduli for homogeneous tilt and for homogeneous compression of the FLL follow from the magnetization curve $B(B_a)$ by thermodynamic arguments:[4]

$$c_{44}(0) = BB_a/\mu_0 \ ,$$

$$c_{11}(0) - c_{66} = (B^2/\mu_0)dB_a/dB \ .$$

The shear modulus, and also $c_{11}(k)$ and $c_{44}(k)$ for $k > 0$, cannot be obtained in this way.[6] However, a simple geometric argument shows that the compressional strain with shortest possible wavelength (the lattice spacing a times 2 or times $\sqrt{3}$ along the two principal orientations of the triangular FLL) is identical with a (standing) shear wave with $\mathbf{k}$ turned by 90°. One thus has $c_{11}(k_B) \approx c_{66}$, where $k_B = (2b)^{1/2}/\xi$ is the radius of the circularized Brillouin zone. The relation $c_{11}(0) \gg c_{66}$, which allows us to

TABLE I. The linear elastic displacements of the flux-line lattice at the point of application of four model forces. S_L is the result of local elasticity theory, S_{NL} is the correct, nonlocal response. The flux lines are along z and the forces act along x. For the planar-force example I chose the extension of the specimen, or the distance of two opposing forces, $D \approx 5a$.

Model force on the FLL	S_L local elast. theory	S_{NL} nonlocal elast. theory	Correction factor S_{NL}/S_L general expression	$b=0.8$ $\kappa=3$	$b=0.8$ $\kappa=60$
Planar force $f=\delta(x)$	$\sim\frac{1}{c_{11}}\sim\frac{1}{b^2}$	$\frac{\kappa^2}{(1-b)^2}$	$1+\frac{d}{D}\kappa^2\left[\frac{b}{1-b}\right]^2$	30	12000
Line force $\parallel$ FL's $f=\delta(x)\delta(y)$	$\sim\frac{1}{c_{66}}\sim\frac{2}{b(1-b)^2}$	$\frac{\kappa^2}{b(1-b)^2}$	1	1	1
Line force $\perp$ FL's $f=\delta(x)\delta(z)$	$\sim\frac{1}{\sqrt{c_{11}c_{44}}}\sim\frac{1}{b^2}$	$\frac{\kappa^2}{\sqrt{b}(1-b)^3}$	$1+\frac{2\kappa^2}{3\ln(D/d)}\left[\frac{b}{1-b}\right]^{3/2}$	30	12000
Point force $f=\delta(x)\delta(y)\delta(z)$	$\sim\frac{1}{\sqrt{c_{66}c_{44}}}\sim\frac{\kappa}{b^{3/2}(1-b)}$	$\frac{\kappa^2}{b(1-b)^{3/2}}$	$1+\frac{\kappa}{\sqrt{2}}\left[\frac{b}{1-b}\right]^{1/2}$	5	180

treat the FLL as an "incompressible solid" for many purposes, is thus closely connected with the pronounced nonlocality of its elasticity.

The moduli derived in Refs. 5, 7, and 8 from the GL theory (valid near the critical temperature T_c) have been rederived from the microscopic theory of Gorkov (valid at arbitrary temperature).[1] The modifications of the GL result are negligibly small even at low temperatures in most cases. We summarize the (slightly improved) GL results for the rather general case $0.707 \le \kappa < \infty$, $(2\kappa^2)^{-1} < b < 1$:

$$c_{66} \approx \frac{B_{c2}^2}{\mu_0}\frac{b(1-b)^2}{8\kappa^2}\left[1-\frac{1}{2\kappa^2}\right](1-0.58b+0.29b^2) ,$$

$$c_{11}(k) = \frac{B_{c2}^2}{\mu_0}\left[1-\frac{1}{2\kappa^2}\right](1+k^2\lambda'^2)^{-1}(1+k^2\xi'^2)^{-1} ,$$

$$c_{44}(k) = \frac{B_{c2}^2}{\mu_0}[(1+k^2\lambda'^2)^{-1}+k_B^{-2}\lambda'^{-2}] .$$

In $c_{11}(k)$ the factor containing the effective coherence length $\xi'=\xi/(2-2b)^{1/2}$ becomes important only very close to B_{c2} and may usually be omitted. For many applications one may put $c_{11}=\infty$; the FLL is then incompressible, not liquid, since c_{66} is finite.

The linear elastic displacements caused by four different model forces of unit strength at the point of application are compiled in Table I. Note that the correction to the result of the usual local elastic theory caused by the correct nonlocal treatment is typically large except when the model force is a line force parallel to the FL's, which causes only shear deformations. The complete displacement fields generated by point forces or by planar forces (which in some cases yield oscillatory displacement fields) are given in Ref. 9. The generalization of the above bulk elastic behavior of the FLL to the presence of a planar (or weakly curved) surface is treated in Ref. 10.

III. EFFECTIVE INTERACTION POTENTIAL BETWEEN FLUX LINES

The above expressions for the elastic moduli apply if the induction is not too small; then each FL interacts with many neighboring FL's. At small induction, $b \lesssim 1/2\kappa^2$ corresponding to $a \gtrsim 4\lambda$, the moduli decrease exponentially with the FL spacing. In this case the FLL is usually strongly perturbed even by weak pinning. It is then appropriate not to use the linear elastic moduli but to start calculations from the structure-dependent part of the energy. For straight, parallel FL's this reads

$$U_2=\tfrac{1}{2}\sum_i \sum_{j(\neq i)} V(|\mathbf{r}_i-\mathbf{r}_j|) .$$

The elastic moduli derived from this interaction are

$$c_{11}-c_{66}=(B/8\phi_0)\sum_i [R_i^2V''(R_i)-R_iV'(R_i)] ,$$

$$c_{66}=(B/16\phi_0)\sum_i [R_i^2V''(R_i)+3R_iV'(R_i)] .$$

In these sums $\mathbf{r}_i$ and $\mathbf{r}_j$ are the FL positions and $\mathbf{R}_i$ are the vectors of an ideal triangular lattice, $R_i=|\mathbf{R}_i|$. The interaction

$$V(r)=(\phi_0^2/2\pi\lambda'^2\mu_0)[K_0(r/\lambda')-K_0(r/\xi')]$$

(K_0 is a modified Bessel function) reproduces the correct linear elastic properties to a good approximation. For $b \ll 1$ the repulsive part of $V(r)$ reduces to the London potential $\sim K_0(r/\lambda)$, and its attractive part to the FL core interaction caused by the change in condensation energy when the cores overlap. This attractive term was first obtained for well-separated FL's in Ref. 11. Here we derived it from the nonlocal moduli.

A more general, three-dimensional version of the structure-dependent energy applies to arbitrarily curved FL's, even to FL's forming loops, cutting other FL's, or merging to FL's carrying more than one quantum of flux, though it was originally constructed to reproduce $c_{11}(k)$, $c_{44}(k)$, and c_{66}. It reads

$$U_3=\frac{1}{2}\sum_i\sum_j\frac{\phi_0^2}{2\pi(\lambda')^2\mu_0}\left[\int d\mathbf{r}_i\int d\mathbf{r}_j\frac{e^{-r/\lambda'}}{r}-\int|d\mathbf{r}_i|\int|d\mathbf{r}_j|\frac{e^{-r/\xi'}}{r}\right].$$

Here the sums are over all FL's and the line integrals are along each FL. Note the different nature of the "vectorial" repulsion ($\sim d\mathbf{r}_i d\mathbf{r}_j$) and the "scalar" attraction ($\sim|d\mathbf{r}_i|\,|d\mathbf{r}_j|$). The terms $i=j$ are included in U_3 but not in U_2. These terms give the interaction between all line elements of the same FL, i.e., the self-energy of the (in general, curved) FL's. In principle, these terms could also have been included in U_2 but since for straight FL's the self-energy does not depend on the lattice structure it may be omitted without loss of generality. Note that the self-energy in U_3 does not diverge (in contrast to the London model) since ξ' effectively acts as a cut-off radius.

The line integrals in U_3 may be expressed explicitly by using the z-coordinate as a line parameter and writing $\mathbf{r}_i(z)=(x_i(z),y_i(z),z)$, $r=|\mathbf{r}_i(z_1)-\mathbf{r}_j(z_2)|$, $d\mathbf{r}_i/dz_1=\mathbf{r}_i'$, and $d\mathbf{r}_j/dz_2=\mathbf{r}_j'$. This gives us

$$U_3=\frac{1}{2}\frac{\phi_0^2}{2\pi\lambda'^2\mu_0}\sum_i\int dz_1\sum_j\int dz_2\left\{\mathbf{r}_i'\mathbf{r}_j'\frac{e^{-r/\lambda'}}{r}-[1+(\mathbf{r}_i')^2]^{1/2}[1+(\mathbf{r}_j')^2]^{1/2}\frac{e^{-r/\xi'}}{r}\right\}.$$

The above approximate expressions U_2 and U_3 prove to be very useful. Not only do they circumvent the necessity of solving the GL equations (even for a regular FLL this requires a computer[12]), but they also provide us with a transparent picture of a repulsive and attractive interaction and "explain" the b, κ, and k dependences of the elastic moduli. These expressions are useful in computer simulations of flux pinning[13] and, in principle, also allow analytical and numerical calculations of the plastic properties of the FLL.

IV. DISLOCATIONS IN THE FLUX-LINE LATTICE

A plastic deformation of the FLL may be described by the presence of edge and/or screw dislocations.[14,15] When the local elastic description is replaced by the correct nonlocal one the displacement field $\mathbf{s}$ and the line energy J_{edge} of the edge dislocation is not seriously modified since its strain is a mere shear strain and the shear modulus exhibits only weak dispersion. For FL's along z one has for an edge dislocation centered at the origin and with Burgers vector $\mathbf{b}$ along x (e.g., $b=|\mathbf{b}|=a$),

$$\mathbf{s}(x,y)=-(b/2\pi)[\hat{\mathbf{x}}(xy/r^2+\varphi)+\hat{\mathbf{y}}y^2/r^2],$$

$$J_{\text{edge}}=(c_{66}b^2/4\pi)\ln(R/b).$$

Here $r^2=x^2+y^2$ and $\varphi=\arctan(y/x)$. R is an outer cut-off radius.

In contrast to edge dislocations,[16] the screw dislocation in the FLL (which is oriented parallel to the FL's) possesses several peculiarities compared with screw dislocations in atomic lattices: (a) The extension of the dislocation core along the FL's, l, is typically much larger than its extension perpendicular to the FL's, $\approx a$, since $c_{44}(0)\gg c_{66}$, (b) the nonlocality (the dispersion of c_{44}) should be accounted for, and (c) the screw dislocation can move freely along the FL's since there is no Peierls potential (no FL structure) in this direction. In my opinion all three points are important for a quantitative explanation of the observed abrupt transition.[3]

Minimizing the sum of the shear energy (which tends to decrease l and comes from the region outside the dislocation core) and the tilt energy (which tends to increase l and is concentrated inside the core) one finds

$$l\approx a(c_{44}/c_{66})^{1/2}\approx a(1-b)/3\kappa b^{1/2}.$$

The nonlocality reduces the core length l. For simple estimates one may replace $c_{44}(0)$ by $c_{44}(1/l)$ in the local expressions. For a more rigorous treatment one may approximate the displacement field of a screw dislocation passing through the origin and oriented along x by

$$\mathbf{s}(z,y)=\hat{\mathbf{x}}(a/\pi)\arctan[z/c(y)],$$

where $c(y)$ is a trial function which satisfies $c(y)=-c(-y)$, $c(a/2)=l$, and $c(y\to\infty)=\infty$. The elastic energy should then be minimized with respect to $c(y)$. The resulting line energy will not depend crucially on $c(y)$. The effective inner cut-off radius entering the logarithm is λ', even if $l_0\ll\lambda'$, since strains varying over a shorter length do not contribute much to the tilt energy. The self-energy of the screw dislocation is thus expected to be

$$J_{\text{screw}}\approx(c_{44}c_{66})^{1/2}(a^2/4\pi)\ln(R/\lambda').$$

For a further investigation of the plastic properties of the FLL a detailed treatment of the production and anisotropic interaction of straight or curved dislocations is required. Such calculations should be based on the ideas outlined above and on the theory of crystal lattice dislocations.[17] The force between dislocations with well-separated cores is obtained from the Peach-Koehler formula when the stress field far from a dislocation is known. This stress field may be obtained from the linear theory of elasticity in its local approximation, since the relevant wavelengths are large and linear superposition of the stress fields is allowed in this case. When the dislocation cores overlap, both nonlinearity and nonlocality become important. A quantitative treatment of dislocations with overlapping cores requires numerical calculations based on the above interaction potentials between FL's or FL elements.

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