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Effective Landau theory for disordered interacting electron systems: Specific-heat behavior

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The frequency renormalization parameter of the generalized nonlinear σ model introduced to describe the interacting disordered electron system is identified in terms of the specific heat. This allows us to complete the effective Landau Fermi-liquid picture for this system and to give the asymptotic behavior of the electronic specific heat in the various universality classes of the metal-insulator transition.

The electron-electron interaction in a disordered medium has been found[1] to introduce relevant corrections to the Landau theory of the normal Fermi liquid. Many of these corrections are logarithmically divergent in two dimensions as the temperature T decreases to zero.[1] Since that discovery,[2] theorists have been looking for a renormalization-group approach which could sum the correction terms in the physical quantities to obtain power-law behaviors near the metal-insulator transition, at least in the ε expansion ($\varepsilon = d-2$, d being the dimensionality).

Theorists have had to face the problem of deriving the renormalization group equations for the couplings describing the electron-electron interaction in the different channels.[3] This problem was first solved by Finkelstein[4] in the simplified case where particle-particle (hole-hole) channels are suppressed, so that only the singlet (Γ_s) and triplet (Γ_t) interaction amplitudes must be considered.

Finkelstein[4] mapped the interacting disordered electron system into an effective nonlinear σ model. As in the standard weak-localization regime,[1] the expansion parameter of the model is the dimensionless inverse conductivity $t = \Lambda^{\varepsilon}/(2\pi)^2 \nu_0 D$, where D is the diffusion coefficient, ν_0 is the bare density of states, and Λ is the ultraviolet cutoff of order $(D_0\tau_0)^{-1/2}$, τ_0 being the bare scattering time. Together with Γ_s, Γ_t, and t, the effective nonlinear σ model is specified by the frequency or temperature renormalization parameter Z which is needed to take care, in a consistent way, of the corrections introduced by the electron-electron interaction in the diffusive mode (the ladder in the particle-hole channels).[4,5]

We shall show in this report that, at least to lowest order in ε and to one-loop expansion, Z is related to the electronic specific-heat renormalization induced by the interaction in the presence of disorder

$$Z = \frac{\gamma}{\gamma_0}, \quad c_V = \gamma T \ , \tag{1}$$

where $\gamma_0 = (2\pi^2 \nu_0/3)$ is the coefficient of the linear term of the Landau specific heat in the absence of disorder ($c_V^L = \gamma_0 T$). This completes the identification of the renormalization parameters of the effective nonlinear σ model in terms of physical quantities. It has, in fact, been shown[4-8] that, while Z renormalizes the frequency in the diffusive mode, the two combinations $Z_1 = Z - 2\nu_0(\Gamma_s - \Gamma_0)$ (Γ_0 being the static screened Coulomb amplitude) and $Z_2 = Z + \nu_0\Gamma_t$ renormalize the frequency associated with the density and the spin-fluctuation modes, respectively. They can be expressed in terms of the thermodynamic density of states $\partial n/\partial\mu$ and the spin susceptibility χ:

$$Z_1 = \frac{1}{\nu_0}\frac{\partial n}{\partial \mu}, \quad Z_2 = \frac{\chi}{\chi_0} \ , \tag{2}$$

where $\chi_0 = \mu_B^2 \nu_0/2$ is the Pauli susceptibility. If one substitutes Z_1 and Z_2 in Eq. (2) with their bare values $Z_1^0 = 1 - 2\nu_0(\Gamma_s^0 - \Gamma_0^0)$, $Z_2^0 = 1 + \nu_0\Gamma_t^0$ one recovers the standard Landau expressions $(\partial n/\partial\mu)^L$ and χ^L for $\partial n/\partial\mu$ and χ in the absence of disorder. According to the analysis carried out in Refs. 4–8, Z_1 remains unrenormalized and equal to its initial value Z_1^0. Z_2 instead is strongly renormalized, leading to a pronounced spin susceptibility enhancement in the nonmagnetic impurity case.

The problem of disordered interacting electrons may therefore be modeled in terms of a "renormalized" Landau theory

$$c_V = Z c_V^L, \quad \chi = \frac{Z_2}{Z_2^0}\chi^L, \quad \frac{\partial n}{\partial \mu} = \left(\frac{\partial n}{\partial \mu}\right)^L \ , \tag{3}$$

where the renormalization parameters Z and Z_2 are expressed in terms of the renormalized couplings of Finkelstein's nonlinear σ model[4] and can be analyzed by the renormalization-group approach.

To identify γ/γ_0 with Z we shall first evaluate the leading corrections to c_V in terms of the renormalized parameters of the effective nonlinear σ model to lowest order in t, thus generalizing the perturbative results of Ref. 9. For this purpose we use the standard procedure of multiplying the interaction amplitudes appearing in the Lagrangian of the effective nonlinear σ model[4] by a parameter η. By integrating the derivative with respect to η of the logarithm of the grand partition function in the interval (0,1), we obtain the variation of the thermodynamic potential per unit volume. In the one-loop approximation (i.e., to lowest order in the disorder) we have

$$\frac{\Delta F}{V}=-T\sum_m\int\frac{d^dq}{(2\pi)^2}\int_0^1 d\eta\left[\frac{v_0\Gamma_s\,|\,\omega_m\,|}{|\,\omega_m\,|\,(Z-2\eta v_0\Gamma_s)+Dq^2}-\tfrac{3}{2}v_0\Gamma_t\frac{|\,\omega_m\,|}{|\,\omega_m\,|\,(Z+\eta v_0\Gamma_t)+Dq^2}\right]$$

$$=-T\sum_m\int\frac{d^dq}{(2\pi)^d}\{2\ln(Z\,|\,\omega_m\,|+Dq^2)-\tfrac{3}{2}\ln[(Z+v_0\Gamma_t)\,|\,\omega_m\,|+Dq^2]-\tfrac{1}{2}\ln(Dq^2)\}\ , \tag{4}$$

where we have used the condition $Z-2v_0\Gamma_s=0$ which was shown to hold in the case of long-range forces.[4] The last term in the curly bracket of the second expression does not contribute to the specific heat. The sum over the Matsubara frequency can be carried out by a contour integration via the Bose function. The resulting expression for $\Delta F/V$ in d dimensions agrees with the perturbative result of Ref. 9 when the combination of their bare singlet and triplet couplings

$$\lambda^{j=0}+3\lambda^{j=1}=4/d\,[4-3(1+\Gamma_t^0)^{d/2}]$$

is replaced by the renormalized quantity $4/d\,[4Z^{d/2}-3Z_2^{d/2}]$. At $d=2$ the leading logarithmic correction term for $\Delta\gamma/\gamma_0=-(1/\gamma_0)(\partial^2\Delta F/\partial T^2)$ is given by

$$\frac{\Delta\gamma}{\gamma_0}=t\,(2Z-\tfrac{3}{2}Z_2)\ln(T\tau_0)=t\,(v_0\Gamma_s-\tfrac{3}{2}v_0\Gamma_t)\ln(T\tau_0)\ . \tag{5}$$

Equation (5) coincides with the correction ΔZ to Z_0 evaluated in Ref. 4 in the framework of the Wilson renormalization-group procedure (which integrates out the "fast" degrees of freedom in the region $\lambda^2\Lambda^2<q^2<\Lambda^2$, $\lambda^2\Lambda^2<Z\,|\,\omega_m\,|/D<\Lambda^2$) with the logarithm of the square of the rescaling parameter λ taking the place of $\ln(T\tau_0)$. To leading order, γ/γ_0 has therefore the same renormalization-group equation as Z:

$$\frac{\partial(\gamma/\gamma_0)}{\partial\ln(\lambda^2)}=\frac{\partial Z}{\partial\ln(\lambda^2)}=t\,[v_0\Gamma_s-\tfrac{3}{2}v_0\Gamma_t]\ , \tag{6}$$

where γ/γ_0, Z, Γ_s, Γ_t, and $t=\lambda^\epsilon\Lambda^\epsilon/(2\pi)^2D(\lambda)$ are intended to be functions of λ. Equation (1) then follows. Our analysis, which has been carried out in the absence of particle-particle channels, is correct to first order[10] in ϵ when a fixed point for t of order ϵ exists.[5,7,11,12] No weak-coupling assumption is made for the interaction couplings, provided they are finite. We suggest, however, that the result $\gamma/\gamma_0=Z$ has a more general validity and we shall use it to derive the asymptotic power-law behavior of the specific heat also when the interaction couplings scale to infinity (strong spin-fluctuation range, see below) or when singlet particle-particle channel must be considered (spin-orbit imputity scattering).

Equation (1) can be used to sum the leading corrections to γ (or c_V) whenever the renormalization-group analysis of the effective nonlinear σ model produces a scaling theory for $\lambda\to 0$. We recall that at finite temperature the elimination of the "fast" degrees of freedom in the renormalization-group procedure has to stop when $D\lambda^2\tau_0^{-1}$ is of order of ZT.[5,8] Since $D\sim\lambda^\epsilon$ at the metal-insulator transition,[1,5,7,11] and assuming that Z behaves as λ^{x_Z} for $\lambda\to 0$, one gets the following scaling relation:

$$T\sim\lambda^{x_T},\ x_T=d-x_Z\ , \tag{7}$$

which allows us to translate the power-law behavior as a function of λ to a power-law behavior as a function of T.

From $\gamma/\gamma_0=Z\sim\lambda^{x_Z}$ we now obtain

$$\gamma\sim T^{x_Z/x_T},\ c_V\sim T^{1+x_Z/x_T}=T^{d/x_T}\ . \tag{8}$$

This result is in agreement with the following simple scaling argument. As usual, the logarithm of the partition function is an invariant of the renormalization group. In ordinary critical phenomena the thermodynamic potential F, being multiplied by T around its critical value T_c, is also an invariant. By contrast, in the present case, the temperature T goes to zero and scales according to Eq. (7). F is no longer invariant. On the other hand, the specific heat per unit volume $c_V=-(T/V)(\partial^2F/\partial T^2)$ has an additional compensating T dependence and will, in this case, obey the same equation as the thermodynamic potential per unit volume in ordinary critical phenomena:

$$c_V(T;\{\mu\})=\lambda^d c_V(T/\lambda^{x_T};\{\mu_\lambda\})\ , \tag{9}$$

where Eq. (7) has been used for the scaling dimension of T, and $\{\mu_\lambda\}$ indicates the full set of the flowing couplings. Equation (8) follows by taking, in Eq. (9), $T=\lambda^{x_T}\tau_0^{-1}$, $c_V(\tau_0^{-1};\{\mu_\lambda\})$ being a well-defined finite quantity when evaluated at the fixed point $\{\mu^*\}$ of the group transformation. This supports the suggestion that Eq. (1) is valid quite generally.

In the noninteracting case $Z=Z_0=1$ and $x_z=0$; therefore the specific heat is linear in T. In this case γ remains unrenormalized and equal to γ_0. Quite different is the situation in the interacting system, where the combined effect of interaction and disorder leads to various cases which we are going to analyze.

We first recall that a bonafide metal-insulator transition is obtained whenever the triplet contribution is at least partially suppressed as in the physically relevant situations when magnetic impurity, external magnetic field, or spin-orbit interaction is present.[5,7,11,12]

(i) For the magnetic impurity case $x_Z=\epsilon/2$.[5,7] According to Eqs. (7) and (8), the specific-heat factor γ acquires, therefore, a temperature dependence and goes to zero at the transition as

$$\gamma\sim T^{\epsilon/(4+\epsilon)}\ . \tag{10}$$

(ii) When the magnetic field is large enough ($\mu_BH\gg k_BT$) to introduce a Zeeman splitting (and no magnetic impurities are present), a line of fixed points appears in $d=2+\epsilon$ with a finite value Z^* for Z.[5,7] In this case the coefficient γ of the specific heat is rescaled by a constant

$$\gamma = Z^* \gamma_0 \ . \tag{11}$$

We note that in the two previous cases the particle-particle channels are indeed irrelevant and our initial assumption of suppressing their contributions turns out to be appropriated.

(iii) In the spin-orbit impurity case the problem is more complex since the singlet particle-particle channel has also to be included. Assuming Eq. (8) is still valid, the specific-heat coefficient γ is again approaching zero at the transition, as $T \rightarrow 0$, since $Z^* = 0$.[11] At lowest order in ε the exponent x_Z is estimated to be ε and

$$\gamma \sim T^{\varepsilon/2} \ . \tag{12}$$

In the general case of nonmagnetic impurities the situation is complicated by the appearance of strong spin fluctuations. At $d = 2+\varepsilon$ an instability line $t_0 = t_c(\Gamma_t^0)$ is present.[8,13] Once again the presence of the particle-particle channel is not likely to modify the general content of the theory.[13,14] In the region of $t_0 < t_c$ the model system scales to a conductor as it should . As the instability line is reached, Z and Z_2 tend to infinity as $\lambda^{-3\varepsilon}$ and $\lambda^{-4\varepsilon}$, respectively, while the system remains metallic. The dimension of Z and Z_2 are in this case $x_Z = -3\varepsilon$, $x_{Z_2} = -4\varepsilon$. Since we stay in the metallic region, the scaling relation Eq. (7) for x_T has to be modified by substituting d (appropriated to the metal-insulator transition) with the ordinary bare dimension of T in terms of an inverse length, which is equal to two. It follows that at $t_0 = t_c$, χ and γ behave as

$$\begin{aligned} \chi &\sim T^{x_{Z_2}/x_T} = T^{-4\varepsilon/(2+3\varepsilon)} \ , \\ \gamma &\sim T^{x_Z/x_T} = T^{-3\varepsilon/(2+3\varepsilon)} \ . \end{aligned} \tag{13}$$

There are indications of an anomalous increase of the spin susceptibility[15,16] and of the specific-heat parameter γ (Ref. 17) in Si:P at low temperature. In Ref. 16 the spin susceptibility is analyzed at donor concentration $n = 1.09 n_c$ (n_c being the critical concentration at the metal-insulator transition) and shows an enhancement of a factor 10 in the range 1 K–30 mK, roughly in agreement with Eq. (13). The specific-heat data refer instead to a sample at $n = 1.6 n_c$ far from the metal-insulator transition. Experiments of χ and c_V at the same donor concentration would distinguish between the disorder-interaction-induced enhancement [which leads to Eq. (13)] and the enhancement coming from the Brinkman-Rice liquid picture.[18] In the former case, in fact, χ and γ renormalize differently, while in the latter case χ and γ are equally enhanced. By increasing the disorder in these systems, one should switch from a weak-localization regime, where the perturbative expressions for the physical quantities are valid, to the instability region where, after an initial enhancement of χ and γ according to Eq. (13), a crossover mechanism should take place.[6,8,13] Eventually the system should flow into one of the universality classes previously considered, where random magnetic moments or spin orbit coupling play a role and a bona fide metal-insulator transition is obtained.

In conclusion, the specific-heat parameter $\gamma = c_V/T$ should show an initial anomalous increase as $T \rightarrow 0$ in the case of nonmagnetic impurities, a temperature decreasing power-law behavior in T at critical concentration of the metal-insulator transition in systems with magnetic impurities or spin-orbit coupling, and remain finite when an external magnetic field is present.[19] The available data do not yet allow a full comparison with the predictions of the theory. More detailed experiments on the specific heat and on the spin susceptibility in the various materials will test the validity of the general content of the theory and allow for a more specific classification of the physical systems in different universality classes.

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