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Theory of the microwave-soliton or antisoliton interaction in Josephson tunnel junctions

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Recently, microwave-induced harmonic and subharmonic structures were observed on the resistive branches associated with soliton or antisoliton propagations in a long Josephson tunnel junction. We have calculated the microwave-power dependence of the induced step-structure heights using a perturbation method. It is found that for different steps, the power dependences are different. Furthermore, for a given step, the height as a function of the microwave power also depends upon the nature of the coupling of the microwaves to the junction which can be either electric or magnetic.

I. INTRODUCTION

Microwave-induced current steps in the current-voltage (I-V) characteristics of short Josephson junctions have been observed in experiments as well as computer simulations.[1] A recent experiment[2] has also revealed step structures on the resistive branches of the I-V curves in long Josephson tunnel junctions. They occur at voltages given by

$$V_{n/m}=\frac{n}{m}\frac{h\nu}{2e} \tag{1}$$

with ν being the microwave frequency, h the Planck constant, e the magnitude of the electronic charge, and n and m positive integers. It is well known that in long junctions, the resistive branches are associated with soliton or antisoliton (vortex or antivortex) propagations.[3] For short junctions in zero applied magnetic field, these branches do not appear because a soliton has a finite size of order $2J\lambda_J$, with λ_J being the Josephson penetration depth[4] typically one-tenth of a millimeter *and J the critical current density.* Therefore, the physical mechanism leading to the step structures in long and short junctions are different. It was suggested that for long junctions, *these steps* are due to the synchronous coupling between the oscillatory soliton or antisoliton and the microwaves.[2] This can be understood as follows. Synchronous coupling occurs when the soliton or antisoliton makes n' round trips across the junction during the time interval the microwaves oscillate m times, thus the condition $m\nu_v=n'\nu$. Since the dc voltage of the first resistive branch (associated with the motion of a single soliton or antisoliton) is related to the frequency of the soliton or antisoliton ν_v by

$$V=\frac{h}{2e}(2\nu_v)\ , \tag{2}$$

the relationship between V and ν *is* given by Eq. (1) except for the replacement of n by $2n'$ on the right-hand side.[5]

In addition to the positions of the step structure, the dependences of the step heights on the microwave power for some of the steps have also been measured.[2] The largest step structure is found for the $\nu_v=\frac{1}{2}\nu$ step. It has a square-root power dependence. This is an interesting result because the largest structure does *not* correspond to the resonant condition. Furthermore, there are subharmonic as well as harmonic structures. To gain insight into the coupling process and to check the validity of the synchronous-coupling idea, we have calculated the power dependence of the step heights for several steps. We use the model first developed by McLaughlin and Scott[6] to describe the soliton or antisoliton propagations and perform perturbative calculation using the microwave power as the expansion parameter. It is found that for different steps the power dependences of the step heights are different. Furthermore, for a given step, *the height* depends on the nature of the microwave-soliton coupling. Wherever a comparison between theory and experiment is possible, the agreement is good.

II. CALCULATION

We consider long overlap junctions shown in Fig. 1 with length $L\gg\lambda_J$. The microwave frequency is taken to be much smaller than the plasma frequency $\nu_p=c/(2\pi\lambda_J)$. Here c is the speed of the electromagnetic wave in the junction. Therefore, the electromagnetic field of the microwaves cannot penetrate deeply into the junction, and the soliton- or antisoliton-microwave interaction occurs only near the junction edges. Due to the Meissner effect,[7] the interaction of soliton or antisoliton and the dc bias current *also occurs* near the edges as has been substantiated by recent theoretical and experimental studies.[8]

Following Ref. 6, we treat the soliton or antisoliton as a point particle and use the equation of motion

$$\frac{dP}{dt}=-\alpha P \tag{3}$$

to describe its propagation inside the junction. Here α is a damping parameter representing the energy loss associated with the quasiparticle current, and $P=8U\gamma(U)$ is the soliton or antisoliton momentum normalized by $(\hbar/2e)J\lambda_J/c$, where U is the soliton or antisoliton speed in units of c and $\gamma(U)=(1-U^2)^{-1/2}$ is the Lorentz factor. Moving inside the junction, the soliton or antisoliton loses its speed and energy. However, they are replenished at the

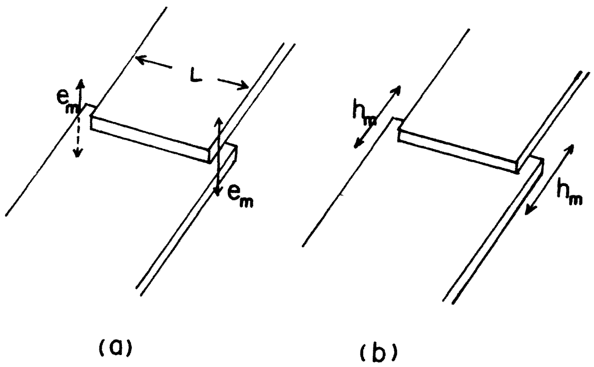

FIG. 1. Schematic drawings of the overlap junctions and their couplings with the microwave field. (a) Electric coupling. (b) Magnetic coupling.

junction edges. A periodic steady state can be established and an example is shown schematically in Fig. 2 for the case without microwaves. To obtain the junction voltage, one calculates the frequency of the soliton $\nu_v=[(t_2-t_1)+(t_4-t_3)]^{-1}$. The time of flights can be obtained from Eq. (3). One has

$$\alpha(t_j-t_i)=\tfrac{1}{2}\ln\left[\frac{\sinh a_i}{\sinh a_j}\right] \text{ for } (i,j)=(1,2),(3,4) \quad (4)$$

with $a\equiv\cosh^{-1}[\gamma(U)]$. The four unknown a's can be calculated from the boundary conditions which take into account the junction length:

$$\alpha L=a_i-a_j \text{ for } (i,j)=(1,2),(3,4) \quad (5)$$

and the energy input at the edges:

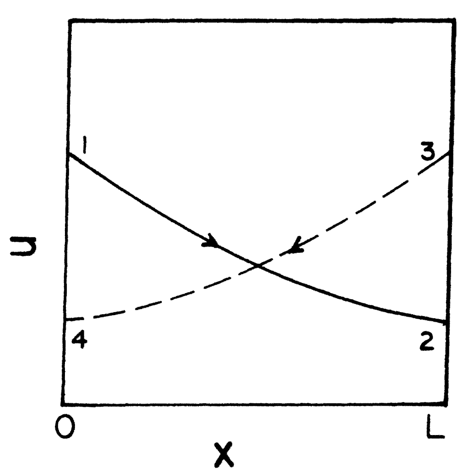

FIG. 2. The spatial dependence of the soliton or antisoliton speed for the periodic motion of the soliton or antisoliton in a Josephson junction of length L. At time t_1 a soliton moves from $x=0$ to the right (solid curve) and reaches $x=L$ at t_2. It is then reflected from the junction edge as an antisoliton at $t_3=t_2$ and moves to the left (dashed curve). After reaching $x=0$ at time t_4 it is reflected again as a soliton to start a new cycle.

$$8\gamma(U_i)-8\gamma(U_j)=4\pi(2I\pm H) \text{ for } (i,j)=(1,4),(3,2)\ . \quad (6)$$

Here I is the injection current at *each* junction edge measured in units of $2J\lambda_J$. By symmetry, the injection current at the $x=0$ edge has the same value as that at $x=L$ and is equal to $I_{dc}^0/2$, one-half of the total dc bias current. Here the superscript 0 represents the case of microwave power being zero. H is the applied magnetic field in units of $(4\pi/c)J\lambda_J$. In the presence of the microwaves, parameters I and H may be modified differently for the edges at $x=0$ and L, and will be represented by $I_{0,L}$ and $H_{0,L}$, respectively. How these parameters are modified depends upon the orientation and the position of the junction in the microwave cavity. We consider here two situations. General situations can be obtained as a combination of these two. The calculation is greatly simplified by observing the fact that the period of the microwave oscillation is much longer than the time spent by the soliton or antisoliton at the junction edges where the interaction takes place. This means that the soliton or antisoliton experiences an effective dc field, although the magnitude of the field can be different at the two edges. As soon as we have calculated the effective field, we can use the results obtained previously[6] to extract the I-V characteristics and hence the induced-current step heights.

A. Electric coupling

As shown in Fig. 1(a), this is the situation where the microwaves contribute an ac electric field across the junction edges. The junction is small compared with the wavelength of the microwaves so that the ac fields at the two edges are in phase. There is no magnetic field affecting the soliton motion. The electric field induces an ac current to flow,[9] which then modifies the energy gain of the soliton or antisoliton when it collides with the junction edges. This changes the I-V characteristics and current steps appear. In the followings, we consider various steps separately. For simplicity, the dc applied magnetic field is set to zero and hence $H_{0,L}=0$.

1. The V_1 step ($\nu_v=\frac{1}{2}\nu$ and $n/m=1$)

For voltage set at $V_1=h\nu/2e$, the fundamental frequency of the combined microwaves and soliton or antisoliton system is $\nu_v=\frac{1}{2}\nu$. The motion of the soliton or antisoliton during the time interval of one period is shown in Fig. 3(a), essentially the same as Fig. 2. Using the notation in Fig. 3(a), one has

$$I_{L,0}=\frac{I_{dc}}{2}+i_m\sin(2\nu t_{2,4}+\phi_0)\ . \quad (7)$$

Here I_{dc} is the dc bias current of the junction and can be different from the unperturbed value I_{dc}^0. We emphasize that *in one period ac currents affect the soliton motion only at times t_2 and t_4*. These values depend on phase ϕ_0. As an example, they are indicated by arrows in Fig. 3(b). Evidently $\nu t_4=2$. Therefore we need to solve the same set of equations [Eqs. (4)–(6)] using (I_0,H_0) to replace (I,H) in Eq. (6) for $(i,j)=(1,4)$. For $(i,j)=(3,2)$ we use

(I_L, H_L). The situation seems complicated. However, we can put the right-hand side into the form:

$$2I_0 + H_0 = 2I + H \tag{8a}$$

and

$$2I_L - H_L = 2I - H \tag{8b}$$

with

$$2I = I_{dc} + i_m[\sin(2\pi\nu t_2 + \phi_0) + \sin\phi_0] \tag{8c}$$

and

$$H = -i_m[\sin(2\pi\nu t_2 + \phi_0) - \sin\phi_0] \ . \tag{8d}$$

Thus the situation is as if the junction were not in an ac microwave field but an effective dc magnetic field H and were biased at an effective current $2I$. Note that I_{dc} is the actual (measured) dc bias current and can be different from the unperturbed value I^0_{dc}. It is evident that the leading term of correction for $2I$ is $O((i_m)^1)$ while that for H is $O((i_m)^2)$ because $\nu t_2 = 1 + O((i_m)^1)$. Therefore to order $(i_m)^1$, the magnetic field can be ignored and $2I = I^0_{dc}$. We obtain from Eq. (8c)

$$I_{dc} = I^0_{dc} - 2i_m \sin\phi_0 \ . \tag{9}$$

The difference of the maximum and minimum values of I_{dc} can be calculated from Eq. (9) to give the step height of the V_1 step:

$$(\Delta I_{dc})_1 = 4i_m \sim \sqrt{P} \ . \tag{10}$$

It has a square-root power dependence. Furthermore, the unperturbed I-V curve will pass through the center of the current step.

For the case of large microwave power, one cannot use the perturbative method. However, from Eqs. (4)—(8), one obtains the following equations relating V_1, I, and H:

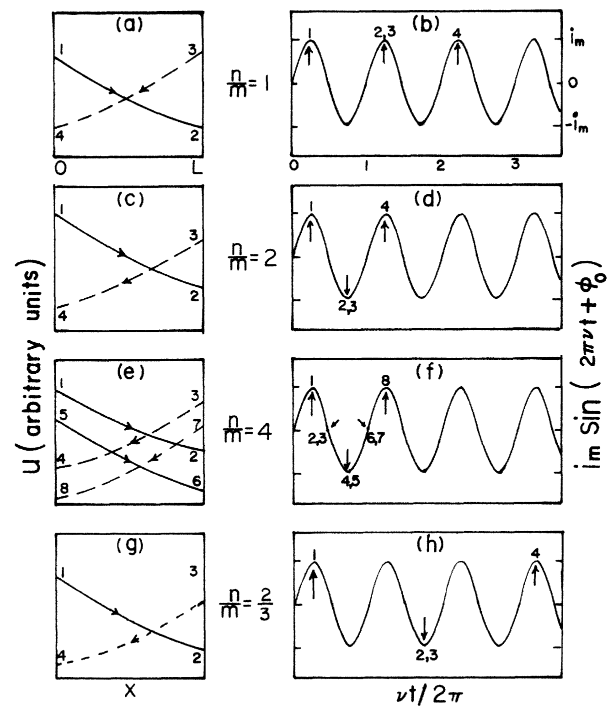

FIG. 3. Left-hand figures: the spatial dependences of the soliton or antisoliton speed in the time interval of one period for steps of $n/m = 1$, 2, 4, and $\frac{2}{3}$. Solid (dashed) curves represent soliton (antisoliton) propagations. Right-hand figures: microwave-induced ac current at the junction edges. The arrows and the associated number K indicate the current value at time t_K when the soliton or antisoliton reaches one of the junction edges. We have arbitrarily chosen ϕ_0 so that $2\pi\nu t_1 + \phi_0 = \pi/2$.

$$e^{4\pi\alpha/V} = \frac{[\sqrt{A-B}\cosh(\alpha L/2) + \sqrt{1+A-B}\sinh(\alpha L/2)]^2 - B/(A-B)}{[\sqrt{A-B}\cosh(\alpha L/2) - \sqrt{1+A-B}\sinh(\alpha L/2)]^2 - B/(A-B)} \tag{11}$$

with

$$A \equiv [-I/2\sinh(\alpha L/2)]^2$$

and

$$B \equiv [\pi H/4\cosh(\alpha L/2)]^2 \ .$$

To obtain the current-voltage relationship, one has to find t_2 which can be shown to satisfy

$$e^{\alpha t_2} = \frac{[(\sqrt{A} - \sqrt{B})/(\sqrt{A-B})]^2(\sqrt{A-B} + \sqrt{1+A-B})^2 - e^{-\alpha L}}{[(\sqrt{A} - \sqrt{B})/(\sqrt{A-B})]^2(\sqrt{A-B} - \sqrt{1+A-B})^2 e^{-\alpha L} - 1} \ . \tag{12}$$

It is easy to show that the unperturbed values of $t_2 = \nu^{-1}$ and $H = 0$ satisfy Eqs. (11) and (12) as well as Eq. (8b) provided we choose $2I = I^0_{dc}$. This means that Eq. (10) is valid even for strong microwave power as long as the assumption that the soliton or antisoliton experiences a dc field holds.

2. *The V_2 step ($\nu_v = \nu$ and $n/m = 2$)*

In this case, the soliton or antisoliton motion is in *resonance* with the microwave oscillation. The variation of the speed and the ac currents experienced by the soliton or antisoliton in one period are shown schematically in Figs. 3(c) and 3(d). Since, as shown, the soliton or antisoliton experiences a current of different directions at the two edges of the junction, the symmetry of the soliton or antisoliton propagation is broken. To calculate the step height we can still use Eqs. (11) and (12) except for the replacement of V_1 by $V_2 \equiv (h/2e)(2\nu)$. The parameters $2I$ and H are still given by Eq. (8b). However, in this cases the period is $t_4 = \nu^{-1}$ and hence $\nu t_2 = \frac{1}{2} + O((i_m)^1)$. This gives $I = I_{dc}/2 + O((i_m)^2)$ and $H = 2i_m \sin\phi_0$. Therefore, the dominant effect of the ac microwave electric field is to give an effective dc magnetic field. Since a small dc magnetic field is known to give the current of the resistive branch a modification proportional to H^2, one expects the induced step height to be of the same order and hence to have a linear power dependence.

We have calculated the modified current at V_2 to the leading order $(i_m)^2$ and find

$$2I = 2I_{dc}^0 + \pi i_m \nu \delta t_2 \cos\phi_0 + \tfrac{1}{2}[H \tanh(\alpha L/2)]^2 \left[1 + \frac{2(1+A_0)}{A_0\cosh^2(\alpha L/2) - (1+A_0)\sinh^2(\alpha L/2)}\right] + O((i_m)^3) \tag{13}$$

with

$$\delta t_2 = \frac{i_m}{\alpha} |\sin\phi_0| \frac{(A_0)^{1/2}}{\cosh(\alpha L/2)} [1+(1+1/A_0)^{1/2}]^2 \times \left[\frac{1}{[(A_0)^{1/2}+(1+A_0)^{1/2}]^2 - e^{-\alpha L}} - \frac{1}{[(A_0)^{1/2}+(1+A_0)^{1/2}]^2 - e^{\alpha L}}\right] + O((i_m)^3) . \tag{14}$$

Here A_0 is the unperturbed value for A. The second term is due to the modified t_2 and contributes a leading correction to $2I$ of the form $(i_m)^2 |\sin\phi_0| \cos\phi_0$. This term, acting alone, would induce a current step with the unperturbed I-V curve passing through the center of the step as in the previous case. On the other hand, the third term, associated with the effective dc magnetic field, contributes a leading correction proportional to $(i_m \sin\phi_0)^2$. Therefore, the modification of the junction current due to this term alone, can be either positive or negative depending on the sign of the term within large parentheses in Eq. (13). Hence, in general, the current steps above and below the unperturbed current value are not the same as long as the contribution of the effective field is significant. However, the step size should have a linear power $(i_m)^2$ dependence for low microwave power.

3. *The V_4 step ($\nu_v = 2\nu$ and $n/m = 4$)*

In this case the fundamental frequency of the combined system of the soliton or antisoliton and the microwaves is ν. During one period, the vortex makes *two* round trips across the junction and one has to consider the complicated soliton or antisoliton motion shown in Figs. 3(e) an 3(f). A perturbation method similar to that for the previous cases can be employed, although the calculation becomes very complicated. It was found that the dc current of the junction is modified by a value proportional to $(i_m)^2$. However, the current-step height has a dependence of $(i_m)^3$ and hence a $P^{3/2}$ power dependence.

4. *The $V_{2/k}$ steps ($\nu_v = \nu/k$ and $n/m = 2/k$)*

It is possible to obtain information about other steps with the results obtained above. For *example*, consider steps resulted from soliton or antisoliton moving slowly so that the synchronous coupling condition $\nu_v = k^{-1}\nu$ holds $(n/m = 2/k)$. Here k is an integer. If k is even, we have a situation similar to case (i), where the soliton or antisoliton experiences a microwave field of the same strength and direction at the two edges. If k is odd, we have a situation similar to case (ii) where the directions are different and the symmetry of the soliton or antisoliton motion is broken [see, for example, Figs. 3(g) and 3(h)]. The step *heights* therefore *have* the same power dependence as those calculated before; namely $(i_m)^1$ for steps with even k's and $(i_m)^2$ for those with odd k's. These are subharmonic steps. Here a word of caution is in order. When k is large, the motion of the soliton or antisoliton is very slow, the time *it spends* near the junction edge may be comparable with the period of the microwave oscillation. If this is the case, the assumption of the soliton or antisoliton experiencing a time-independent field during its collision with the edges is no longer valid, and the result given above has to be modified.

B. Magnetic coupling

We now discuss the junction configuration shown in Fig. 1(b). The junction is coupled to the microwaves through the oscillatory magnetic field. Again, because of the long microwave wavelength, the ac magnetic field is uniform over the junction. No electric field is present to affect the soliton or antisoliton motion. Therefore, we have $I_{0,L} = I_{dc}/2$.

1. *The V_1 step ($\nu_v = \frac{1}{2}\nu$ and $n/m = 1$)*

We remind ourselves that in this case the soliton makes one round trip across the junction while the microwaves oscillate twice during the time interval of one period. The effective magnetic field is given by

$$H_{L,0} = h_m \sin(2\pi\nu t_{2,4} + \phi_0) \tag{15}$$

with h_m and ϕ_0 being the amplitude and phase of the ac field, respectively, and $\nu t_4 = 2$. Again, the relevant quantities are $2I_0 + H_0$ and $2I_L - H_L$ and they can be put into the forms given by Eqs. (8a) and (8b) with the effective junction current and applied magnetic field given by

$$2I = I_{dc} + \tfrac{1}{2} h_m[\sin\phi_0 - \sin(2\pi\nu t_2 + \phi_0)] \tag{16a}$$

and

$$H = \tfrac{1}{2} h_m[\sin\phi_0 + \sin(2\pi\nu t_2 + \phi_0)] \ . \tag{16b}$$

For low microwave powers, $\nu t_2 = 1 + O(h_m)$ thus $2I = I_{dc} + O((h_m)^2)$ and $H = h_m \sin\phi_0 + O((h_m)^2)$. The situation is similar to case (ii) of the electric couplings where the junction is as if in an effective dc magnetic field.[10] Therefore we anticipate a $(h_m)^2$ dependence of the step height. The perturbation calculation gives

$$(\Delta I_{dc})_1 = (2I - 2I_{dc})_1 = [(\pi/2) h_m \nu \delta t_2]\cos\phi_0 + \tfrac{1}{2}[H \tanh(\alpha L/2)]^2 \left[1 + \frac{2(1+A_0)}{A_0 \cosh^2(\alpha L/2) - (1+A_0)\sinh^2(\alpha L/2)} \right] + O((h_m)^3) \tag{17}$$

with $H = h_m \sin\phi_0$ and δt_2 given by Eq. (14) except for the replacement of i_m by $h_m/2$. Therefore $(\Delta I_{dc})_1$ has a linear power dependence and the unperturbed resistive branch should not pass the center of the microwave-induced current step at V_1 in general.

2. The V_2 step ($\nu_v = \nu$ and $n/m = 2$)

This is the resonance case. When the microwaves oscillate once, the soliton or antisoliton makes one round trip. Equations (15) and (16a) again apply. However, we have $\nu t_4 = 1$ and thus for low microwave powers (where $\nu t_2 \cong \frac{1}{2}$)

$$2I = I_{dc} + h_m \sin\phi_0 + O((h_m)^2) \tag{18a}$$

and

$$H = O((h_m)^2) \ . \tag{18b}$$

This case is similar to case (i) of electric couplings. We therefore anticipate a large step with a square-root power dependence of the step height. Detailed calculation shows that this is indeed the case and the step height is given by

$$(\Delta I_{dc})_2 = 2h_m \ . \tag{19}$$

We there have a means to measure the local amplitude of the microwave field at the junction edges.

3. The V_4 step ($\nu_v = 2\nu$ and $n/m = 4$)

Again, for this step we found that the junction current is modified by the microwaves by a value proportional to $(h_m)^2$ and the step height is proportional to $(h_m)^3$. Properties of this step do not depend on the nature of the coupling.

4. The $V_{2/k}$ steps ($\nu_v = \nu/k$ and $n/m = 2/k$)

Since $\nu_v t_4 = 1$, we have $\nu_v t_2 = \frac{1}{2} + O(h_m)$ and thus $\nu t_2 = k/2 + O(h_m)$. Therefore if k is even, the situation is similar to that of $n/m = 1$, and the junction is as if in a dc magnetic field. The step height has a linear power dependence [i.e., $\Delta I_{dc} \sim (h_m)^2$]. On the other hand, if k is odd, the situation is similar to that of $n/m = 2$, and the junction is as if biased by an additional current of $h_m \sin\phi_0$. The step height therefore has a square-root power dependence (i.e., $\Delta I_{dc} \sim h_m \sim \sqrt{P}$).

III. DISCUSSION AND CONCLUSION

The model for the soliton or antisoliton propagation in Josephson junctions developed by McLaughlin and Scott has been extended to investigate the microwave-induced harmonic and subharmonic current steps in the current-voltage characteristics. It was shown that because of the fact that the microwave-soliton or -antisoliton interaction occurs at the junction edges, the effect of the ac microwave field can be represented by a dc perturbation if the soliton or antisoliton moves fast enough so that the time it spends near the junction edges is smaller than the microwave period. This observation simplifies the calculation greatly.

Before we compare our results with the experimental observation, we briefly summarize our findings. We found that the size of the microwave-induced current step depends upon the nature of the coupling. For electric couplings and low microwave power, we found the following.

(i) The $\nu_v = \frac{1}{2}\nu$ ($n/m = 1$) step is the largest; its height has a square-root power dependence, and the unperturbed *I-V* curve passes through the center of the step.

(ii) The $\nu_v = \nu$ ($n/m = 2$) step has a linear power dependence, and the unperturbed curve, in general, does not go through the center of the step.

(iii) The $\nu_v = 2\nu$ ($n/m = 4$) step has a $p^{3/2}$ power dependence for the step height. However, the step is shifted from the unperturbed current value by an amount proportional to P.

(iv) The subharmonic steps of $\nu_v = \nu/k$ are similar to that of (ii) if k is an odd integer and (i) if k is an even integer.

For magnetic couplings, the situation is different. In the cases of low microwave power, we found the following.

(i) The $\nu_v = \frac{1}{2}\nu$ step has a linear power dependence, and the unperturbed *I-V* curves does not, in general, pass through the center of the step in a way similar to case (ii) of the electric coupling.

(ii) The $\nu_v = \nu$ step is the largest, its height has a square-root power dependence, and the unperturbed *I-V* curve passes through the center of the step in a way similar to case (i) of the electric coupling.

(iii) The $\nu_v = 2\nu$ step has the same behavior regardless of the nature of the coupling.

(iv) The $\nu_v = \nu/k$ steps have a square-root power dependence for the step height if k is an odd integer. When k

is an even integer, the step height depends on power linearly with properties similar to that of case (i).

Experiments were done on junctions in the electric-coupling situation only. All the features of the $\nu_v=\frac{1}{2}\nu$ $(n/m=1)$ step listed agree excellently with the observation, except for the cases of high microwave powers. In these cases, the microwaves induce an instability at the lower end of the current step leading to a switch causing the step size below the unperturbed curve to be smaller than that above the unperturbed curve. This instability is not considered in our model and should be interesting to investigate. As for the $\nu_v=\nu$ $(n/m=2)$ step, experiment shows that the unperturbed I-V curve passes through the lower end of the current step, in agreement with our result for a junction with $A \gg 1$. There is no detailed power dependence recorded for this step. The $\nu_v=\nu/3$ $(n/m=\frac{2}{3})$ step shows features similar to that of the $\nu_v=\nu$ step as expected. Finally, the only other step covered by our calculation and studied experimentally is the $\nu_v=\nu/4$ step $(n/m=\frac{1}{2})$. Unfortunately, this step is not sharp. It also appears to suffer from the same prematured switch induced by the strong microwave power discussed above. We believe that these features are indications that the soliton or antisoliton is no longer affected by dc-field representation of the microwaves. A comparison with the calculated result is therefore unwarranted.

In summary, we have found that the microwave-induced current steps on the resistive branch of a long junction can be understood by the synchronous couplings of the soliton or antisoliton motion and the microwave oscillation. The properties of the current steps depends on the nature of the coupling. Furthermore, using microwave coupling, one can obtain the local amplitude of the microwave at the junction edges by measuring the microwave-induced current-step height at voltage $V_2=h\nu/e$.

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