

Reanalysis of "Dilute random-field Ising models and uniform-field antiferromagnets"

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The susceptibility series presented by Aharony, Harris, and Meir [Phys. Rev. B **32**, 3203 (1985)] for the random-field Ising model and dilute antiferromagnet in a field are reanalyzed. This reanalysis utilizes improved methods of power-series analysis, more recent p_c estimates, and a redefined constant term. We also invoke updated exponent estimates for comparison with our results, and find that new estimates of γ for the two models are consistent with each other and with the scaling value from the literature estimates.

Aharony, Harris, and Meir¹ discussed the dilute random-field Ising model (RFIM) and dilute antiferromagnet in uniform field (DAFF) for general dimension, presenting an exact solution on the Cayley tree, field theoretic arguments near and above six dimensions, and series for general dimension. While exact results show that these models do not have the same critical behavior at $d=1$, they do in the Cayley tree, and Aharony, Harris, and Meir presented scaling arguments to show that this equivalence should extend down to $d=2$. Their series analysis, however, did not seem to support the extension of this equivalence to $d=2$, since they found $\gamma=0.7$ (RFIM) and $\gamma=1.25$ (DAFF), where γ is the critical exponent of the susceptibility χ ,

$$\chi \sim (p_c - p)^{-\gamma}.$$

In the present Brief Report we report on a reanalysis of

both series that uses improved methods of series analysis, allowing both for the effect of nonanalytic corrections to scaling,² and for the fact that the exponent γ is expected to be zero at $d=6$ and is small for $d > 4$. We also use the latest p_c estimates³ for bond percolation for $d > 3$, and more recent literature values of the percolation exponents γ_p (Ref. 4) and β_p (Ref. 5) for purposes of comparison. We study the series for the susceptibility

$$\chi(d, p) = \sum_{k=1}^{\infty} \sum_{l=1}^k A_{kl} d^l p^k,$$

which should behave as

$$\chi \sim \begin{cases} (p_c - p)^{-\gamma} [1 + a(p_c - p)^{\Delta_1} + \dots] & \text{for } d < 6, \quad (1a) \\ |\ln(p_c - p)|^z & \text{for } d = 6. \quad (1b) \end{cases}$$

The coefficients A_{kl} are given in Ref. 1. We note that Ref. 1 considered the series with an additional constant term,

TABLE I. γ estimates for $d < 6$.

d	p_c	$\frac{1}{2}(\gamma_p - \beta_p)$	γ_{RFIM}	γ_{DAFF}	Method
5	0.1182 (Ref. 4)	~0.19 (Refs. 3,4)	0.37 ± 0.05 0.37 ± 0.05 0.25 ± 0.05	0.38 ± 0.5 0.32 ± 0.05 0.18 ± 0.05	DA, χ DA, χ AMP II, χ^a
4	0.1603 (Ref. 4)	~0.41 (Refs. 3,4)	0.48 ± 0.05 0.48 ± 0.05 0.48 ± 0.05 0.35 ± 0.05	0.48 ± 0.05 0.48 ± 0.05 0.48 ± 0.05 0.40 ± 0.05	DA, χ AMP II, χ DA, χ AMP II, χ^a
3	0.2486 (Ref. 3)	~0.67 (Refs. 3,4)	0.67 ± 0.03 0.66 ± 0.03 0.66 ± 0.03 0.70 ± 0.10	0.735 ± 0.015 0.725 ± 0.05 0.67 ± 0.05 0.84 ± 0.15	DA, χ AMP II, χ AMP I, χ^a DA, χ
2	$\frac{1}{2}$ Exact	$\frac{9}{8}$ Exact	0.90 ± 0.15 0.95 ± 0.15 ...	1.20 ± 0.15 1.16 ± 0.15 1.20 ± 0.4	DA, χ AMP II, χ^a DA, χ
1	1	$\frac{1}{2}$	0.50 ± 0.05^b		

^aThe technique that we believe is most reliable for the chosen dimension.

^bThe exact result is $\frac{1}{2}$.

i.e., studied $1 + \chi(d, p)$, but both in order to reproduce the exact results at $d = 1$ and for consistency with the susceptibility definition this constant is unnecessary and retards convergence, and we analyze $\chi(d, p)$ as defined above.

We use several different methods of Padé-based analysis. First, we repeat the differential approximant (DA) analysis of Ref. 1, and in addition study differential approximants to χ .

While providing the best possible fit to the form $(p_c - p)^{-\gamma}$, the DA analysis is equivalent to assuming $a = 0$ in Eq. (1a). Thus we also use two methods that allow for the effect of confluent corrections to scaling ($a \neq 0$)^{2,6} and assume that Δ_1 takes the same value here as for usual percolation.³ The method used for all $d < 6$ (Ref. 2) is a generalized Roskies transform and is denoted here by AMP II. The method⁶ used for $d = 3$ is good when $\Delta_1 \sim 1$ and is denoted here by AMP I. For the higher dimensions we study

$$\chi'' \sim (p_c - p)^{-(2+\gamma)} [1 + a''(p_c - p)^{\Delta_1} + \dots]$$

instead of χ , convergence being much improved since Padé-based methods seem to be able to estimate exponents of order 2 better than exponents that are close to zero.

We quote results for $d < 6$ in Table I (which updates Table I of Ref. 1). We compare the calculated values with $(\gamma_p - \beta_p)/2$ from literature values of γ_p (Ref. 4) and β_p .⁵

The technique that is believed to be the most reliable for each dimension is indicated. For $3 \leq d \leq 5$ the results of the most reliable method agree with the literature estimate to within ± 0.02 in all cases, and $\gamma_{\text{RFIM}} = \gamma_{\text{DAFF}}$ to within the accuracy of the estimates. For $d = 2$ the results of AMP II are marginally better than those from DA. The γ_{RFIM} estimate is substantially higher than that of Ref. 1. Both γ_{RFIM} and γ_{DAFF} are now much closer to the scaling value $(\gamma_p - \beta_p)/2$.

We have also studied the logarithmic corrections at $d = 6$. Here we studied χ with the Adler and Privman⁷ method and found $1.21 < z_{\text{DAFF}} < 1.45$ and $1.21 < z_{\text{RFIM}} < 1.33$ as p_c was varied from 0.0936 to 0.0946, in agreement with the predicted 9/7.

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²J. Adler, M. Moshe, and V. Privman, Phys. Rev. B **26**, 1411 (1983).

³See the discussion in J. Adler, J. Phys. A **18**, 307 (1983) for $d > 3$; for $d = 3$ we use $p_c = 0.24586$, Grassberger (private communication via D. Stauffer).

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⁷J. Adler and V. Privman, J. Phys. A **14**, L4633 (1981).