PHYSICAL REVIEW B VOLUME 34, NUMBER 6 15 SEPTEMBER 1986

Electron heating in a multiple-quantum-well structure below 1 K

A. K. M. Wennberg, S. N. Ytterboe, C. M. Gould, and H. M. Bozler
Department of Physics, University of Southern California, Los Angeles, California 90089-0484

J. Klem and H. Morkoç
Coordinated Sciences Laboratory, University of Illinois, 1101 West Springfield Avenue, Urbana-Champaign, Illinois 61801

(Received 24 March 1986; revised manuscript received 16 June 1986)

We have measured the electron-energy-loss rate in an n-doped GaAs-$Al_xGa_{1-x}As$ heterostructure by finding the power required to establish a steady-state temperature offset between the electrons and the lattice. The measurement utilizes the temperature dependence of the resistance as a self-thermometer; heat is injected by an additional dc current. We find that for our sample the energy relaxation rate below 1 K is $\tau_\varepsilon^{-1} = (2.5\times10^6)T^3$ sec^{-1}K^{-3}.

The study of two-dimensional electron gases at very low temperatures has gained a great deal of interest because of the effects of weak localization and electron-electron interactions on the transport properties of these systems.[1] These transport studies have found that temperature-dependent effects such as the logarithmic increase in resistance appear to saturate at temperatures below ~100 mK. Only with great care can measurements be extended below this range because the electron-phonon coupling becomes very weak and the electrons remain at a substantially higher temperature than the lattice.

In this paper we will describe our measurement of the temperature dependence of the electron-phonon coupling in an n-doped GaAs-$Al_xGa_{1-x}As$ heterostructure. This is done by applying a known power to the electrons and measuring the resulting increase in their temperature above the lattice temperature. The resistance of the heterostructure has a temperature dependence that is used as the thermometer for the electrons. The temperature dependence of the electron-phonon coupling is extracted using a "two-bath" model.

In our experiment we used a 25-layer modulation-doped GaAs-$Al_xGa_{1-x}As$ structure. Each layer is comprised of the sequence 250 Å GaAs, 375 Å $Al_{0.28}Ga_{0.78}As$ (100 Å undoped, 175 Å n-doped to $n = 5\times10^{17}$ cm^{-3}, 100 Å undoped). The mobility is 11600 cm^2/Vs at 77 K. The number of carriers corresponds to an areal density of 1.7×10^{11} cm^{-2} for a single electron sheet. Thus the Fermi temperature should be about 70 K so the electrons will be degenerate below 1 K. We have studied two sections of this material. The first was a Hall pattern sample 100 μm long by 50 μm wide in the voltage-measuring region. This sample has diffused Sn contacts. The resistance of this sample is shown in Fig. 1(a). The second sample taken from the same wafer, and whose data are shown in Figs. 1(b) and 2, is a 6 mm-by-6 mm square section with four diffused indium contacts placed at the corners. The In contacts were heat treated at 430° C for approximately 5 min in order to obtain Ohmic connections. We found that the contact resistance was never greater than 25 Ω at the lowest powers and temperatures used. There was, however, some alteration in the effective shape of the sample due to the superconducting transition of the In at the contacts.

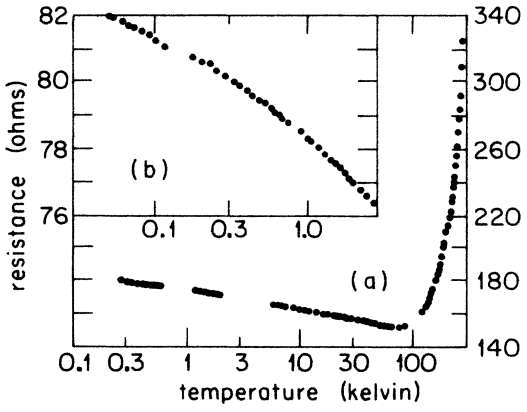

FIG. 1. The measured sample resistance vs temperature. The sample in (a) was prepared with a Hall pattern and was used to measure the mobility. The sample in (b) was a much larger piece of the same wafer with a scribed H pattern in the surface. The larger sample was used to reduce the effects of unintended heating. The temperature dependence of the two samples is presumed to be similar, with the exception of a fixed geometric factor.

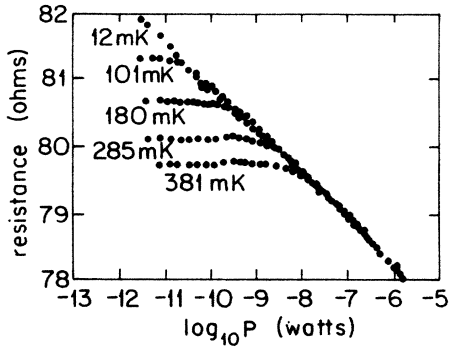

FIG. 2. The measured resistance of the large sample at a number of lattice temperatures vs the applied power. The values of the lattice temperature are indicated close to each curve.

 © 1986 The American Physical Society

In fact, below T_c for the In (~ 3.2 K) there was a complex current- and magnetic-field-dependent artifact in the resistance in the region of the contacts. For this reason, we isolated the contact regions with two parallel scratches on two opposite sides that extended 1.0–1.5 mm from the edge near each corner. For later purposes of calculating surface area, we assume that the entire area is at a constant temperature, since the effective electron thermal diffusion length as derived from our measurements below is larger than a millimeter at the temperatures used in this experiment.

Resistance measurements were performed with a four-wire active bridge utilizing two matched and temperature-compensated instrumentation amplifiers for current and voltage inputs.[2] We were able to achieve $1:10^4$ resolution and long-term stability with a measuring power of 1 pW. We injected dc current into the sample current leads in order to produce heating. The applied heating rate P was calculated from the measured voltage drop across the current leads, including the contacts. There is also a residual power due to noise and other extraneous heat leaks, as well as the power required for the resistance measurements. We lump these together as P_0 so that the total heat input is $\dot{Q}=P+P_0$.

We cooled the sample in direct contact with pure ^{3}He in a sample cell attached to a dilution refrigerator. Below 1 K the thermal boundary resistance becomes small compared with the electron-phonon resistance so we could take the lattice temperature to be equal to the surrounding ^{3}He during the heating experiment.

Figure 1(a) shows the resistance of the Hall pattern sample measured with no additional heating. The resistance appears to be reasonably well described by $\ln T$ below 30 K, but the slope decreases as T is reduced. The origin of the $\ln T$ behavior is related to the two-dimensional behavior of the conducting layers, but the details are not well understood. Our own longitudinal magnetoresistance measurements[2] as well as those of others on similar structures show that it is negative.[3] The magnetoresistance of these samples is not understood at this time. We can, however, use the resistance as a self-thermometer for the electron gas. For the purposes of our experiment it is not necessary to understand the origin of the $\ln T$ behavior, and in addition it is not necessary to know the slope of the resistance in advance. This is fortunate since the measured slope is affected by residual heating below ~ 100 mK and eventually goes to zero.

Figure 1(b) shows the resistance at low temperatures of the large (6-mm-square) sample. We see that although the resistance is roughly logarithmic, the slope decreases with decreasing temperature. Eventually, the residual power causes the resistance to saturate, but this is not the cause of the curvature above ~ 100 mK. Thus, the effects of heating and the intrinsic temperature dependence of the resistance are difficult to separate. Fortunately, with a self-consistent analysis as described below, using a combination of heating rates and lattice temperatures, the effect of heating can be readily determined.

Figure 2 shows the measured resistance versus applied power for several lattice temperatures. Except at the lowest temperature, we see that there is a range of applied power where the resistance is almost power independent. At these low powers the electron temperature is dictated by the lattice temperature. Then at high power the resistance approaches a universal curve. This universal curve represents the situation where the electron temperature is dominated by the effects of the power.

The thermal relaxation of electrons to the lattice can be well represented by a "two-bath" model where the electrons are at a well-defined elevated temperature T_e above the lattice bath T_l. The temperature rise is given by the ratio between the electron-phonon energy relaxation rate τ_ε^{-1} and the electron heat capacity C_e. We assume an equilibrium has been reached in which the rate that heat enters the electron system, $d\dot{Q}$, equals the rate at which it leaves: $d\dot{Q}=\tau_\varepsilon^{-1}C_e dT_e$. If we take $C_e=\gamma T$ and assume that τ_ε^{-1} follows a power law, $\tau_\varepsilon^{-1}=\alpha T^p$, we get

$$\dot{Q}=\alpha'(T_e^\beta-T_l^\beta)\ , \tag{1}$$

where $\alpha'=\alpha\gamma/(p+2)$ and $\beta=p+2$.

There is a crossover between the high- and low-power regime that represents the bends in each curve. If we assume that at least over a limited temperature range the resistance is given by

$$R=-A\ln T_e+B\ , \tag{2}$$

then from Eq. (1), we find that $R-B\propto\ln(\dot{Q}/\alpha'+T_l^\beta)$. Since it turns out that $\beta\gg 1$ we move rapidly from the low-power behavior with R being given by T_l^β to high-power behavior with R depending on the power. The crossover is at $\dot{Q}\approx\alpha' T^\beta$.

Since deviations of our low-power resistance curve [Fig. 1(b)] from Eq. (2) involve both intrinsic effects and the effect of residual heating, we cannot determine the parameters A and B in Eq. (2) with sufficient accuracy from the low-power measurement alone. However, since the determination of the crossover power does not depend on our choice of slope in the resistance versus temperature curve, we can then confidently perform a least-squares curve fit to the entire set of data shown in Fig. 2, utilizing Eq. (1) and assuming that the resistance follows Eq. (2). We fit the data simultaneously to Eq. (1), substituting Eq. (2) for the electron temperature, minimizing the squared deviations for R. This fit allows α', β, A, and B to be free parameters and results in a minimum χ^2. P_0 was determined separately from the lowest-lattice-temperature curve, to be 2.8×10^{-12} W. The fit is quite good with an average error for each point of 0.04 Ω leaving a reduced χ^2 of close to 1. Using standard statistical techniques,[4] we can estimate the accuracy of the determination of β by calculating the variation of the reduced χ^2 with β holding the other parameters fixed. The error limits correspond to the points where the reduced χ^2 doubles. With this analysis we get $\beta=5.0\pm0.2$ or $p=3.0\pm0.2$. To emphasize that this fit is not sensitive to the parameters in Eq. (2) we also made a much cruder analysis, consisting of simply estimating the power at the bend for each curve in Fig. 2 so that $\dot{Q}=\alpha' T^\beta$. This analysis also gives $\beta=5$.

The best fit to our data allows us to estimate the product $\alpha\gamma$ as defined in the discussion of Eq. (1) to be $\approx 5\times10^{-6}$ W/K[5]. We can get an estimate of the electron-phonon re-

laxation time if we assume a two-dimensional electron gas with an effective mass[5] $m^* = 0.0665 m_0$. We use 18 cm^2 for our effective sample area (assuming 50 two-dimensional electron sheets) and estimate $\gamma = 1.9 \times 10^{-12}$ J/K^2. Then the energy relaxation rate becomes $\tau_\varepsilon^{-1} = \alpha T^p = (2.5 \times 10^6) T^3 \sec^{-1} K^{-3}$.

For GaAs quantum wells Price[6] has calculated the theoretical rate of energy loss from the electron system to acoustic phonons in the low-temperature limit. He finds that a simple deformation coupling to LA phonons is negligibly small in our temperature range. However, the piezoelectric coupling of acoustic phonons to electrons gives the correct temperature dependence and (after converting to our notation) an energy relaxation rate for our sample density of $\tau_\varepsilon^{-1} = 6.6 \times 10^8 T^3 \sec^{-1} K^{-3}$. As for the two-order-of-magnitude discrepancy with experiment we note that the precise prefactor of the calculation is known to be unreliable. Price assumed a Fermi function for the electron distribution, but then demonstrated that this is an unsubstantiated assumption. He expects that a proper numerical evaluation will merely change the prefactor while retaining the same temperature dependence.

We should also make clear the distinction between our results and the much-higher-temperature work by Shah, Pinczuk, Gossard, and Wiegmann[7] and Yang, Carlson-Swindle, Lyon and Worlock.[8] In their temperature range ($T > 30$ K) the dominant cooling mechanism is electron–LO-phonon coupling. In our range ($T < 0.5$ K) this mechanism is exponentially suppressed and expected to be many orders of magnitude smaller than the piezoelectric coupling to acoustic phonons we observe.

A recent experiment by Roukes *et al.*[9] measured the energy relaxation rate in a metal film. Since the resistance of their metal film was essentially temperature independent at low temperatures they could not utilize the resistance as a simple self-thermometer, and instead measured Johnson noise to find the temperature. Despite the considerably different material they also found that $p = 3$. Roukes *et al.* found that their corresponding energy relaxation rate was $\tau_\varepsilon^{-1} = 9 \times 10^7 T^3 \sec^{-1} K^{-3}$ which is a factor of 36 higher than our value.

In conclusion, we find that the thermal relaxation of electrons in this quantum well system is sufficiently slow to account for the difficulty in obtaining temperature-dependent effects at very low temperatures. An extrapolation of our results shows that we will need to reduce all measuring and noise powers to less than 10^{-16} W in order to extend measurements to near 10 mK.

We wish to thank A. L. Thomson and L. J. Friedman for their assistance during this experiment. This work is supported by the National Science Foundation through Grants No. DMR85-01090 (A.K.M.W., S.N.Y., H.M.B.), No. DMR82-00661 and No. DMR85-19970 (C.M.G.), and the U.S. Air Force Office of Scientific Research (J.K., H.M.).

[1]G. Bergmann, Phys. Rep. **107**, 1 (1984).

[2]A. K. M. Wennberg, Ph.D. dissertation, University of Southern California, 1985 (unpublished).

[3]B. J. Lin, M. A. Paalanen, A. C. Gossard, and D. C. Tsui, Phys. Rev. B **29**, 927 (1984).

[4]The deviation in a parameter required to increase the reduced χ^2 from 1 to 2 can be used to estimate the corresponding diagonal component of curvature matrix. See P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969).

[5]Z. Zawadaski and R. Lassnig, Solid State Commun. **50**, 537 (1984).

[6]P. J. Price, J. Appl. Phys. **53**, 6863 (1982); Phys. Rev. B **30**, 2236 (1984).

[7]J. Shah, A. Pinczuk, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 2045 (1985).

[8]C. H. Yang, J. M. Carlson-Swindle, S. A. Lyon, and J. M. Worlock, Phys. Rev. Lett. **55**, 2359 (1985).

[9]M. L. Roukes, M. R. Freeman, R. S. Germain, R. C. Richardson, and M. B. Ketchen, Phys. Rev. Lett. **55**, 422 (1985).