Disordered electron systems with Hubbard interaction

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(Received 24 February 1986)

A Hubbard system in the presence of disorder is considered. This extreme-short-range-force case is explicitly shown to behave as the ordinary short-range-force case of disordered interacting electron systems previously considered.

Spin fluctuations in interacting electron systems have been shown to be strongly enhanced by the presence of disorder.[1-3] This discovery led many authors[4-8] to reconsider the itinerant fermion model with a Hubbard-like contact repulsion U among electrons with opposite spins. Since the contact repulsion introduces some constraints on the spin structure of the interaction amplitudes, the disordered Hubbard model has been approached differently than the ordinary short-range-force model. This has somewhat masked the physical equivalence of the two models in both the weak- and strong-localization regimes. It is therefore worth showing explicitly the correspondence between the approach devised for dealing with disordered interacting electron systems with either long- or short-range forces[1,9,10] and the one specific to the Hubbard model.[4-8] We show here that the use of the contact repulsive interaction does not introduce new features and that the results previously obtained in general for the short-range forces[1,2,10] can be transferred to this extreme-short-range case.

In order to simplify our analysis we do not consider the effect of maximally crossed diagrams and of the Cooper channel. Within this simplified model,[9,10] in addition to the inverse conductance t_0, specifying the amount of disorder present in the system, the other two effective couplings due to the interaction are the singlet and the triplet interaction amplitudes Γ_S and Γ_T. They are related to the spin structure of the total two-particle interaction scattering amplitude in the particle-hole channel, in the small-k limit (Fig. 1):

$$\Gamma_{\alpha\beta\gamma\delta}(k)=\Gamma_S(k)\delta_{\alpha\delta}\delta_{\beta\gamma}+\Gamma_T(k)\sigma_{\alpha\delta}\cdot\sigma_{\beta\gamma}\ . \quad (1)$$

The p and $\tilde{p}$ dependence of the nonrelevant external momenta has been integrated out by averaging over the Fermi sphere. The small- and large-momentum-transfer amplitudes Γ_1 and Γ_2 were alternatively used in place of Γ_S and Γ_T, to which they are related by $\Gamma_S=\Gamma_1-\frac{1}{2}\Gamma_2$, $\Gamma_T=-\frac{1}{2}\Gamma_2$.

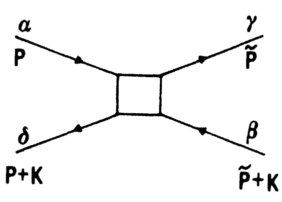

FIG. 1. Total two-particle interaction scattering amplitude. The small k limit individuates the particle-hole channel.

In the ladder approximation

$$\Gamma_S=\frac{U_1-U_2/2}{1+\Pi(2U_1-U_2)},\ \Gamma_T=-\frac{U_2/2}{1-\Pi U_2}\ , \quad (2)$$

where U_1 and U_2 are now the bare small and large momentum-transfer interactions, respectively, in the static limit.

In the presence of disorder, to zeroth order in t_0, the full bubble is given by

$$\Pi=\frac{N_0D_0q^2}{D_0q^2-i\omega}\ ,$$

where D_0 is the Drude diffusion constant and N_0 is the free single-particle density of states.

As $\omega\to 0$, one obtains from Eq. (2) the static interaction amplitudes in the ladder approximation

$$\Gamma_S=\frac{U_1-U_2/2}{1+N_0(2U_1-U_2)},\ \Gamma_T=-\frac{U_2/2}{1-N_0U_2}\ , \quad (3)$$

where in the Hubbard case $U_1=U_2=U$. For $\omega\neq 0$ the scattering amplitudes are instead dynamically dressed by impurities according to[9-11]

$$\Gamma_S(q,\omega)=\frac{\Gamma_S(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_S)\omega}\ ,$$
$$\Gamma_T(q,\omega)=\frac{\Gamma_T(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_T)\omega}\ . \quad (4)$$

Corrections appear in higher order in t_0.

In the Hubbard sysem a special role has been assigned to the strictly local nature of the interaction, which introduces constraints on the possible infinite resummations. This has led to consider the following three scattering amplitudes as effective couplings:[4-8]

(5)

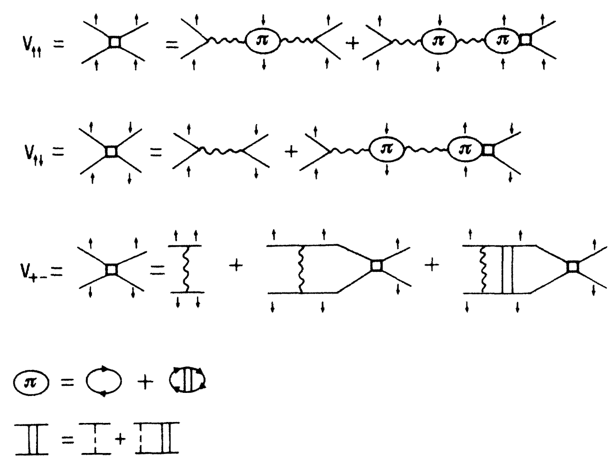

FIG. 2. Ladder resummations for the V's amplitudes. The polarization bubble and the diffusion propagator are also shown. Broken lines represent the scattering on impurity atoms.

In the ladder approximation the three previous amplitudes contain an odd and an even number of bubbles Π and an infinite interaction ladder resummation, respectively, as shown in Fig. 2,

$$V_{\uparrow\uparrow}=-\frac{U^2\Pi}{1-\Pi^2U^2},\quad V_{\uparrow\downarrow}=\frac{U}{1-\Pi^2U^2},\quad V_{+-}=-\frac{U}{1-\Pi U}\ . \tag{6}$$

In the same way as for the Γ's, the V's therefore acquire a frequency dependence via the polarization bubble $\Pi(q,\omega)$.

Actually, the introduction of the amplitudes V is not strictly related to the Hubbard model. They simply represent the total amplitude Γ in a different way with respect to Eq. (1):

$$\Gamma_{\alpha\beta\gamma\delta}(k)=V_{\uparrow\uparrow}(k)\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\alpha\beta}+V_{\uparrow\downarrow}(k)\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{\alpha,-\beta}+V_{+-}(k)\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{\alpha,-\delta}\ . \tag{7}$$

By identifying in Eqs. (1) and (7) the terms with the same spin structure we obtain the amplitudes $V_{\uparrow\uparrow}$, $V_{\uparrow\downarrow}$, $V_{\pm}$ in terms of the two independent amplitudes Γ_S and Γ_T.

$$\begin{aligned}
V_{\uparrow\uparrow}&=\Gamma_S\delta_{\alpha\alpha}\delta_{\alpha\alpha}+\Gamma_T\boldsymbol{\sigma}_{\alpha\alpha}\cdot\boldsymbol{\sigma}_{\alpha\alpha}=\Gamma_S+\Gamma_T=\Gamma_1-\Gamma_2\ ,\\
V_{\uparrow\downarrow}&=\Gamma_S\delta_{\alpha\alpha}\delta_{\beta\beta}+\Gamma_T\boldsymbol{\sigma}_{\alpha\alpha}\cdot\boldsymbol{\sigma}_{\beta\beta}=\Gamma_S-\Gamma_T=\Gamma_1,\ \alpha\neq\beta\ ,\\
V_{+-}&=\Gamma_T\boldsymbol{\sigma}_{\alpha\beta}\cdot\boldsymbol{\sigma}_{\beta\alpha}=2\Gamma_T=-\Gamma_2,\ \alpha\neq\beta\ .
\end{aligned} \tag{8}$$

The dynamically dressed amplitudes V in the presence of disorder are therefore related via Eq. (4) to the Γ's:

$$\begin{aligned}
V_{\uparrow\uparrow}(q,\omega)&=\frac{\Gamma_S(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_S)\omega}+\frac{\Gamma_T(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_T)\omega}\ ,\\
V_{\uparrow\downarrow}(q,\omega)&=\frac{\Gamma_S(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_s)\omega}-\frac{\Gamma_T(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_T)\omega}\ ,\\
V_{+-}(q,\omega)&=\frac{2\Gamma_T(D_0q^2-i\omega)}{D_0q^2-i(1-2N_0\Gamma_T)\omega}\ .
\end{aligned} \tag{9}$$

By using Eq. (3) with $U_1=U_2=U$,

$$\Gamma_T=-\frac{U/2}{1-N_0U},\quad \Gamma_S=\frac{U/2}{1+N_0U} \tag{10}$$

one recovers the expressions for the V's given in Ref. 5 for the Hubbard model.

It is worth noting that according to Eq. (10) Γ_T is going to diverge near the magnetic instability $1-N_0U\ll 1$, whereas Γ_S remains finite. If one neglects the singlet contributions in Eq. (9) one obtains the dynamical amplitudes used in Refs. 6–8 in proximity of the ferromagnetic instability.

The identification in Eq. (8) is fully general and not restricted to the ladder approximation. Each evaluation of a physical quantity in the two different representations, Eqs. (1) and (7), will give the same final answer.

Therefore, all the results previously obtained for disordered interacting electron systems with short-range forces[2,3,9] also apply to the Hubbard model. The Hubbard specific nature manifests itself only via the condition $U_1=U_2=U$, which, however, does not imply any special physical consequence in the present context.

We thank Professor H. Fukuyama for pointing out to us the opportunity to make explicit the present correspondence for the Hubbard system.

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