# Condon domains in the two-dimensional electron gas. III. Dynamical effects

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Recent experiments have observed extremely sharp and complex structures in the ac susceptibility of the Condon domain phase in a high-density two-dimensional electron gas. Understanding of these structures requires a theoretical analysis of the dynamics of domain-wall motion. The present paper offers a theoretical framework for this analysis. Two phenomena are dealt with in detail: domain-wall resonance and a bending-mode resonance similar to the helicon-wave-like state seen in superconductors.

## I. INTRODUCTION

In a strong magnetic field, a dense two-dimensional electron gas (2D EG) can undergo a transition to a nonuniform state with domains in which the highest Landau level is either full or completely empty. While this transition is similar to Condon domain formation in three-dimensional metals,[1] it is predicted to have many unique features characteristic of the 2D EG.[2–4] While the electron gas behaves two-dimensionally, the domains show very different properties depending on the sample thickness,[3,4] and the present results refer to thick samples. Condon domains in such samples have been observed[5] to display a variety of dynamical effects—complex susceptibility, resonance, hysteresis, and chaos—and the present paper provides a background for understanding these effects.

The condition for the electron gas in a physically thick sample to behave two-dimensionally is that the successive layers interact only very weakly—that the dispersion of energy along the $c$ axis (perpendicular to the planes) is sufficiently small. In some acceptor graphite intercalation compounds (GIC's), the dispersion is small enough that successive Landau levels do not overlap,[6] and it is in one of these compounds that the very strong Condon domain formation is observed.[5] While the holes behave two-dimensionally, the domains are presumably coupled over the entire sample thickness, since they are regions of excess magnetic flux. The dynamical effects are due to the physical motion of these domains or of their walls.

## II. DOMAIN STATICS

A general condition for stability in a magnetic field is that $\partial H/\partial B \geqq 0$. With $H = B - 4\pi M$, this is equivalent to $4\pi\chi \leqq 1$, where $\chi = \partial M/\partial B$. For a 2D EG, $\chi$ is a constant within a Landau level. Hence if $4\pi\chi > 1$, the partly filled level is unstable and the gas phase separates into domains in which the highest Landau level is either full or empty. To maintain approximate[3] charge neutrality, the carrier density is uniform, while the magnetic field is inhomogeneous, taking on values $B_N$ and $B_{N-1}$, appropriate to $N$ or $N-1$, filled Landau levels, where $B_N = B^*/N$ and $\hbar e B^*/m^*c = E_F$. The excess energy stored in domains was calculated in paper I (Ref. 3) of this series for striped domains and in paper II (Ref. 4) for cylindrical domains. This excess is distributed among domain-wall surface tension, magnetic field energy associated with inhomogeneous fields outside the sample, and electric fields, due to charging of the domain. The surface tension may be written

$$\Sigma = \frac{(\Delta B)^2}{48\pi}\sqrt{8\pi\chi(4\pi\chi-1)}\,r_c\ , \tag{1}$$

where $\Delta B = B_{N-1} - B_N$, and $r_c = v_F/\omega_c$ ($v_F$ is the Fermi velocity, $\omega_c = eB/m^*c$ the cyclotron frequency). The magnetic field energy depends on the domain configuration. For a single cylinder of radius $a$, it is

$$E_F = \frac{2}{3\pi}(\Delta B)^2 a^3\ . \tag{2}$$

For a thick sample, the Coulomb energy is negligible compared to $E_F$.

## III. DOMAIN-WALL DYNAMICS

Within the domain phase, a number of dynamical effects can be expected to occur. This paper specifically addresses two effects relevant to the experimental observations: (1) In response to a time-varying magnetic field, the sample adjusts its average magnetization by domain-wall motion—the domains growing and shrinking. This motion is discussed in Secs. III—V. (2) At higher frequencies, the domains can undergo a transverse bending at a characteristic frequency, similar to helicon waves in a solid. These vibrations are the subject of Sec. VI.

In explicit calculations, it is necessary to assume a particular configuration of domains. The final results will display the correct functional dependence on parameters, but will all contain a numerical factor of order unity which is sensitive to the actual domain shape. For domain-wall dynamics, the simplest configuration is a one-dimensional striped array, with repeat distance $d$ determined as in I. The domains are assumed to be stripes corresponding to the $(N-1)$st Landau level, of width $x_0$ and $N$th level, of width $d - x_0$. In equilibrium the distance $x_0$ is determined by requiring that the average magnetic field in the sample is equal to the external field, $B_0$:

 

$$B_0=[x_0B_{N-1}+(d-x_0)B_N]/d$$

or

$$B_0=B_N+(\Delta B)x_0/d \ . \tag{3}$$

In the presence of a small, superimposed time-varying field, $B_a$, the wall motion $\delta x(t)=[x(t)-x_0]$ is described by an equation very similar to that found in the study of ferromagnetic domains[7,8] or the intermediate state of a superconductor:[9,10]

$$m\ddot{x}+b\dot{x}+k(x-x_0)=P_a-P_p \ . \tag{4}$$

Here $m$ is the effective wall mass per unit area, $b$ the viscous drag per unit area, $k$ the restoring force constant, $P_a$ the externally applied pressure, and $P_p$ the pinning pressure. These parameters are evaluated in the context of the Condon domain problem, in the remainder of this section and in Secs. III and IV.

Once $\delta x$ is known, the ac susceptibility can readily be calculated. In the domain phase, $H=B-4\pi M$ is constant (a correction due to charging effects is negligible in a thick sample—see II), so the static susceptibility is $\chi=\partial M/\partial B=1/4\pi$. Thus from Eq. (3), the ac susceptibility is

$$\chi=\frac{\Delta B}{4\pi}\frac{\delta x/d}{B_a} \ . \tag{5}$$

If $B_a=B_{a0}e^{i\omega t}$, $\chi$ will in general be complex:

$$4\pi\chi=1/(1-\omega^2/\omega_0^2+i\omega\tau) \ , \tag{6}$$

where $\omega_0^2=k/m$ and $\tau=b/k$.

The applied pressure due to an ac magnetic field is easily calculated. In a field $B$, displacement of the domain wall by a distance $\delta x$ lowers the system energy by $-2MB\delta x$ (per unit area), where $2M=\Delta B/4\pi$ is the change of magnetization in switching from one domain to another. Hence an ac field $B_a$ produces an extra pressure

$$P_a=B_a\,\Delta B/4\pi \ . \tag{7}$$

The restoring force contains a similar contribution. The dc limit of Eq. (4) should be compatible with Eq. (3). From Eq. (3), it is clear that $x$ varies from 0 to $d$ as $B_0$ changes from $B_N$ to $B_{N-1}$, or

$$k=(\Delta B)^2/4\pi d \ . \tag{8}$$

There can be additional contributions to the restoring force, associated with the mechanisms of Sec. II. These contributions depend on the domain distortion—for instance, if the surface area is increased, the energy associated with surface tension will be larger. These changes are in general expected to be small, except at the fields at which domains appear or disappear. At these fields, the loss of all domain walls will cause a sudden jump in $M$ (discontinuity in $\chi$), as observed in the superconducting intermediate state.[11]

The nature of the pinning forces is not well understood. For electrically charged domains, presumably any charged defect can attract a domain and possibly lead to pinning. The form used in Eq. (4) is similar to a static friction force or a coercive force in a ferromagnet:[7] If $P_a<P_p$, no net motion of the domain wall will occur. More recent theories suggest the possibility of a roughening transition,[12,13] where strong pinning can occur beyond a threshold concentration of impurities.

The remaining two material parameters, the viscous drag $b$ and the effective mass $m$, will be discussed in the following sections.

## IV. VISCOUS DRAG

The damping term in Eq. (3) requires special care, since it is associated with the longitudinal conductivity, $\sigma_{xx}$, which should vanish inside a domain.[2–4] This is because the field $B_N$ corresponds to $N$ exactly filled Landau levels, so there are no empty states to scatter into. In principle, there are five potential contributions to a nonzero $\sigma_{xx}$ in the domain phase: (1) $\sigma_{xx}$ is not strictly zero but is thermally activated, and hence finite (but small) if $T>0$. (2) The Fermi level in the domain wall is pinned within a series of interface states, similar to the surface states in the quantum-Hall effect,[14] and these could provide a small $\sigma_{xx}$. (3) In the graphite intercalation compounds in which these effects are observed, there may be small gradients of carrier density near the end faces of the sample. Since the magnetic flux must be continuous throughout the sample, it is possible that the domains must carry around small normal end caps, leading to finite resistance and excess pinning. Such effects could be very sensitive to the cool-down procedure. The samples are sealed in glass tubes with excess $Br_2$ gas, but some deintercalation could occur as the gas freezes out. Indeed we have observed that much more prominent dynamic effects are observed (much less "pinning") if the samples are cooled in a strong field. (4) At lower fields, $c$ axis dispersion may be large enough to cause Landau levels to overlap, but overlap should cease above $\sim 5$ T.[6] It should be pointed out that the bandwidth estimated in Ref. 6 is an upper limit, assuming the entire $c$-axis conductivity $\sigma_c$ is due to band conduction. Sugihara[15] has argued that in these compounds hopping conduction could significantly enhance $\sigma_c$. (5) There is yet another mechanism which may be significant in high-stage intercalation compounds. The experiments of Ref. 5 were done on a stage 2 material, in which two bands of holes contributed to the Fermi surface. Landau levels from the two different bands could overlap, either due to $c$-axis dispersion or to small deviations from thermal equilibrium (exchange of carriers between the two bands could be sluggish). There could then be a competition in domain formation between the two bands. The fields corresponding to the highest Landau level of one band being exactly full (or empty) would in general correspond to a partially filled Landau level in the second band. This partly filled band would then have a finite $\sigma_{xx}$. While such a mechanism would not exist in a strictly two-dimensional system, a careful analysis of the data will be required to assess its importance in the experimental observations.

All of the above mechanisms are expected to provide only small corrections to $\sigma_{xx}$, and therefore $\rho_{xx}=\sigma_{xx}/(\sigma_{xx}^2+\sigma_{xy}^2)$ will be small in the domain state. Hence eddy current effects can be strong at low temperatures. While the present paper discusses how eddy currents affect the domain dynamics, it should be kept in mind that they will have additional importance in determining the ac response of the sample. Eddy currents act to screen out the ac magnetic field from the interior of the sample, and hence should significantly reduce the magnitude of susceptibility oscillations at high frequencies. Analogous effects are seen in superconductors[16] and the quantum-Hall effect.[17] These effects will be discussed further in a separate paper, when the experimental data are analyzed.

Given that $\sigma_{xx} \neq 0$, it is straightforward to calculate the drag coefficient due to eddy current damping. As shown in II, a domain moving at velocity $\mathbf{v}$ induces an electric field, $\mathbf{E}=(\mathbf{v}\cdot\nabla)\mathbf{A}/c$, where $\mathbf{A}$ is the vector potential of the domain, and hence a dissipative current $\mathbf{j}=\sigma_{xx}\mathbf{E}$. (There is also a Hall current, due to $\sigma_{xy}$, but as shown in II, this is simply a reflection of the charge moving with the domain.) For the striped domains, the damping per unit area is

$$b=d(\Delta B)^2/2c^2\rho_{xx} \; . \tag{9}$$

Note that when there is an appreciable Hall effect ($\sigma_{xy}\neq 0$), the damping constant depends not on $\sigma_{xx}$ but on $\rho_{xx}$.

## V. EFFECTIVE MASS

There is an extra contribution to the energy of a moving domain stored in the electric field. Since $E \propto v$, the energy is proportional to $v^2$, and hence can be treated as a kinetic energy term, $\frac{1}{2}m_E v^2$, where $m_E$ is an effective wall mass. For a striped domain, the mass per unit area is

$$m_E=\frac{d(\Delta B)^2}{16\pi c^2} \; . \tag{10}$$

As shown in Ref. 7, any contribution to the viscous damping $b$ will produce a contribution to the effective mass of order $b^2$. This was discussed by Carr[18] for eddy currents in a ferromagnet. In the domain phase the effect is very similar. The dissipative current produces a magnetic field, $\nabla\times\mathbf{B}'=4\pi\mathbf{j}/c$, leading to an energy contribution, $\int(B')^2/8\pi$. Since $\mathbf{j}\propto v$, this appears as a kinetic energy term, with mass

$$m_B=\frac{m_E}{3}\left[\frac{4\pi d}{c\rho_{xx}}\right]^2 . \tag{11}$$

For the normal state $\sigma_{xx}\sim 4\times 10^6$ $(\Omega\,\mathrm{cm})^{-1}$, $d\sim 10$ $\mu$m (from I), and $m_B \gg m_E$. The resonant frequency,

$$\omega_0=\sqrt{3/\pi}\rho_{xx}c^2/4\pi d^2 \cong 2\times 10^7 \text{ s}^{-1} \; ,$$

suggesting that inertial effects can be ignored in the experiments, for which frequencies $\leqq 50$ kHz were used, unless $\rho_{xx}$ is greatly reduced from its normal state value. The damping is also of the same order of magnitude, $\omega_0\tau=\sqrt{3/4\pi}$. See *Note added in proof* for more recent calculations.

## VI. MAGNUS FORCE

There is an additional force on a magnetic vortex state moving through an electron gas in a magnetic field. While the form of the force is very similar to the Lorentz force, it is in origin closer to the Magnus force in superfluid He. This force has been extensively discussed for superconductors,[19,20] and has been shown to produce a heliconlike resonance. A similar effect occurs in Condon domains, as will now be shown.

The form is most easily found by calculating the force on the electrons due to a moving domain. Consider a single domain wall, separating regions of field $B_N$ and $B_{N-1}$, and moving with velocity $v$ in the $x$ direction. As an electron moves from the region of field $B_N$ to that of field $B_{N-1}$, its energy increases by $\hbar\Delta\omega_c=\hbar e(\Delta B)/m^*c$. This is produced by a force $F_y$ in the domain wall, which translates the orbit center in the $y$ direction by a distance $\Delta y$. The average force on the electron is $F_y=\hbar\Delta\omega_c/\Delta y$. When the electron leaves the other side of the domain, its energy is lowered, but its orbit center is translated in the opposite direction, so $F_y$ has the same magnitude and direction. To complete the calculation requires a knowledge of $\Delta y=\bar{v}_y\,\Delta t=\bar{v}_y\,\Delta x/v$, where $\bar{v}_y$ is the average velocity of the electron in the domain wall and $\Delta x$ is the wall width. Since $\Delta x\cong 2r_c$, $\bar{v}_y\cong r_c\,\Delta\omega_c/\pi$, $\Delta y\cong 2r_c^2\,\Delta\omega_c/(\pi v)$, and $F_y\cong\pi\hbar v/2r_c^2=\pi e v\,\Delta B/4c$, per electron. While some of the above calculations were oversimplified, it was felt necessary to give a clear indication of how the Magnus force arises in the domain case. The numerical factor in $F$ can easily be in error by a factor of $\sim 2$, and henceforth the factor $\pi/4$ will be neglected, to agree with the superconducting case: The force on an *electron* entering the domain is just equal to a Lorentz force. The net force on the *domain* is then $-F_y$ times the number of electrons in the domain. The force is at right angles to the velocity $v$.

As de Gennes and Matricon[19] showed, this Magnus force can excite transverse oscillations in a domain (bending modes). This motion is very different from the domain-wall motion discussed in Sec. III, since the volume of the domain remains unchanged, and damping and inertial effects are small. The restoring force will be found in those terms neglected in Sec. III: distortion of the domain shape increases its internal energy. The main effect of bending is to reduce the condensation energy. In the bent tube, the magnetic field will point along the tube axis, thereby reducing the average value of $B_z$. If the domain is bent by an amount $\delta x(z,t)\equiv s=s_0 e^{i(\omega t-kz)}$, $v=i\omega s$, then the domain energy is increased by an amount $\Delta E=\frac{1}{2}\mathcal{T}(\partial s/\partial z)^2$ per unit length, where $\mathcal{T}$ is the average line tension:

$$\mathcal{T}=\frac{(\Delta B)^2}{2}\chi A \; , \tag{12}$$

where $A$ is the cross-sectional area of a domain. Balancing the Magnus force against this restoring force gives a natural frequency:

$$\omega=\frac{ck^2}{2ne}\Delta B\,\chi \; . \tag{13}$$

Using the free electron value[3] for $\chi \cong \hbar n e / m^* c \, \Delta B$ gives

$$\omega = \hbar k^2 / 2m^* \ . \tag{14}$$

This may be rewritten in the form $\omega = (e \Delta B / 4m^* c)(k r_c)^2$, which differs from the superconducting result only in the substitution $r_c/2 \rightarrow \lambda$, the field penetration depth. If the wave vectors are assumed to be $k = k_n = n\pi / L_z$, where $L_z$ is the sample thickness, then the frequencies are field independent and two lowest resonant frequencies will be in the ratio 1:4. Both of these predictions are in agreement with experiment.[5] Actual numerical agreement with experiment is however problematical. The experimental values are about 5 times larger than given by Eq. (14), using the free electron values of $\chi$, but the experiments themselves show that $\chi$ is underestimated by at least a factor of 2. Experimentally, it is found that domain-domain interaction acts to enhance $\omega$: as the field increases above $B_N$, the frequency increases until the field is about halfway between $B_N$ and $B_{N-1}$, then starts to decrease again. A more detailed comparison with experiment will be published separately.

*Note added in proof.* More recent calculations[21] have shown that two-band effects (mechanisms of Sec. IV) have a large influence on $p_{xx}$, making it improbable that $p_{xx}$ is reduced significantly below its zero-field value, for $B \leq 12T$. The observed susceptibility peak is more likely due to bulk eddy curves, as in Ref. 16.


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