Theory of the dynamic spin response function near the Kosterlitz-Thouless transition

René Côté and Allan Griffin
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7

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The pioneering work of Ambegaokar, Halperin, Nelson, and Siggia on dynamical phenomena in superfluid films near the Kosterlitz-Thouless transition is formulated in a more systematic way in the context of the classical two-dimensional (2D) XY model. Specifically, we extend the discussion of Nelson and Fisher on spin dynamics to include the effect of the vortices. The coupled equations of motion for the spin-wave and vortex fields are derived from a Lagrangian analogous to that used in classical electrodynamics of a continuous medium. In the electromagnetic analogy, the spin waves (or third sound) correspond to transverse photons, the renormalization effects of the bound vortex pairs entering through the transverse dielectric function $\epsilon_{\perp}(\mathbf{q},\omega)$ of the equivalent 2D neutral Coulomb gas. The relation between our results for the dynamic response functions and previous work on static response functions (in which the spin-wave—vortex coupling is apparently ignored) is clarified by noting the distinction between the bare (θ_0) and the renormalized ($\theta=\theta_0/\epsilon_{\perp}$) spin-wave fields.

I. INTRODUCTION

For some time, we have had a very thorough understanding (using a variety of techniques whose qualities range from intuitive to rigorous) of the Kosterlitz-Thouless-Berezinskii transition in the classical two-dimensional (2D) XY model insofar as equilibrium and *static* response functions are concerned. (For reviews, see Refs. 1 and 2.) In contrast, there have been very few theoretical studies of *dynamical* response functions near the Kosterlitz-Thouless (KT) transition. The key paper in this field is that by Ambegaokar, Halperin, Nelson, and Siggia[3] (henceforth referred to as AHNS), written with the specific goal of explaining certain experiments in superfluid ^{4}He films. In the present paper, our goal is to give a systematic derivation of the AHNS results within the more general language of the XY model, as well as to expand on the analogy AHNS made to the equations of motion for electrodynamics in a dielectric medium. We put special emphasis on what the theory says about the frequency and weight of the spin-wave pole, as renormalized by the bound vortex pairs. We hope that our work will set the stage for studies on the dynamical properties of XY models with frustration.

For orientation, we briefly recall some standard results for the static[4–7] response functions near the KT transition. The starting point of such studies is the Hamiltonian

$$\overline{H}\equiv\frac{H}{k_BT}=\frac{K_0}{2}\int d\mathbf{r}[\nabla\phi(\mathbf{r})]^2\ , \tag{1.1}$$

where we have introduced the continuous field angle variable $\phi(\mathbf{r})$ since only the long-wavelength properties are of interest. This field is split into two parts

$$\nabla\phi=\nabla\theta+\nabla\psi\ , \tag{1.2}$$

where the spin-wave field $\theta(\mathbf{r})$ describes the small phase fluctuations in the local order parameter while the vortex field $\psi(\mathbf{r})$ is related to the large amplitude fluctuations associated with the zeros of the local order parameter (for a particularly clear discussion, see Ref. 1). The energy (1.1) then splits into two separate contributions, due to the spin waves and vortices,

$$\overline{H}=\overline{H}_{\rm SW}+\overline{H}_v\ . \tag{1.3}$$

The energy $\overline{H}_v$ corresponding to the vortices can be shown to be essentially equivalent to a classical 2D neutral Coulomb gas,

$$\overline{H}_v=-\pi K_0\int d\mathbf{r}\int d\mathbf{r}'\,n(\mathbf{r})n(\mathbf{r}')\ln\left|\frac{\mathbf{r}-\mathbf{r}'}{a_0}\right|+E_c\ , \tag{1.4}$$

where E_c is the vortex core contribution. In this picture, the spin-wave and vortex contributions to the static correlation functions are apparently uncoupled and one finds the power-law behavior at large distances characteristic of a system with only quasi-long-range order,

$$\langle e^{i[\theta(\mathbf{r}')-\theta(0)]}\rangle_{\rm SW}\sim\frac{1}{r^{1/2\pi K_0}}\ ,$$
$$\langle e^{i[\psi(\mathbf{r})-\psi(0)]}\rangle_v\sim\frac{1}{r^{1/2\pi K_V}}\ . \tag{1.5}$$

The usual way of summarizing these results is that the effect of the (bound) vortices is to renormalize the Gaussian spin-wave power-law exponent $K_0\rightarrow K_R$, with

$$\frac{1}{K_R}=\frac{1}{K_0}+\frac{1}{K_V}\ . \tag{1.6}$$

As first shown by Kosterlitz and Thouless,[4] one can express K_V in terms of a size-dependent dielectric function $\tilde{\epsilon}(l)$ describing the screening of the interaction between charges separated by a distance $r\equiv a_0\exp(l)$ arising from the bound pairs of smaller size. One obtains

$$K_R = \frac{K_0}{\tilde{\epsilon}(l \to \infty)} \tag{1.7}$$

as being appropriate in describing long-wavelength phenomena near the KT transition. The famous KT renormalization-group flow diagram shows that in the $l \to \infty$ limit, K_R decreases as the temperature increases until it reaches the critical value $2/\pi$, at which point the bound vortex pairs unbind and the system passes into a phase described by a gas of free vortices $[\tilde{\epsilon}(l=\infty) \to \infty$ or, equivalently, $K_R = 0]$. This abrupt behavior in fact only occurs in the extreme long-wavelength limit. At any finite value of the wave vector q, however, the important vortex pairs have a size approximately $\sim q^{-1}$ and K_R is smoothly varying through $T_{\rm KT}$.

The only detailed studies of the dynamic response functions of the 2D XY model below $T_{\rm KT}$ have been made in the so-called Gaussian spin-wave approximation[8–10] in which one completely ignores the vortices in (1.1). Our major interest in this paper is to understand the origin of the spin-wave—vortex coupling which gives rise to the renormalization of the spin-wave velocity (the analogue of third sound discussed in Ref. 3) and to what extent this renormalization involves the same K_R as found in the *static* response function calculations reviewed above.

In Sec. II, following Nelson and Fisher[9] and others, we argue that in order to discuss the dynamics of the 2D XY model, the energy in (1.1) must be augmented to include a term involving the square of the z component of the local magnetic moment $M_z(\mathbf{r})$. The resulting equations of motion are shown to automatically reproduce a term involving the vortex current, which is ultimately the origin of the renormalization of the spin-wave field. We also show how these equations of motion reduce to those derived by AHNS, who started from the phenomenological theory of the motion of vortices in superfluid ^{4}He films.

In Sec. III, we formalize and develop the electromagnetic analogy discussed by AHNS and Halperin,[1] in which the spin-wave (vortex) field is associated with the transverse (longitudinal) electric field. In particular, we discuss the equivalent Lagrangian for the spin-wave—vortex problem.

In Sec. IV, we set the stage for the calculation of the dynamic spin response function by expressing the vortex current in terms of the dielectric function describing bound vortex pairs. In the electromagnetic analogy, this corresponds to passing from the *microscopic* to the *macroscopic* version of Maxwell's equations. We concentrate on the region just below $T_{\rm KT}$, which is dominated by the unbinding of the vortex pairs. We discuss to what extent one can calculate the response functions using a Gaussian spin-wave approximation of the kind

$$\overline{H} = \frac{K_R}{2} \int d\mathbf{r} [\nabla \theta_0(\mathbf{r})]^2 , \tag{1.8}$$

where $\theta_0(\mathbf{r})$ is the bare spin-wave field but K_R is the renormalized exchange constant given by (1.7). The distinction between $\theta_0(\mathbf{r})$ and $\theta(\mathbf{r})$ is the key to understanding why the spin-wave—vortex coupling is crucial in our work in spite of the fact that in most discussions of static response functions, it is apparently ignored. These results are used in Sec. V to generalize the Nelson-Fisher[9] calculation of the transverse spin response function to include the effect of the bound vortex pairs.

II. COUPLED EQUATIONS OF MOTION

In the two-dimensional classical XY model with fixed-length spins, the only degree of freedom is the angle ϕ_i which the spin at site i makes with some reference axis in the plane. The standard Hamiltonian for such a system is

$$H = -J_0 \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j) , \tag{2.1}$$

where the summation is over nearest neighbors only and $J_0 > 0$ is the exchange constant (the fixed magnitude of the spins is incorporated into J_0). Since it is the long-wavelength response of the system which is of interest, the discreteness of the lattice is irrelevant and we introduce a continuous field $\phi(\mathbf{r})$. Keeping only the quadratic term in the expansion for the potential energy given by (2.1) and adding a kinetic energy for the rotation of the spins, a simple Lagrangian for the dynamic 2D XY model is

$$L = -\frac{J_0}{2} \int d\mathbf{r} [\nabla \phi(\mathbf{r})]^2 + \frac{\alpha}{2} \int d\mathbf{r} \left[\frac{d\phi(\mathbf{r})}{dt} \right]^2 . \tag{2.2}$$

Here α is a constant that can be determined, in principle, from the solution of the quantum-mechanical problem of spin evolution in a planar magnet.[11,12] Defining a conjugate momentum density to $\phi(\mathbf{r})$,

$$S_\phi(\mathbf{r}) = \frac{\delta L}{\delta \left[\dfrac{d\phi(\mathbf{r})}{dt} \right]} = \alpha \frac{d\phi(\mathbf{r})}{dt} \tag{2.3}$$

the corresponding Hamiltonian is found to be[13]

$$H = \frac{J_0}{2} \int d\mathbf{r} [\nabla \phi(\mathbf{r})]^2 + \frac{1}{2\alpha} \int d\mathbf{r}\, S_\phi^2(\mathbf{r}) . \tag{2.4}$$

$S_\phi(\mathbf{r})$ is in fact proportional to the spin component $S_z(\mathbf{r})$ as discussed by Villain[12] as well as Nelson and Fisher.[9] The inclusion of a term $(\nabla S_\phi)^2$ can be shown to be irrelevant in the critical region of a 2D XY model close to the Kosterlitz-Thouless transition. More precisely, our model deals with a 2D system of three component spins with an easy plane anisotropy.[9] As is well known, a fully isotropic 2D Heisenberg model exhibits no quasi-long-range order at any temperature.[14]

Using Hamilton's equations of motion, (2.4) gives us immediately

$$\begin{aligned} -\frac{\delta H}{\delta \phi(\mathbf{r})} &= \frac{dS_\phi(\mathbf{r})}{dt} = J_0 \nabla^2 \phi(\mathbf{r}) , \\ \frac{\delta H}{\delta S_\phi(\mathbf{r})} &= \frac{d\phi(\mathbf{r})}{dt} = \frac{1}{\alpha} S_\phi(\mathbf{r}) . \end{aligned} \tag{2.5}$$

If $\phi(\mathbf{r})$ was a well-behaved, smoothly varying field, the solution of this set of equations would be trivial. However, this is not the case when we include the vortex configurations. More precisely, if we denote the local order parameter as

$$\mathbf{M}(\mathbf{r}) = |\mathbf{M}(\mathbf{r})| \exp[i\phi(\mathbf{r})] ,$$

then the phase $\phi(\mathbf{r})$ is continuous and differentiable except at the points where $|\mathbf{M}(\mathbf{r})|=0$. These points are associated with vortices. In using (2.4), we are ignoring the small fluctuations in the amplitude of $\mathbf{M}(\mathbf{r})$ but are keeping the large fluctuations associated with the vortices.

As mentioned in the Introduction, we can separate the phase $\phi(\mathbf{r})$ into a spin-wave part $\theta(\mathbf{r})$ and a part associated with vortices $\psi(\mathbf{r})$,

$$\phi(\mathbf{r})=\theta(\mathbf{r})+\psi(\mathbf{r}) \ . \tag{2.6}$$

If the vortex part $\psi(\mathbf{r})$ is ignored, the equations of motion (2.3) lead to spin-wave oscillations with the dispersion relation

$$\omega=\left[\frac{J_0}{\alpha}\right]^{1/2} q \ . \tag{2.7}$$

This is the approximation used by Nelson and Fisher[9] in their evaluation of dynamic response functions for the 2D XY model. These authors used a "hydrodynamic fixed-length" free-energy functional

$$F=\frac{K_0}{2}\int d\mathbf{r}[(\nabla M_x)^2+(\nabla M_y)^2]+\frac{K_0}{2}\int d\mathbf{r} M_z^2 \ , \tag{2.8}$$

where $K_0 \equiv J_0/k_B T$ and $M_x^2+M_y^2=1$. In contrast with (2.4), this constraint means that all magnitude fluctuations are neglected, including those associated with vortices (apart from being included implicity in K). With regard to (2.4), we also remark that it is assumed that $\alpha=1/J_0$ in (2.8). While this may be true (see, for example, the microscopic calculations of Villain[12]), it is only the coefficient of $(\nabla\psi)^2$ which is renormalized by the bound vortex pairs.

The vortex field $\psi(\mathbf{r})$ at point $\mathbf{r}$ due to vortices of charge $n_i=\pm 1$ at $\mathbf{r}_i$ is given by

$$\psi(\mathbf{r})=\sum_i n_i \arctan\left[\frac{y-y_i}{x-x_i}\right] . \tag{2.9}$$

The gradient of this is easily shown to be

$$\nabla\psi(\mathbf{r})=\sum_i n_i \hat{\mathbf{z}}\times\nabla G(\mathbf{r}-\mathbf{r}_i) \ , \tag{2.10}$$

where

$$\nabla^2 G(\mathbf{r}-\mathbf{r}_i)=2\pi\delta(\mathbf{r}-\mathbf{r}_i) \tag{2.11}$$

and $\hat{\mathbf{z}}$ is a unit vector normal to the xy plane. Since we only need (2.10) in the region $r \gg a_0$ (a_0 is the vortex core radius) where the field is smoothly varying, we take $G(r)\simeq \ln(r/a_0)$. We note that $\nabla\psi(\mathbf{r})$, as defined in (2.10), is purely transverse since

$$\nabla\cdot\nabla\psi(\mathbf{r})=0 \ . \tag{2.12}$$

In summary, then, we see that

$$\nabla\phi=\nabla\theta+\nabla\psi \tag{2.13}$$

involves a purely longitudinal component

$$\nabla\times\nabla\theta=\mathbf{0} \tag{2.14}$$

associated with spin-wave field and a purely transverse component $\nabla\psi(\mathbf{r})$ associated with the vortex field. However, as we shall show, these two fields are coupled and this leads to a renormalized value of the spin-wave frequency due to the presence of bound vortex pairs.

The number density of vortices is given by

$$n(\mathbf{r})=\sum_i n_i \delta(\mathbf{r}-\mathbf{r}_i) \tag{2.15}$$

and the vortex current density associated with it

$$\mathbf{j}_v(\mathbf{r})=\sum_i n_i \dot{\mathbf{r}}_i \delta(\mathbf{r}-\mathbf{r}_i) \ . \tag{2.16}$$

These satisfy the continuity equation

$$\frac{dn(\mathbf{r},t)}{dt}+\nabla\cdot\mathbf{j}_v(\mathbf{r},t)=0 \ . \tag{2.17}$$

A straightforward calculation gives

$$\nabla\times\nabla\psi(\mathbf{r})=2\pi n(\mathbf{r})\hat{\mathbf{z}} \ , \tag{2.18}$$

$$\frac{d}{dt}[\nabla\psi(\mathbf{r})]=\nabla\left[\frac{d\psi(\mathbf{r})}{dt}\right]-2\pi\hat{\mathbf{z}}\times\mathbf{j}_v(\mathbf{r}) \ , \tag{2.19}$$

$$\frac{d}{dt}[\nabla\psi(\mathbf{r})]=-\sum_i n_i\left[\hat{\mathbf{z}}\times\left[\frac{d\mathbf{r}_i}{dt}\times\nabla\right]\nabla G(\mathbf{r}-\mathbf{r}_i)\right] . \tag{2.20}$$

Combining all the results in Eqs. (2.9)—(2.20) with some elementary vector algebra, one obtains

$$\nabla\cdot(\nabla\phi\times\hat{\mathbf{z}})=\hat{\mathbf{z}}\cdot(\nabla\times\nabla\phi)=2\pi n(\mathbf{r}) \ , \tag{2.21}$$

$$\nabla\times(\nabla\phi\times\hat{\mathbf{z}})=-\hat{\mathbf{z}}\nabla^2\phi=-\frac{1}{J_0}\frac{dS_\phi}{dt}\hat{\mathbf{z}} \ , \tag{2.22}$$

$$\nabla\times(S_\phi\hat{\mathbf{z}})=-\hat{\mathbf{z}}\times\nabla S_\phi=\alpha\frac{d}{dt}(\nabla\phi\times\hat{\mathbf{z}})+2\pi\alpha\mathbf{j}_v \ . \tag{2.23}$$

We have written these equations of motion in this way in order to facilitate comparison with the analogous microscopic Maxwell's equations in Sec. III. As we shall discuss there, the coupling between spin waves and vortices has its origin in the vortex current $\mathbf{j}_v$ in (2.23). It is gratifying that this crucial term arises naturally in our analysis based on the Hamiltonian in (2.4).

Taking (2.12) and (2.14) into account, we note that (2.21) reduces to an equation only involving the vortex field

$$\nabla\cdot(\nabla\psi\times\hat{\mathbf{z}})=2\pi n(\mathbf{r}) \ , \tag{2.24}$$

while (2.22) reduces to an equation only involving the spin-wave field

$$\nabla\times(\nabla\theta\times\hat{\mathbf{z}})=-\hat{\mathbf{z}}\nabla^2\theta=-\frac{1}{J_0}\frac{dS_\phi}{dt}\hat{\mathbf{z}} \ . \tag{2.25}$$

At this point, we can make contact with the work of Ambegaokar *et al.*,[3] on the dynamics of superfluid ^{4}He films. As we discussed in the Introduction, these authors combined the phenomenological theory of vortex motion in superfluid films with the Kosterlitz-Thouless analysis of bound vortex pairs. To the extent that superfluid ^{4}He films are a realization of the 2D classical XY model, the superfluid velocity is given by

$$\nabla\phi(\mathbf{r})=\frac{M}{\hbar}\mathbf{v}_s(\mathbf{r}) \tag{2.26}$$

and $S_\phi(\mathbf{r})$ is identified with the variable $m(\mathbf{r})$ related to the deviation in film thickness (for details, see Ref. 3)

$$S_\phi(\mathbf{r})=\frac{M}{\hbar g}m(\mathbf{r}) . \tag{2.27}$$

In terms of these variables, (2.22) and (2.23) are completely equivalent to Eqs. (5.15) and (5.16) of AHNS, namely,

$$\begin{aligned}\frac{d\mathbf{v}_s(\mathbf{r})}{dt}&=g\nabla m(\mathbf{r})-\hat{\mathbf{z}}\times\mathbf{J}(\mathbf{r}) ,\\ \frac{dm(\mathbf{r})}{dt}&=g\rho_s^0\nabla\cdot\mathbf{v}_s(\mathbf{r}) ,\end{aligned} \tag{2.28}$$

where $g^2\equiv 1/\alpha$, $\mathbf{J}=\hbar\mathbf{j}_v/M$. The superfluid density in the absence of vortices is given by $\rho_s^0=J_0$. Thus we see that the equations of vortex motion in superfluid films used by AHNS are in fact generated by

$$H=\frac{M^2}{2\hbar^2}\rho_S^0\int d\mathbf{r}\,\mathbf{v}_s^2(\mathbf{r})+\frac{M^2}{2\hbar^2}\int d\mathbf{r}\,m^2(\mathbf{r}) . \tag{2.29}$$

The equivalent free-energy functional was written down by Hohenberg, Halperin, and Nelson[15] in their review of AHNS, although they did not emphasize that it gives a description of vortices in both the statics and dynamics of 2D superfluid films.

III. ANALOGY TO ELECTROMAGNETIC THEORY

As pointed out by AHNS in their discussion of superfluid films, there is a formal relation between the equations of motion for the 2D XY model summarized in (2.21)–(2.23) and the microscopic Maxwell's equations. In this section, we wish to develop this equivalence in somewhat more detail since it is crucial in understanding in a more systematic fashion how bound vortices below $T_{\rm KT}$ lead to a renormalized spin-wave dispersion relation.

Introducing an electric field and a magnetic field,

$$\begin{aligned}\mathbf{E}&\equiv(2\pi J_0)^{1/2}(\nabla\phi\times\hat{\mathbf{z}}) ,\\ \mathbf{B}&\equiv\left[\frac{2\pi}{\alpha}\right]^{1/2}S_\phi\hat{\mathbf{z}} ,\end{aligned} \tag{3.1}$$

Eqs. (2.21)–(2.23) may be written as

$$\nabla\cdot\mathbf{B}(\mathbf{r})=0 , \tag{3.2}$$

$$\nabla\cdot\mathbf{E}(\mathbf{r})=2\pi\rho(\mathbf{r}) , \tag{3.3}$$

$$\nabla\times\mathbf{E}(\mathbf{r})=-\frac{1}{c_0}\frac{\partial\mathbf{B}(\mathbf{r})}{\partial t} , \tag{3.4}$$

$$\nabla\times\mathbf{B}(\mathbf{r})=\frac{1}{c_0}\frac{\partial\mathbf{E}(\mathbf{r})}{\partial t}+\frac{2\pi}{c_0}\mathbf{j}(\mathbf{r}) . \tag{3.5}$$

The speed of light c_0 is equivalent to the bare spin-wave velocity in (2.7) in our 2D XY model. The mapping is thus to a system of Maxwell equations on a surface (since $\mathbf{B}=B_0\hat{\mathbf{z}}$) in three dimensions such that all quantities have variation in the xy plane only: $\nabla\equiv[(\partial/\partial x),(\partial/\partial y),0]$. Here $\mathbf{r}$ is a two-dimensional vector in the plane so that a vortex in our system is equivalent to a line of charges along the $\hat{\mathbf{z}}$ axis. This accounts for the logarithmic potential of a vortex charge and for the factor 2π instead of the usual 4π in the above equations. The charge of a vortex is now $q_i=(2\pi J_0)^{1/2}n_i$ and thus

$$\begin{aligned}\rho(\mathbf{r})&=(2\pi J_0)^{1/2}n(\mathbf{r}) ,\\ \mathbf{j}(\mathbf{r})&=(2\pi J_0)^{1/2}\mathbf{j}_v(\mathbf{r}) .\end{aligned} \tag{3.6}$$

In this electromagnetic analogy, we see that[1,3] the *transverse* part of the electric field is related to the gradient of the *longitudinal* spin-wave field

$$\mathbf{E}_T(\mathbf{r})=(2\pi J_0)^{1/2}\nabla\theta(\mathbf{r})\times\hat{\mathbf{z}} , \tag{3.7}$$

while the longitudinal part of the electric field is related to the gradient of the transverse vortex field

$$\mathbf{E}_L(\mathbf{r})=(2\pi J_0)^{1/2}\nabla\psi(\mathbf{r})\times\hat{\mathbf{z}} . \tag{3.8}$$

Using this equivalence, one can use the standard Lagrangian for the electromagnetic field in the nonrelativistic limit[13] to find the analogous Lagrangian for our XY model in terms of vector and scalar potentials. This will be used in Sec. IV in our study of dynamic response functions. The Lagrangian is given by

$$L=\int\frac{d\mathbf{r}}{4\pi}[\,|\mathbf{E}(\mathbf{r})|^2-|\mathbf{B}(\mathbf{r})|^2]-\sum_i q_i\Phi(\mathbf{r}_i)+\sum_i\frac{q_i}{c_0}\dot{\mathbf{r}}_i\cdot\mathbf{A}(\mathbf{r}_i) , \tag{3.9}$$

where

$$\begin{aligned}\mathbf{E}&=-\nabla\Phi(\mathbf{r})-\frac{1}{c_0}\frac{\partial\mathbf{A}(\mathbf{r})}{\partial t} ,\\ \mathbf{B}&=\nabla\times\mathbf{A}(\mathbf{r}) ,\end{aligned} \tag{3.10}$$

and $\mathbf{r}_i$ ($\dot{\mathbf{r}}_i$) gives the position (velocity) of the ith vortice.

Using the Coulomb gauge $\nabla\cdot\mathbf{A}=0$, we obtain

$$\begin{aligned}\mathbf{E}_L&=(2\pi J_0)^{1/2}(\nabla\psi\times\hat{\mathbf{z}})=-\nabla\Phi ,\\ \mathbf{E}_T&=(2\pi J_0)^{1/2}(\nabla\theta\times\hat{\mathbf{z}})=-\left[\frac{\alpha}{J_0}\right]^{1/2}\frac{\partial\mathbf{A}}{\partial t} .\end{aligned} \tag{3.11}$$

We can find $\Phi(\mathbf{r})$ explicitly using (2.10), namely,

$$\Phi(\mathbf{r})=-\sum_i q_iG(\mathbf{r}-\mathbf{r}_i) \tag{3.12}$$

and thus the Lagrangian of our XY model can be written in the form

$$L=\tfrac{1}{2}\int d\mathbf{r}\left[J_0(\nabla\phi\times\hat{\mathbf{z}})^2-\frac{1}{\alpha}(S_\phi\hat{\mathbf{z}})^2\right]-\sum_{\substack{i,j\\(i\neq j)}}q_iq_jG(\mathbf{r}_i-\mathbf{r}_j)+\sum_i\left[\frac{\alpha}{J_0}\right]^{1/2}q_i\dot{\mathbf{r}}_i\cdot\mathbf{A}(\mathbf{r}_i) . \tag{3.13}$$

The equations of motion discussed in Sec. II can be derived from this Lagrangian, where $\Phi(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ are now considered as the generalized coordinates along with the positions of the vortices $\mathbf{r}_i$. We can also derive an equation of motion for the vortices using

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\mathbf{r}}_i}\right]+\nabla\cdot\frac{\partial L}{\partial(\nabla \mathbf{r}_i)}=\frac{\partial L}{\partial \mathbf{r}_i} \quad (3.14)$$

to obtain the equivalent of the Lorentz force

$$\mathbf{F}=m\ddot{\mathbf{r}}_i=q_i\mathbf{E}(\mathbf{r}_i)+\frac{q_i}{c_0}\mathbf{v}_i\times\mathbf{B}(\mathbf{r}_i) . \quad (3.15)$$

Using the fact that vortices have no mass, this gives

$$\dot{\mathbf{r}}_i=\frac{-J_0}{S_\phi(\mathbf{r}_i)}[\nabla\theta(\mathbf{r}_i)+\nabla\psi(\mathbf{r}_i)] . \quad (3.16)$$

That is, $\nabla\phi$ at $\mathbf{r}_i$ acts as the equivalent of a force which produces a velocity, in agreement with the work of Nikiforov and Sonin.[16] This equation of motion for the vortices shows in an explicit manner how the spin-wave field $\nabla\theta$ and the vortex field $\nabla\psi$ at the position of a vortice are coupled through the velocity $\dot{\mathbf{r}}_i$ of the vortice.

While the equations of motion (3.2)–(3.5) are formally identical to Maxwell's equations, certain differences should be kept in mind. For example, we note that a vortice is itself a particular configuration of the fields, in contrast to the charges and currents in ordinary electrodynamics which are entities independent of the fields they create. The vortices having no mass means that *all* the energy is in the fields **E** and **B**. Another difference is that the equivalent longitudinal electric field as defined in (3.8) involves $\nabla\psi$ which is not an ordinary gradient in that $\nabla\times\nabla\psi(\mathbf{r})\neq\mathbf{0}$ at the vortex centers $\mathbf{r}_i$. This is easily verified using (2.10) and arises from the singularities associated with the vortices. As a consequence, one cannot easily extract information about correlation functions involving the vortex field ψ from the well-known correlation functions for longitudinal electric fields.

IV. RENORMALIZATION OF THE SPIN-WAVE FREQUENCY BY VORTEX PAIRS

The analysis in Secs. II and III may be viewed as the equivalent of deriving the microscopic Maxwell's equations. That is to say, we are dealing with the fields in free space treating the *full* charge and current densities as sources. In the present section, we go over to the equivalent of macroscopic Maxwell's equations in that we shall shift the contribution of the bound vortex pairs from the sources into the effective electric fields they give rise to. The bound vortex pairs will be treated as a dielectric medium in which the unbound vortices move, in complete analogy with the introduction of dielectric functions in ordinary electrodynamics. Thus (3.3) is now written as (for an isotropic system)

$$i\,\mathbf{q}\cdot\epsilon_L(\mathbf{q},\omega)\mathbf{E}_L(\mathbf{q},\omega)=2\pi\rho_{\text{free}}(\mathbf{q},\omega) , \quad (4.1)$$

while (3.4) and (3.5) can be combined to give

$$q^2\mathbf{E}_T(\mathbf{q},\omega)=\frac{\omega^2}{c_0^2}\epsilon_T(\mathbf{q},\omega)\mathbf{E}_T(\mathbf{q},\omega)+\frac{2\pi i}{c_0^2}\mathbf{j}_{T,\text{free}}(\mathbf{q},\omega) . \quad (4.2)$$

Here $\epsilon_{L,T}(\mathbf{q},\omega)$ are the wave-vector- and frequency-dependent dielectric functions describing the effect of the bound pairs, while $\mathbf{j}_{T,\text{free}}(\mathbf{q},\omega)$ is the transverse part of the current related with the free vortices.

Since we are mainly interested in how the bound pairs renormalize the spin-wave excitations, we shall concentrate on the equation of motion for the transverse electric field $\mathbf{E}_T(\mathbf{q},\omega)$. The equivalent of the displacement field can be associated with the unperturbed spin-wave field (denoted by θ_0)

$$\mathbf{D}_T\equiv(2\pi J_0)^{1/2}\nabla\theta_0\times\hat{\mathbf{z}}=\epsilon_T\mathbf{E}_T . \quad (4.3)$$

θ_0 is renormalized by the transverse current of vortex pairs which it generates and we have $\theta_0\rightarrow\theta=\theta_0+\theta_{\text{ind}}$. In terms of ϵ_T, this is given by

$$\theta(\mathbf{q},\omega)=\frac{\theta_0(\mathbf{q},\omega)}{\epsilon_T(\mathbf{q},\omega)} . \quad (4.4)$$

If we ignore the presence of any free vortices ($T\leq T_{\text{KT}}$), the effective transverse Hamiltonian is now

$$H_T=\int d\mathbf{r}\frac{\mathbf{D}_T\cdot\mathbf{E}_T}{4\pi}+\int d\mathbf{r}\frac{\mathbf{B}^2}{4\pi} . \quad (4.5)$$

The transverse normal modes have the usual dispersion relation

$$\omega^2=\frac{c_0^2q^2}{\epsilon_T(\mathbf{q},\omega)} , \quad (4.6)$$

as can be seen most directly by expressing (4.2) in the form

$$\mathbf{E}_T(\mathbf{q},\omega)=\frac{2\pi i\omega\mathbf{j}_{T,\text{free}}(\mathbf{q},\omega)}{c_0^2[q^2-(\omega^2/c_0^2)\epsilon_T(\mathbf{q},\omega)]} . \quad (4.7)$$

Thus we see that the renormalized longitudinal spin-wave modes are equivalent to the (transverse) photons in a dielectric media composed of dipoles. In the long-wavelength limit which we are interested in, the renormalized spin-wave speed is given by

$$c=\frac{c_0}{[\epsilon_T(q=0,\omega)]^{1/2}}\equiv\left[\frac{J_T}{\alpha}\right]^{1/2} , \quad (4.8)$$

since the renormalized exchange constant is

$$J_T(\mathbf{q},\omega)=\frac{J_0}{\epsilon_T(\mathbf{q},\omega)} . \quad (4.9)$$

In the $q\rightarrow0$ limit, the longitudinal and transverse dielectric functions are equal in an isotropic system and we defer further discussion of $\epsilon(\mathbf{q}\rightarrow\mathbf{0},\omega)$ to the end of this section.

The calculation of the dynamic correlation functions for transverse electromagnetic fields $\chi^T_{\mathbf{EE}}(\mathbf{q},\omega)=\langle\mathbf{E}_T\cdot\mathbf{E}_T\rangle(\mathbf{q},\omega)$ is discussed in the literature[17] and one easily obtains

$$\chi^T_{\mathbf{EE}}(\mathbf{q},\omega)=\frac{\omega^2}{c_0^2}\chi_{\mathbf{AA}}(\mathbf{q},\omega)$$
$$=\frac{\pi k_B T}{\epsilon_T(\mathbf{q},\omega)}[\delta(\omega-cq)+\delta(\omega+cq)] . \quad (4.10)$$

Using (3.7), this immediately gives

$$\chi_{\theta\theta}(\mathbf{q},\omega)=\frac{k_BT}{2q^2J_0\epsilon_T(\mathbf{q},\omega)}[\delta(\omega-cq)+\delta(\omega+cq)]\ , \tag{4.11}$$

where $\chi_{\theta\theta}(\mathbf{q},\omega)\equiv\langle\theta\theta\rangle(\mathbf{q},\omega)$.

If one defines the *new* transverse electric field

$$\mathbf{E}'_T\equiv(2\pi J_T)^{1/2}\nabla\theta_0\times\hat{\mathbf{z}}\ , \tag{4.12}$$

treating J_T in (4.9) as a constant, then one can rewrite (3.4) and (3.5) in the form

$$\nabla\times\mathbf{E}'_T=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\ ,\qquad \nabla\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}'_T}{\partial t}\ , \tag{4.13}$$

where c is the renormalized value defined in (4.8). These equations of motion are generated by an effective transverse Hamiltonian of the kind

$$H_T=\int d\mathbf{r}\left[\frac{\mathbf{E}'^2_T+\mathbf{B}^2}{4\pi}\right]=\tfrac{1}{2}J_T\int d\mathbf{r}(\nabla\theta_0)^2+\frac{1}{2\alpha}\int d\mathbf{r}\,S_\phi^2\ . \tag{4.14}$$

Thus we see that in terms of this new transverse field $\mathbf{E}'_T$, the equations of motion and the associated Hamiltonian look precisely as if we were dealing with a *pure* spin field, with no vortex configurations included. The effect of the bound vortices is completely buried in the renormalized exchange constant. A straightforward calculation of the correlation function for this bare spin-wave field gives (see also Ref. 17)

$$\chi_{\theta_0\theta_0}(\mathbf{q},\omega)=\frac{k_BT}{2q^2J_T(\mathbf{q},\omega)}[\delta(\omega-cq)+\delta(\omega+cq)]\ , \tag{4.15}$$

where c is defined in (4.8). Our work shows how this renormalization of J_0 to $J_T(\mathbf{q},\omega)$ arises from the spin-wave—vortex coupling which enters through the transverse vortex current. The correlation function (4.15) is consistent with our earlier results for the renormalized spin variable given by (4.11) and (4.4), namely,

$$\chi_{\theta_0\theta_0}(\mathbf{q},\omega)=\epsilon_T^2(\mathbf{q},\omega)\chi_{\theta\theta}(\mathbf{q},\omega)\ . \tag{4.16}$$

We recall that with the equations of motion (2.28) appropriate to ^{4}He superfluid films, the unrenormalized superfluid density is given by $\rho_s^0=J_0$. In this connection, one has two equivalent expressions for the longitudinal superfluid current density which includes the effects of the bound pairs (see also p. 70 of Halperin[1])

$$\mathbf{g}_s=\rho_s^0\nabla\theta \tag{4.17}$$

$$=\rho_s\nabla\theta_0\ . \tag{4.18}$$

Here $\rho_s=J_T$ is the superfluid density including the effects of the vortex pairs. In the approach used in most of the literature, it is common to work with the unrenormalized *longitudinal* superfluid current

$$\mathbf{g}_L=\rho_s^0\nabla\theta_0 \tag{4.19}$$

and the *transverse* superfluid current contribution

$$\mathbf{g}_T=\rho_s^0\nabla\psi\ , \tag{4.20}$$

where ψ still includes the bound vortex pairs. In terms of $\mathbf{g}_L$ and $\mathbf{g}_T$, the superfluid density is given by the rigorous expression[2,18,19]

$$\rho_s=\lim_{q\to0}\frac{1}{k_BT}[\chi^L_{gg}(\mathbf{q})-\chi^T_{gg}(\mathbf{q})]\ . \tag{4.21}$$

This can be shown to be related to the helicity order parameter.[20] There is no contradiction with the alternative formula

$$\rho_s=\lim_{q\to0}\frac{1}{k_BT}\chi^L_{g_sg_s}(\mathbf{q}) \tag{4.22}$$

since the significant part of the transverse superfluid current associated with vortex pairs has already been incorporated into $\mathbf{g}_s$ as defined in (4.18). To be precise, the static correlation functions in (4.21) are calculated using (1.3) in which the spin waves and vortices are uncoupled. In (4.22), in contrast, the correlation function is calculated using the Hamiltonian (1.8).

For completeness, we briefly consider the renormalized *transverse* vortex field associated with the *longitudinal* electric field $\mathbf{E}_L$ in (4.1). Of course, recalling

$$\mathbf{D}_L=\epsilon_L\mathbf{E}_L\ , \tag{4.23}$$

we see that (4.1) is equivalent to [compare (3.3)]

$$\nabla\cdot\mathbf{D}_L(\mathbf{r})=2\pi\rho_{\text{free}}(\mathbf{r})\ . \tag{4.24}$$

The effect of the bound vortices on the dynamics of the free vortex gas is completely contained in the longitudinal dielectric function ϵ_L. [As defined, we note that the zeros of $\epsilon_L(\mathbf{q},\omega)$ would only describe the longitudinal collective modes[1,3] in a gas of *bound* vortex pairs.] The longitudinal part of the Hamiltonian is given by [compare with (4.5)]

$$H_L=\int d\mathbf{r}\frac{\mathbf{D}_L\cdot\mathbf{E}_L}{4\pi}\ , \tag{4.25}$$

which is equivalent to a 2D gas of free vortices with a renormalized coupling,

$$H_L=-\pi\frac{J_0}{\epsilon_L}\int d\mathbf{r}\int d\mathbf{r}'\,n_f(\mathbf{r})n_f(\mathbf{r}')\ln\left|\frac{\mathbf{r}-\mathbf{r}'}{a_0}\right|\ . \tag{4.26}$$

(Here we treat ϵ_L as a constant for simplicity of notation.) For a discussion of correlation functions describing free vortices *above* T_{KT}, we refer to Huber.[21]

The result in (4.26) shows that two vortices added to the system interact via a renormalized stiffness constant $J_L\equiv J_0/\epsilon_L$ due to the screening of the interaction by the "dipoles" in the system. As we mentioned earlier, $\epsilon_T(\mathbf{q}=\mathbf{0},\omega)=\epsilon_L(\mathbf{q}=\mathbf{0},\omega)$ so in the long-wavelength limit, the renormalized exchange constants for the renormalized vortex field problem (J_L) is the same as in the renormalized spin-wave problem (J_T). In this limit, we can denote the renormalized exchange interaction simply as J.

As discussed at length in Refs. 3 and 22, the real part of $\epsilon_L(\mathbf{q}=0,\omega)$ can be related to the Kosterlitz-Thouless scale-dependent dielectric constant $\tilde{\epsilon}_{KT}(\bar{l})$ with $\bar{l} \equiv \ln(r_D/a_0)$, where r_D is the appropriate diffusion length of the vortices. In the case of ^{4}He films, it has been estimated[22] that $r_D \sim (14D/\omega)^{1/2}$, where D is the vortex diffusion constant. According to the preceding results, then, the renormalized spin-wave speed just below the Kosterlitz-Thouless transition is given by[1,3]

$$c(\omega) \simeq \frac{c_0}{[\tilde{\epsilon}_{KT}(\bar{l})]^{1/2}} . \tag{4.27}$$

The dependence on $\bar{l}$ means that the dominant screening effects arise from bound pairs with a size which decreases as the frequency increases (or, equivalently, as the wavelength decreases). However, the classic renormalization-group analysis of the KT transition shows that $\tilde{\epsilon}_{KT}^{-1}(l)$ only exhibits a discontinuous drop at $T=T_{KT}$ for infinitely large bound pairs ($l \to \infty$). The smaller $\bar{l}$ is (i.e., the larger the frequency ω is), the smoother is the behavior of $\tilde{\epsilon}_{KT}^{-1}(\bar{l})$ as we pass through the KT transition. This fact is well understood in the context of third sound studies on ^{4}He films[3] but it should also be kept in mind when discussing how the spin-wave dispersion relation will behave near T_{KT} in layered magnetic systems. One expects that in the range of frequencies accessible to inelastic neutron scattering, the KT renormalization of the spin-wave velocity in 2D XY magnetic systems would be quite small. One would have to use some other sort of experimental technique which would probe spin waves in the true hydrodynamic region if one wanted to see the sudden disappearance of spin waves due to the KT unbinding of vortex pairs (see, however, Ref. 23).

V. ORDER-PARAMETER CORRELATION FUNCTION

The transverse spin-correlation function is defined as

$$S(\mathbf{q},\omega) = \int dt\, e^{-i\omega t} \int d\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}} \langle \cos[\phi(\mathbf{r},t)-\phi(\mathbf{0},0)] \rangle . \tag{5.1}$$

Nelson and Fisher[9] have given a detailed evaluation of $S(\mathbf{q},\omega)$ within the bare spin-wave Gaussian approximation. This corresponds to taking $\phi(\mathbf{r})$ to be $\theta_0(\mathbf{r})$ as defined in Sec. IV, the thermal average and time dependence being determined by

$$H = \frac{J_0}{2} \int d\mathbf{r} (\nabla\theta_0)^2 + \frac{\alpha}{2} \int d\mathbf{r} \left[\frac{d\theta_0}{dt} \right]^2 . \tag{5.2}$$

In this harmonic approximation, one has

$$\begin{aligned} \langle \cos[\theta_0(\mathbf{r},t)-\theta_0(\mathbf{0},0)] \rangle &= \exp\{ -\tfrac{1}{2} \langle [\theta_0(\mathbf{r},t)-\theta_0(\mathbf{0},0)]^2 \rangle \} \\ &= \exp[\chi_{\theta_0\theta_0}(\mathbf{0},0) - \chi_{\theta_0\theta_0}(\mathbf{r},t)] , \end{aligned} \tag{5.3}$$

where [using (4.15)]

$$\chi_{\theta_0\theta_0}(\mathbf{r},t) = \frac{k_B T}{J_0} \int d\mathbf{q} \frac{e^{-i\mathbf{q}\cdot\mathbf{r}}}{q^2} \cos(c_0 q t) . \tag{5.4}$$

Using these results in (5.1), a lengthy but well-documented calculation[8–10] shows that $S(\mathbf{q},\omega)$ exhibits a peak at the bare spin-wave frequency $\omega = c_0 q$ with a characteristic power-law exponent

$$S(\mathbf{q},\omega) \sim \frac{1}{(\omega^2 - c_0^2 q^2)^{1-\eta_0}} . \tag{5.5}$$

The exponent $\eta_0 = (2\pi K_0)^{-1}$ is the same one which occurs in the static spin response function given in (1.5).

In considering $S(\mathbf{q},\omega)$ using the full Hamiltonian (2.4), there may be new dynamical structure arising from the presence of vortices in their own right. However, we limit ourselves to understanding how the spin-wave resonance exhibited by $S(\mathbf{q},\omega)$ in (5.5) is renormalized by the coupling to the bound vortex pairs. This part will still be given by

$$\langle \cos[\theta_0(\mathbf{r},t)-\theta_0(\mathbf{0},0)] \rangle_R$$

but now the thermal average and the dynamics will be controlled by the renormalized Hamiltonian (4.14) instead of (5.2). Since (4.14) is a quadratic form, (5.3) is still correct but now (5.4) is replaced by

$$\chi_{\theta_0\theta_0}(\mathbf{r},t) = k_B T \int d\mathbf{q} \frac{e^{-i\mathbf{q}\cdot\mathbf{r}}}{q^2} \frac{\cos(cqt)}{J_T(\mathbf{q},cq)} , \tag{5.6}$$

where c and $J_T(q,\omega)$ are defined in (4.8) and (4.9), respectively. With this result, it is easy to see that the spin-wave resonance in $S(\mathbf{q},\omega)$ will be described by

$$S(\mathbf{q},\omega) \sim \frac{1}{(\omega^2 - c^2 q^2)^{1-\eta_R}} , \tag{5.7}$$

where in the long-wavelength limit $\eta_R = (2\pi K_R)^{-1}$ and[24]

$$c = \left[\frac{k_B T K_R}{\alpha} \right]^{1/2} , \tag{5.8}$$

where K_R is defined by (1.7).

In Sec. IV, we emphasized how the bound vortex pairs led to renormalization of θ_0 to θ. One might be tempted to use

$$\langle \cos[\theta(r,t)-\theta(0,0)] \rangle_R \tag{5.9}$$

in calculating the spin-wave resonance in $S(\mathbf{q},\omega)$ but this would be quite incorrect. The effect of the bound pairs is to modify the interaction between the θ_0 variables, not to alter their definition. To use an analogy, consider a system of interacting atoms. The full Hamiltonian may be diagonalized by the introduction of new quasiparticle excitations. However, this does not alter the fact that physical observables are given in terms of the original atoms.

We note that the equal-time (static) correlation function is easily obtained from (5.6), namely,

$$\begin{aligned} \lim_{q\to 0} \langle \theta_0\theta_0 \rangle_R(\mathbf{q},t=0) &= \lim_{q\to 0} \frac{k_B T}{\pi q^2 J_T(q,cq)} \\ &= \frac{1}{\pi q^2 K_R} . \end{aligned} \tag{5.10}$$

This is in perfect agreement with the classic Kosterlitz-

Thouless result, although the method of calculation is somewhat different. In the usual approach reviewed in Sec. I, the static correlation function is calculated using the decomposition

$$\lim_{q\to 0}\langle\phi\phi\rangle=\langle\theta_0\theta_0\rangle_{\rm SW}+\langle\psi\psi\rangle_v =\frac{1}{\pi q^2 K_0}+\frac{1}{\pi q^2 K_V}=\frac{1}{\pi q^2 K_R} . \quad (5.11)$$

The spin-wave and vortex contributions are completely decoupled. In contrast, the average in (5.10) involves the spin-wave Hamiltonian with an exchange constant renormalized by the bound vortex pairs. Recalling our discussion of the superfluid density in Sec. IV, we see that (5.10) and (5.11) are equivalent to (4.22) and (4.21), respectively. The discussion there is relevant in the present context.

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