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Effect of dissipation on the phase transition in granular superconductors

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The phase diagram of the self-charging model of an array of resistively shunted junctions is calculated in the mean-field approximation using the functional integral formulation. The dissipation tends to diminish the effects of the charging energy U on the phase boundary: The degree of reentrance decreases and the boundary approaches the classical ($U=0$) limit as $g\rightarrow\infty$.

There has recently been a great deal of interest in the problem of phase ordering in Josephson junction arrays, taking into account the effects of the electrostatic charging energy.[1-7] In general, the quantum phase fluctuations, associated with the charging energy, tend to inhibit the long-range phase order leading to the possibility of a fluctuation-driven phase transition.[3,4] Calculations of the transition temperature for this phase ordering have been, until now, confined to the case of the arrays of undamped junctions. It is well known that dissipation tends to suppress the quantum fluctuations. This effect has been thoroughly investigated in connection with the problem of macroscopic quantum tunneling.[8,9] Applying this concept to a Josephson junction, we see that the phase fluctuations in a damped junction are reduced, compared to the undamped case. In an array of junctions, it is the competition between the phase fluctuations and the Josephson coupling energy which determines the phase-ordering transition temperature. Consequently, we expect some changes in the phase diagram to take place in the presence of dissipation in the junctions. Quantitative predictions of such modifications are of interest especially in view of the possibility of the reentrant phase transition in granular superconductors.[3,4,6]

The purpose of the present work is to calculate the modifications of the mean-field phase diagram of an array of Josephson junctions caused by the dissipation. We consider the simplest possible model, namely, the self-charging model with a diagonal term for the charging energy.[3,6] To calculate the transition temperature, we use the Feynman functional integral formulation of the mean-field theory, introduced recently by one of us.[7] The partition function of the array is given by

$$Z=\int_{\rm per}D\phi(\tau)\exp\left[-\int_0^\beta L(\phi)d\tau\right], \tag{1}$$

where the subscript on the path integral indicates that the paths must satisfy the periodic boundary condition $\phi(0)=\phi(\beta)$. The function $L(\phi)$ is obtained by incorporating the dissipative term (derived for a single junction in Ref. 10) into the diagonal model of Ref. 7:

$$L(\phi)=\frac{1}{8U}\sum_i\left(\frac{\partial\phi_i(\tau)}{\partial\tau}\right)^2+E_J\sum_{\langle ij\rangle}[1-\cos\phi_{ij}(\tau)] -\sum_{\langle ij\rangle}\int_0^\beta d\tau'\alpha_{ij}(\tau-\tau')\cos\{\tfrac{1}{2}[\phi_{ij}(\tau)-\phi_{ij}(\tau')]\}\ , \tag{2}$$

where $\phi_{ij}=\phi_i-\phi_j$ is the phase difference of the nearest-neighbor superconductors in the array, which are coupled by the Josephson energy E_J. The charging energy, proportional to the inverse capacitance, is denoted by U. The function $\alpha_{ij}(\tau)$ describes the effect of the dissipation produced by quasiparticle tunneling or by the shunting resistance.[10] We proceed by introducing the mean-field approximation which replaces the interactions between the ith and jth superconductors in Eq. (2) by an effective Gorkov field and an effective damping acting on a single superconductor with phase $\phi_i(\tau)$. Formally, this can be achieved by generalizing the approach of Ref. 7 to include the damping. We start from the variational principle for the free energy:[11]

$$F\le F_0+\frac{1}{\beta Z_0}\int_{\rm per}D\phi\exp(-S_0[\phi])(S-S_0)=F_t\ , \tag{3}$$

where

$$S=\int_0^\beta L(\phi)d\tau\ , \tag{4}$$

and

$$Z_0=e^{-\beta F_0}=\int_{\rm per}D\phi\exp(-S_0[\phi])\ . \tag{5}$$

The trial action $S_0=\int_0^\beta L_0 d\tau$ is defined by the following choice for L_0:

$$L_0=\sum_i\left[\frac{1}{8U}\left(\frac{\partial\phi_i}{\partial\tau}\right)^2-z\gamma E_J\cos\phi_i(\tau)-z\int_0^\beta d\tau'\Gamma(\tau-\tau')\alpha(\tau-\tau')\cos\{\tfrac{1}{2}[\phi_i(\tau)-\phi_i(\tau')]\}\right]\ , \tag{6}$$

where z is the coordination number in the array. The variational parameters γ and $\Gamma(\tau)$ are determined from Eq. (3), by requiring that $\delta F_t=0$ as $\gamma\rightarrow\gamma+\delta\gamma$ and $\Gamma\rightarrow\Gamma+\delta\Gamma$. This yields

$$\gamma=\frac{2}{Z_0}\int_{\rm per}D\phi\exp(-S_0[\phi])\cos\phi=2\langle\cos\phi\rangle\ , \tag{7}$$

and

$$\Gamma(\tau-\tau')=\frac{2}{Z_0}\int_{\rm per}D\phi\exp(-S_0[\phi])\cos\{\tfrac{1}{2}[\phi(\tau)-\phi(\tau')]\}. \tag{8}$$

Near the transition temperature $T_c=\beta_c^{-1}$ the quantity $\langle\cos\phi\rangle$ is small and the path integral (7) can be calculated by expanding S_0 in γ.

This results in the self-consistent equation for T_c of the form

$$1 = zE_j \int_0^{\beta_c} d\tau R(\tau) , \quad (9)$$

where $R(\tau)$ is the phase correlator given by

$$R(\tau) = \frac{1}{Z_u} \int D\phi(\tau) \exp(-S_u[\phi]) \exp\{i[\phi(\tau)-\phi(0)]\} . \quad (10)$$

The action $S_u = \int_0^{\beta_c} L_u(\phi) d\tau$ is obtained from Eq. (6) by discarding the Josephson coupling term, so that

$$L_u = \sum_i \left[\frac{1}{8U} \left(\frac{\partial \phi_i}{\partial \tau} \right)^2 - z \int_0^{\beta_c} d\tau' \Gamma_u(\tau-\tau') \alpha(\tau-\tau') \times \cos\{\tfrac{1}{2}[\phi_i(\tau)-\phi_i(\tau')]\} \right] , \quad (11)$$

where

$$\Gamma_u = \frac{2}{Z_0} \int_{\text{per}} D\phi e^{-S_u[\phi]} \times \cos\{\tfrac{1}{2}[\phi(\tau)-\phi(\tau')]\} . \quad (12)$$

Because of the non-Gaussian form of the dissipative term in Eq. (11), further approximation is needed to perform the path integrals (10) and (12). One possibility is to replace Eq. (11) directly by a harmonic functional. Recent calculations[12] of the phase correlator for a normal junction, however, have shown that the harmonic approximation overestimates the role of the damping term, if compared with the self-consistent harmonic approximation (SCHA). Hence, we employ the latter approximation to treat Eqs. (10)–(12). This is done by replacing L_u as follows:

$$L_u \to L_u^{\text{SCHA}} = \sum_i \left[\frac{1}{8U} \left(\frac{\partial \phi_i}{\partial \tau} \right)^2 + \frac{z}{8} \int_0^{\beta_c} d\tau' \Gamma^{\text{SCHA}}(\tau-\tau') \alpha(\tau-\tau') G(\tau-\tau') [\phi(\tau)-\phi(\tau')]^2 \right] , \quad (13)$$

where $G(\tau-\tau')$ is the SCHA "variational parameter" given by

$$G(\tau-\tau') = \exp\{-\tfrac{1}{2}\langle[\phi(\tau)-\phi(\tau')]^2\rangle_{\text{SCHA}}\} . \quad (14)$$

Consistent with the ansatz (13), we obtain Γ^{SCHA} from Eq. (12) by replacing S_u by S_u^{SCHA}, so that

$$\Gamma^{\text{SCHA}}(\tau-\tau') = 2\langle\cos\{\tfrac{1}{2}[\phi(\tau)-\phi(\tau')]\}\rangle_{\text{SCHA}} = 2\exp\{-\tfrac{1}{8}\langle[\phi(\tau)-\phi(\tau')]^2\rangle_{\text{SCHA}}\} . \quad (15)$$

Equations (14) and (15) can be combined to define a single SCHA parameter $\Pi(\tau-\tau')$:

$$\Pi(\tau-\tau') = \tfrac{1}{2}\Gamma^{\text{SCHA}}(\tau-\tau') G(\tau-\tau') = \exp\{-\tfrac{5}{8}\langle[\phi(\tau)-\phi(\tau')]^2\rangle_{\text{SCHA}}\} . \quad (16)$$

Using Eq. (16) in Eq. (13), we have

$$L_u^{\text{SCHA}} = \sum_i \left[\left(\frac{\partial \phi_i}{\partial \tau} \right)^2 + \frac{z}{4} \int_0^{\beta_c} d\tau' \Pi(\tau-\tau') [\phi_i(\tau)-\phi_i(\tau')]^2 \right] . \quad (17)$$

Equations (16) and (17) are coupled self-consistent equations from which the phase correlator $R(\tau)$ can be determined, with the use of Eq. (10), as follows:

$$R(\tau) = \langle\exp\{i[\phi(\tau)-\phi(0)]\}\rangle_{\text{SCHA}} = \exp\{-\tfrac{1}{2}\langle[\phi(\tau)-\phi(0)]^2\rangle_{\text{SCHA}}\} = [\Pi(\tau)]^{4/5} . \quad (18)$$

The path integral in the exponent of Eq. (16) can be done by expanding $\phi(\tau)$ into a Fourier series[11]

$$\phi(\tau) = \sum_{n=-\infty}^{\infty} \phi_n e^{-i\omega_n \tau} , \quad (19)$$

where $\omega_n = 2\pi n/\beta_c$. Performing the multiple Gaussian integrations over the coefficients ϕ_n, we obtain from Eqs. (16) and (17) the following integral equation for $\Pi(\tau)$:

$$\Pi(\tau) = \exp\left[-\frac{5U\beta_c}{\pi^2} \sum_{n=1}^{\infty} \frac{1-\cos(\omega_n \tau)}{n^2 + (U\beta_c^2/\pi^2) z \int_0^{\beta} d\tau \Pi(\tau) \alpha(\tau) [1-\cos(\omega_n \tau)]} \right] . \quad (20)$$

In what follows we confine ourselves to the case where the dissipation is due to the shunting resistance R_N across the junctions. Then the function $\alpha(\tau)$ is given by[10]

$$\alpha(\tau) = \frac{R_0}{2\pi R_N} \left(\frac{\pi}{\beta} \right)^2 \frac{1}{\sin^2(\pi\tau/\beta)} , \quad (21)$$

where $R_0 = \hbar/e^2 = 4.11\ \text{k}\Omega$.

Introducing this expression into Eq. (20) and changing the variable τ to $x = 2\pi\tau/\beta$ we obtain

$$\Pi(x) = \exp\left[-a \sum_{n=1}^{\infty} \frac{1-\cos(nx)}{n^2 + ag \int_0^{\pi} dx\, \Pi(x) [1-\cos(nx)]/[1-\cos(x)]} \right] , \quad (22)$$

where

$$a = \frac{5U\beta_c}{2\pi^2} \text{ and } g = \frac{2\pi^2 R_0 z}{5R_N} . \tag{23}$$

This equation has been solved numerically for $\Pi(x)$ by the method of successive approximations.[12] In view of the slow convergence of the Fourier series in the exponent of Eq. (22), the number of terms was taken $n = 10^3$, thus ensuring the overall accuracy of $\Pi(x)$ of the order of 1%.

Using the relation (18) in Eq. (9), we obtain the implicit equation for T_c:

$$1 = \frac{zE_J}{\pi}\beta_c \int_0^\pi \Pi(x)^{4/5} dx . \tag{24}$$

Following Ref. 3, we introduce the parameter $\alpha = zE_J/U$ and rewrite Eq. (24) as follows:

$$\frac{1}{\alpha} = \frac{2\pi a}{5} \int_0^\pi \Pi(x)^{4/5} dx , \tag{25}$$

where the integral is an implicit function of a and g.

This equation is to be solved for $a = a_c$, regarding α and g as parameters. The transition temperature T_c for the phase ordering is determined from the roots a_c using the following relation:

$$\frac{T_c}{T_c^c} = \frac{5}{2\pi^2 a_c \alpha} , \tag{26}$$

where $T_c^c = zE_J$ is the transition temperature in the classical ($U = 0$) limit. The results of the calculations are plotted in Fig. 1 for several values of the coupling parameter g. On increasing g, the phase boundaries move toward smaller values of α in keeping with the general expectation that dissipation diminishes the importance of the quantum phase fluctuations. We note that the phase diagram is rather sensitive to the presence of large shunting resistances. For instance, taking $z = 6$, we see from Eq. (23) that the $g = 1$ case corresponds to a shunting resistance $R_N = 97$ kΩ and a significant change in the phase boundary. The asymptotic limit of $g \to \infty$ is also correctly reproduced, as the phase boundaries tend to the classical limit $T_c/T_c^c = 1$ for $g \to \infty$. More interesting is the dependence of the reentrant protrusion on g. There is a continuous narrowing of the "reentrant bulge" for $0 < g < 2$, which is followed by a more rapid transition toward the case without reentrance. Such a transition would be expected in view of our recent results for the effective conductance in normal tunnel junctions.[12] We note that the conductance is determined by the phase correlation through a relation similar to Eq. (24). Moreover, the self-consistent integral equation for the phase correlator of Ref. 12 has the same form as Eq. (22). Thus, the rapid crossover toward metallic conduction taking place in a normal junction in the region $2 \leq g \leq 3$ should be reflected in the phase diagram of the present work.

We have recently learned that Chakravarty, Ingold, Kivelson, and Luther[13] had also studied the role of the dissipation in the phase transition of Josephson junction arrays. In their work the dissipative term in the trial action is taken in the harmonic form and the Josephson coupling is treated within the SCHA. Nevertheless, the resulting zero-temperature phase diagram seems to substantiate our conjecture regarding the critical value of the damping parameter g. Specifically, taking $z = 2d$, where d is the dimensionality of the array, Eq. (23) yields $g = 8\pi/5(\tilde{a}d)$, where $\tilde{a}$ corresponds to the parameter a of Ref. 13. The important conclusion of the latter work is that the zero-temperature phase diagram exhibits a vertical boundary at a critical value $(\tilde{a}d)_{\text{crit}} = 1$, corresponding to $g_{\text{crit}} \approx 5$. At this critical dissipation the system enters a regime in which the phase transition is dictated entirely by the parameter g, so that the *resistance* R_N becomes the *only relevant variable* for the onset of the global phase coherence.[14] The qualitative change in the T_c/T_c^c curves obtained in the present work may be related to this criticality of g. There is a difference in the present mean-field approach as compared with the SCHA method of Ref. 13. The MFA phase diagram is determined by the phase correlator of the isolated *normal* junction, and the criticality of g stems from the *anharmonicity* of the *dissipative* action. On the other hand, the criticality of $\tilde{a}$ seems to originate from the *anharmonic Josephson term* in conjunction with the dissipative term in the *harmonic* form. The latter is related to the crossover from diffusion to localization which has been previously demonstrated for the quantum Brownian particle with harmonic dissipation in a periodic potential.[15,16]

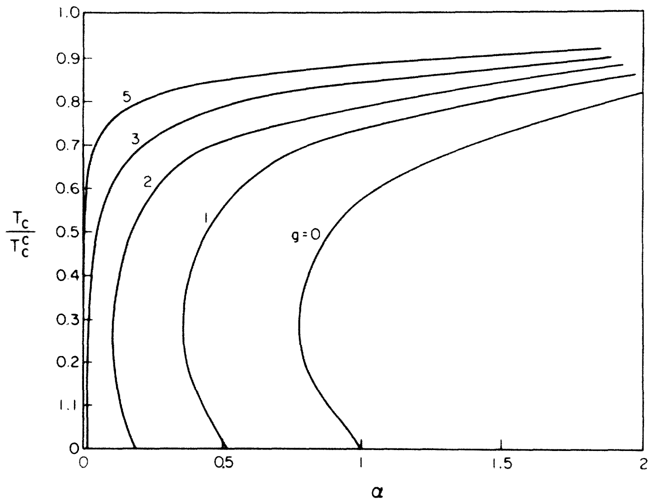

FIG. 1. Phase-ordering temperature ratio T_c/T_c^c plotted as a function of the parameter $\alpha = zE_J/U$ from Eqs. (25) and (26). The dimensionless parameter g given by Eq. (23) is inversely proportional to the shunting resistance R_N.

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