Polar-optical-phonon contribution to the damping of the magnetophonon oscillations in multiple-quantum-well structures

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In the present article we report a theory on the polar-optical-phonon contribution to the damping of magnetophonon oscillations in multiple-quantum-well structures. The present theory is applied to calculate the magnetophonon-oscillation damping (MPOD) in the $GaAs/Al_xGa_{1-x}As$ superlattice. Numerical results on the variations of the MPOD with the width of the quantum well, interquantum-well spacing, height of the quantum well, the magnetic field, the temperature, and the Landau-level number are presented near the magnetophonon resonance. The effect of the screening of the electron—polar-optical-phonon interaction on the decay rate (MPOD) is investigated in a Thomas-Fermi-type mean-field approximation. The polaron decay width (i.e., MPOD) is sensitive to quantum-well parameters and the screening of the electron—polar-optical-phonon interaction.

I. INTRODUCTION

In recent years effects of the polar-optical-phonon interaction with the electrons in a two-dimensional electron gas (2-D EG) such as occur in GaAs heterostructures,[1] superlattices of $GaAs/Al_xGa_{1-x}As$ (Ref. 2), and InSb space-charge layers[3] have been investigated by several authors.[4] Brummell *et al.*[5] have recently investigated experimentally the effect of the electron—polar-optical-phonon scattering on the cyclotron resonance line shape in 2D EG occurring in GaInAs/InP heterojunctions and superlattices in a high magnetic field. A dramatic increase in the cyclotron resonance linewidth and the deviation of the effective mass from its low-frequency values at higher frequencies has been attributed to the resonant interaction of the polar-optical phonons with the electrons. Similar effects of the resonant polaron coupling have been observed by Portal *et al.*[6] and Kido *et al.*[7] in magnetophonon resonance experiments and by Nicholas and co-workers[8] in the measurement of the variation of the effective mass extracted from the position of the peak of the cyclotron resonance line as a function of frequency.

Theoretical investigations of the effect of the electron—polar-optical-phonon interaction on the single-particle spectra of a two-dimensional electron in a high magnetic field have been made by Das Sarma,[9] Larsen,[10] Zawadzki,[11] and Peeters and Devreese[12] earlier.

The effect of screening of the electron-phonon interaction on the effective mass, binding energy, and the polaron emission rate in the *absence of magnetic field* in two-dimensional heterostructures has been investigated by different workers in varying approximations earlier.[13–17] Das Sarma[13] calculated the variation of the effective-mass and the polaron binding energy with screening length using Thomas-Fermi approximation. Das Sarma and Mason[14,15] in their other papers dealt with the same problem but including the screening in different approximations. Effect of the screening is appreciable for static quantities such as binding energy and the effective mass. It is also noted that the difference between static and dynamic screening results for polaron emission rate is only a few percent, whereas on the binding energy and effective mass it is larger. Similar conclusions are also reached by Wu, Peeters, and Devreese[16] and by Yang and Lyon.[17] However, to our knowledge the polaron decay rate and the effect of the screening on the polaron decay rate in the presence of a magnetic field in multiple-quantum-well structures (MQWS) has not been calculated before.

In an earlier paper[18] we developed a theory of relaxation time due to electron-phonon scattering for a quasi-two-dimensional electron gas subjected to a high magnetic field applied perpendicularly to the layers in the presence of an infinitely deep quantum well. The results of our finding of the dependence of the relaxation time on the temperature and density of states for acoustic phonon scattering has been verified recently by Komiyama and co-workers.[19]

The purpose of the present paper is to develop a theory on the polar-optical-phonon contribution to the damping of the magnetophonon oscillations in the MQWS. The effect of the screening has also been included in the electron-phonon interaction by replacing $(1/Q^2)$ by $1/(Q^2+q_0^2)$ in the Thomas-Fermi approximation. The theory is applied to calculate the magnetophonon-oscillation damping (MPOD) in the $GaAs/Al_xGa_{1-x}As$ superlattice. Numerical results on the variations of the magnetopolaron decay rate with the width of the quantum well (a), interquantum well spacing (b), height of the quantum well (W), magnetic field (B), temperature (T), the Landau quantum number (N), and the screening length (q_0^{-1}) near the magnetophonon resonance are presented. Polaron decay width is sensitive to the quantum-well parameters and the screening of the polar-optical-phonon interaction, and is enhanced in the quantized layers.

The expression for the magnetopolaron decay rate due to the interaction of the electrons with the screened polar-optical phonons is presented in Sec. II. Section III contains a summary of the numerical results and discussions of the present paper.

II. THEORY

A theoretical treatment of the relaxation and damping of the magnetophonon resonances in three-dimensional semiconductors has been given earlier.[20] It has been observed that a self-consistent treatment of the electron-phonon scattering in determining the tetradic self-energy operator removes the divergences occurring at the magnetophonon resonance peaks; $\omega_L = P\omega_C$ in which ω_L and ω_C are the longitudinal polar-optical-phonon frequency and the cyclotron resonance frequency, respectively, and P is an integer. Suzuki and Dunn[21] have also developed a similar approach for the calculation of the dynamical conductivity and the cyclotron resonance linewidth in the presence of the electron-phonon interaction. The electron-optical-phonon interaction Hamiltonian in the presence of a magnetic field for the multiple-quantum system is written as

$$H_{ep} = \sum_{N,X,p} \sum_{N',X',p'} \sum_{\mathbf{Q}} V_{\mathbf{Q}} \langle N',X',p' | e^{i\mathbf{Q}\cdot\mathbf{r}} | N,X,p \rangle \times (b_{\mathbf{Q}}^{\dagger} + b_{\mathbf{Q}}) a^{\dagger}_{N',X',p'} a_{N,X,p} , \quad (1)$$

$$|V_{\mathbf{Q}}|^2 = \frac{2\pi^2 \alpha W_L^3}{Q^2 + q_0^2} ,$$
$$\alpha = \frac{e^2}{2} \left[\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0} \right] \left[\frac{2m^*}{\omega_L} \right]^{1/2} , \quad (2)$$

where N, X, and p are the Landau quantum, center of the cyclotron orbit and the subband quantum number, respectively. $\mathbf{Q}(q_\perp, q_z)$ is the phonon propagator vector with component $q_\perp$ parallel to the layer and q_z perpendicular to the layer. a_{NXp} and $a^{\dagger}_{NXp}$ are the annihilation and creation operators of the electron in the Landau state $|N,X,p\rangle$, respectively. $b^{\dagger}_{\mathbf{Q}}$ and $b_{\mathbf{Q}}$ are the creation and annihilation operators for the phonons in the state $|N_{\mathbf{Q}}\rangle$, respectively. ε_∞, ε_0, and ω_L are the high- and low-frequency dielectric constants, respectively, and the phonon frequency. m^* is the effective mass of the electron and q_0 is the screening wave vector.

Using Eq. (1), we have calculated the imaginary part ($\Gamma_{N,p}$) of the tetradic self-energy $[\Sigma_{N,p}(\omega)]$ in the Landau-level representation at the magnetophonon resonance peak ($\omega_L = \omega_C$) for the MQWS. In the high magnetic field approximation we obtained the following expression, $\Gamma_{N,p}$, for the magnetophonon resonance transition between Landau levels N and $N+1$:

$$|\Gamma_{N,p}(\omega_C)|^2 = \int \frac{d^3Q}{(2\pi)^3} [1 + 2N_0(\omega_L)] |V(\mathbf{Q})|^2 \times [K_{N,N}(q_\perp) + K_{N,N+1}(q_\perp)] \times |F_{pp}(q_z)|^2 , \quad (3)$$

where $N_0(\omega_L)$ is the Bose factor for the polaron modes;

$$N_0(\omega_L) = [\exp(\hbar\omega_L | k_B T) - 1]^{-1} .$$

K functions are given by[22]

$$K_{N,N'}(x) = \begin{cases} \dfrac{N!}{N'!} x^{N'-N} e^{-x} L_N^{N'-N}(x) L_{N+1}^{N'-N-1}(x) , & N' > N \\ \dfrac{N!}{(N+1)!} x^{N-N'+1} e^{-x} L_N^{N-N'}(x) L_N^{N-N'+1} , & N' \le N \end{cases} \quad (4)$$

where $L_n^m(x)$ are the associated Laguerre polynomials

$$L_n^m(x) = \frac{e^x}{n!} x^{-m} \frac{d^n}{dx^n} (x^{n+m} e^{-x}) ,$$
$$x = \frac{q_\perp^2 l^2}{2} = \frac{(q_x^2 + q_y^2) l^2}{2} , \quad l = \left[\frac{\hbar}{m^* c} \right]^{1/2} . \quad (5)$$

$|F_{pp}(q_z)|^2$ contains the multiple-quantum-well characteristics. Following Calecki *et al.*[2] for the ground electric subband ($p = 1$) we obtained the following expression for $|F_{11}(q_z)|^2$:

$$|F_{11}(q_z)|^2 = 2\cos^2\left[\frac{k_1 a}{2}\right] e^{-x_1 b} \left[\frac{a}{2} + \frac{k_1^2}{x_1(x_1^2 + k_1^2)} \right]^{-1} \times \left[\frac{\sin\left[\frac{q_z b}{2}\right]}{q_z} + \frac{2x_1(x_1^2+k_1^2)\cos\left[\frac{q_z b}{2}\right] + q_z(k_1^2 - 3x_1^2 - q_z^2)\sin\left[\frac{q_z b}{2}\right]}{(x_1^2 + k_1^2 - q_z^2)^2 + 4x_1^2 q_z^2} \right] ,$$
$$x_1 = \left[\frac{2m^*(W - \varepsilon_1)}{\hbar^2} \right]^{1/2} , \quad k_1 = \left[\frac{2m^* \varepsilon_1}{\hbar^2} \right]^{1/2} . \quad (6)$$

ε_1 is found by solving the following transcendental equation

$$k_1 a + 2\tan^{-1}(k_1/x_1) = \pi \ . \tag{7}$$

W and ε_1 stand for the depth of the well and the energy of the first electric subband, respectively. a and b are the width of the well and the interwell spacing. $d = a + b$ is the superlattice period. Making use of the equations (4) and (6), Eq. (3) for MPOD for the transition $N \to N+1$ is reduced to the following expression:

$$\Gamma_N^2(\omega_c) = A \int_0^{x_{\max}} dx \int_0^{z_{\max}} dz \left[\frac{K_{N,N}(x) + K_{N,N+1}(x)}{2x + z^2 + q_0^2 l^2} \right] \left[\frac{\sin(zb/2l)}{z} + \frac{t_1 \cos(zb/2l) + (t_2 - z^2)\sin(zb/2l)}{[(t_3 - z^2)^2 + 4x_l^2 l^2 z^2]} \right] \tag{8}$$

$$A = \frac{8e^2\omega_L x_1^2 k_1^4 \cos^4(k_1 a/2) e^{-2x_1 b}(\varepsilon_0 - \varepsilon_\infty)}{\pi l \varepsilon_0 \varepsilon_\infty [a x_1 (x_1^2 + k_1^2) + 2k_1^4]} [1 + 2N_0(\omega_L)] \ , \tag{9}$$

$$z = q_z l \ , \quad t_1 = 2x_1^2(k_1^2 - x_1^2)l^2, \quad t_2 = (k_1^2 - 3x_1^2)l^2, \quad t_3 = (x_1^2 + k_1^2)l^2 \ , \tag{10}$$

where we dropped the subband index 1 from $\Gamma N,1$.

We have obtained a numerical solution of the above equation for different values of N. The results on the variations of the magnetopolaron decay width with the multiple-quantum-well parameters (a, b, W, d), magnetic field (B), and temperature (T) in GaAs/Al_xGa_{1-x}As superlattices are discussed in Sec. III.

III. RESULTS AND DISCUSSIONS

In this section we have calculated the magnetophonon-oscillation damping (MPOD) for the GaAs/Al_xGa_{1-x}As superlattice. The physical parameters used in the calculation are taken as $\omega_L = 36.2$ meV, $\varepsilon_\infty = 11.1$, $\varepsilon_0 = 13.2$, and $m^*/m_0 = 0.0667$. We denote the magnetophonon-oscillation damping due to the transition between Landau level N to Landau level $N+1$ by Γ_N. Figure 1 shows the variation of the MPOD for the Landau level $N = 0$ against the magnetic field. The curves A, B, and C correspond to $T = 100$, $T = 200$, and $T = 300$ K, respectively, at $W = 0.15$ eV, $a = b = 50$ Å, and $q_0^{-1} = 70$ Å. The MPOD increases with the increase of the magnetic field and temperature, respectively. We found similar curves for $N = 1$, $N = 2$, and $N = 3$, respectively. Portal *et al.*[6] reported the measurement of magnetophonon resonance in $Ga_{1-x}In_x$As-InP and measurements of the temperature dependence of the oscillations. They plotted the amplitudes of the magnetophonon oscillation for Landau level $N = 2$ between temperature 80 to 300 K. The amplitude decreases with increasing temperature. According to them, the decrease of the magnetophonon-oscillation amplitude is due to the increase of the Landau-level broadening, or, in other words, the increase of MPOD. Brummel *et al.*[5] measured the cyclotron resonance (CR) and polaron effects in a two-dimensional electron gas in $Ga_{1-x}In_x$As-InP and $Ga_{1-x}In_x$As-$Al_{1-x}In_x$As. They found that the CR linewidth increases with the increase of the magnetic field. We know that the CR linewidth and the MPOD are correlated to each other. The present results are in qualitative agreement with the above experimental results.

In Fig. 2, the variation of the MPOD versus the quantum-well thickness (a) for Landau level $N = 0$ is

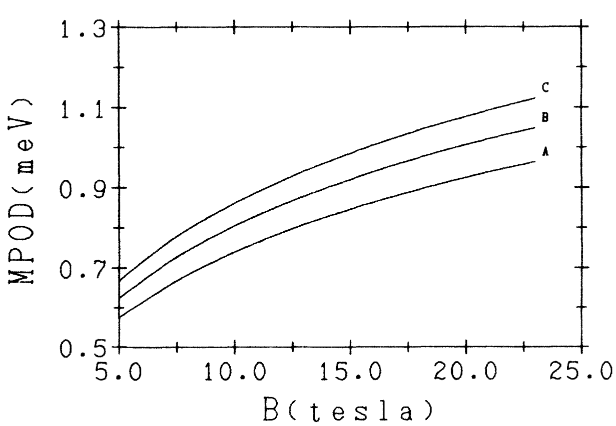

FIG. 1. Plot of the magnetophonon-oscillation damping (MPOD) vs magnetic field (B) for Landau level $N = 0$, with $T = 100$ K (curve A), $T = 200$ K (curve B) and $T = 300$ K (curve C), respectively, at $W - 0.15$ eV, $a = b = 50$ Å, and $q_0^{-1} = 70$ Å.

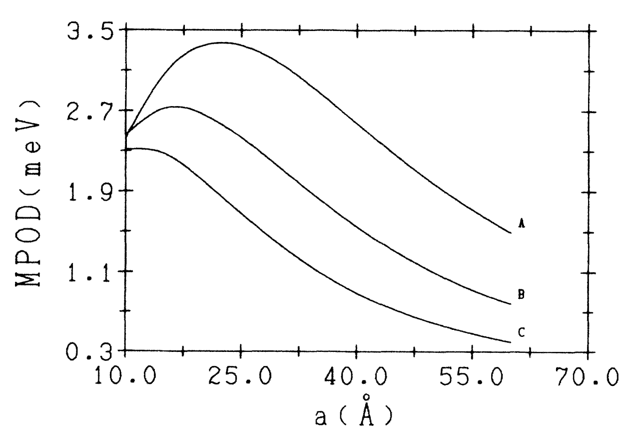

FIG. 2. Plot of the magnetophonon-oscillation damping at $N = 0$ vs the width of the quantum well, a. The inter-quantum-well spacing is $b = 20$, 40, and 60 Å for curves A, B, and C, respectively, at $W = 0.15$ eV, $B = 12$ T, $T = 100$ K and $q_0 l = 1$.

shown. The curves A, B, and C correspond to the inter-quantum well $b=20$, $b=40$, and $b=60$ Å, respectively, at $B=12$ T, $W=0.15$ eV, and $q_0 l=1$. The MPOD decreases as the quantum-well thickness increases. The increase of the quantum well means that there is a slow transition from a two-dimensional to a three-dimensional system. Therefore, the present theoretical analysis predicts the enhancement of the electron-optical phonon scattering (i.e., increase of the MPOD) in a two-dimensional system. The experimental results of Holonyak *et al.*[23] suggest that the electron-optical phonon interaction is enhanced in two-dimensional systems, since they measure phonon sidebands in the quantum well $GaAs/Al_xGa_{1-x}As$. The present results indeed confirm the experimental prediction of Holonyak *et al.*, enhancement of electron-phonon interaction in 2D systems. It is also interesting to note that as we increase the separation between the quantum well, the MPOD decreases. Increasing b means decreasing the interaction between the quantum wells. The present theory predicts the enhancement of the electron-phonon interaction (the increase of the MPOD) due to the interaction of the quantum wells (i.e., decrease of b). Because of the special separation between mobile carriers and ionized impurities, the Coulomb scattering is reduced in modulated doped superlattices. Hence the electron-phonon scattering plays a very important role in the modulated doped semiconductors.

In Fig. 3 we present the variation of the MPOD as a function of the superlattice half period $A(=d/2)$ for the Landau level $N=0$ (curve A) $N=1$ (curve B), N=2 (curve C), and N=3 (curve D), respectively. The other physical parameters are the same as in Fig. 2. The MPOD is decreasing with the increase of the Landau-level and the superlattice period.

Figure 4 shows the plot of the MPOD as a function of the magnetic field for the Landau-level transitions $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively. The values of the magnetic field, temperature, and $q_0 l$ are taken as 12 T, 100 K, and 1, respectively. It is found that the difference of MPOD between two Landau levels decreases as N increases. These results are consistent with the experimental results. In Fig. 5, we presented the calculation of the MPOD as a function of barrier height, W, for the Landau levels $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively. The other parameters are taken as $B=12$ T, $T=100$ K, $(q_0 l)=1.0$, and $a=b=50$ Å. Gossard[24] correlated the barrier height W with x as $W \sim 1.1x$ eV, where x is the fractional Al content in $Al_xGa_{1-x}As$. The MPOD decreases as W or x increases. It is an interesting result. Increasing the barrier height means considering only a single quantum well. In other words, the MPOD in enhanced in MQWS.

Finally, we calculated the variation of the MPOD versus the $(q_0 l)^2$ at $B=12$ T, $T=100$ K, $a=b=50$ Å, and $W=0.15$ eV. The results are presented in Fig. 6. The curves A, B, C, and D correspond to $N=0$, $N=1$, $N=2$, And $N=3$, respectively. The MPOD decreases as

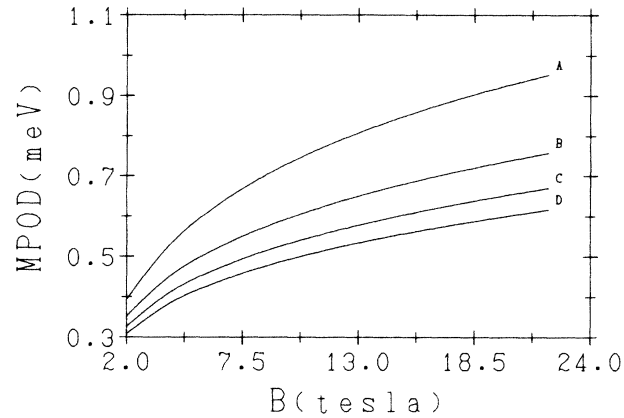

FIG. 4. Plot of the magnetophonon-oscillation damping (MPOD) vs magnetic field (B) for the Landau levels $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively, at $T=100$ K, $W=0.15$ eV, $a=b=50$ Å, and $q_0=70$ Å.

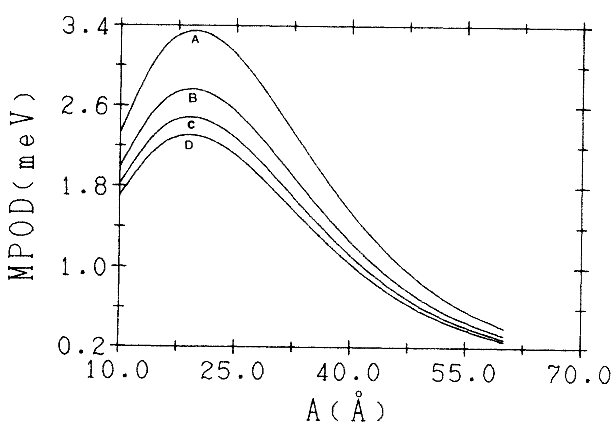

FIG. 3. Plot of the magnetophonon-oscillation damping (MPOD) vs the half periodicity ($A=d/2$) for the Landau levels $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively, at $B=12$ T, $W=0.15$ eV, $T=100$ K, $a=b=d/2$, and $q_0 l=1$.

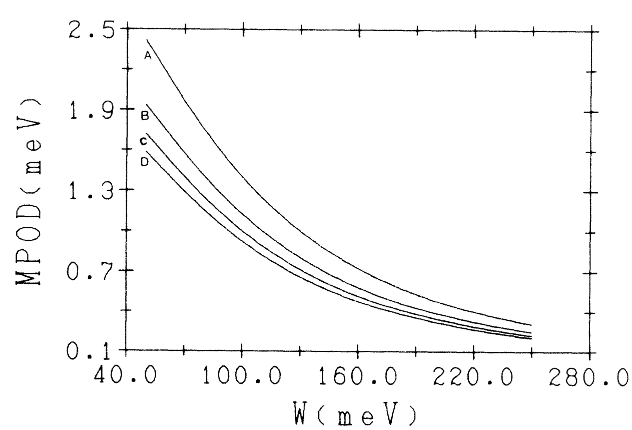

FIG. 5. Plot of the magnetophonon-oscillation damping (MPOD) vs the barrier height (W) for the Landau levels $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively, at $B=12$ T, $a=b=50$ Å, $T=100$ K, and $q_0 l=1$.

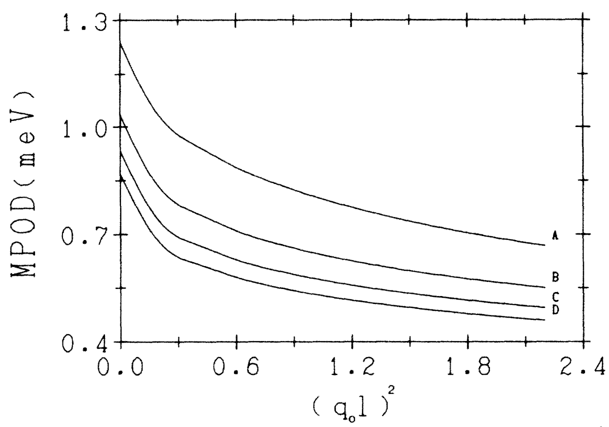

FIG. 6. Plot of the magnetophonon-oscillation damping (MPOD) vs the screening length [i.e., $(q_0 l)^2$] for the Landau levels $N=0$ (curve A), $N=1$ (curve B), $N=2$ (curve C), and $N=3$ (curve D), respectively, at $B=12$ T, $T=100$ K, $a=b=50$ Å, and $W=0.15$ eV.

$(q_0 l)^2$ increases but decreases slowly after $(q_0 l)^2=0.1$. This means that electron-phonon interaction decreases as the electron concentrations increase (i.e., electron-electron interaction increases). We find that screening effects are important when the cyclotron radius lies within the screening radius, i.e., when $l \leq l_0$, where $l_0(=q_0^{-1})$ is the screening radius (see Fig. 6).

The effect of dynamic screening as calculated by the other authors[14–17] on the polaron emission rate in two-dimensional heterostructures in the absence of magnetic field has indicated that the Thomas-Fermi screening results on the effective mass and binding energy are altered significantly where as the polaron emission rate changes only by a few percent. Therefore we believe that our results will remain qualitatively the same as reported here differing quantitatively by a few percent if the dynamic screening is included. To keep the calculations tractable and to obtain a qualitative picture of the effect of the screening on the MPOD, we considered Thomas-Fermi type of screening. In principle, one should include the effect of dynamic screening.

In all the above calculations $N=0$ results corresponds to the extreme magnetic quantum limit, which implies that the magnetic field has to be strong that only the $N=0$ level is populated. Therefore the result for $N=0$ level are valid for $B \geq 8$ T and the lower magnetic field portion of the results should be disregarded.

In conclusion, for the first time we investigated the variation of MPOD with multiple-quantum-well characteristics in a strong magnetic field. MPOD has been found to increase with magnetic field and temperature. Increasing the width of the quantum well (a) decreases the MPOD for a fixed inter-quantum-well distance (b) similar observation is made when a is fixed and b is varied. Screening in Thomas-Fermi approximation reduces the MPOD significantly which is in qualitative agreement with the result of Das Sarma and Mason[14,15] although their calculation is done in the zero magnetic field in heterostructures. For a more accurate calculation, q dependent q_0, screening should be included in high magnetic field which is not expected to effect our results qualitatively anyway.

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