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Integral quantum Hall effect in superlattices

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It is shown that an integral quantization of the Hall conductivity σ_{yx} occurs in superlattices when a magnetic field is applied in their direction, z, provided that (i) only the lowest miniband is occupied and its width is smaller than the energy separation between neighboring Landau levels, and (ii) the Fermi level lies in the resulting energy gaps. For very wide barriers there is no dispersion in the z direction and at zero temperature σ_{yx} is equal to $2n(N+1)e^2/h$, where n is the number of the wells and N the Landau-level index. This is also the case for very high but not too thin barriers. Corrections due to finite dispersion and finite temperatures are evaluated. Tunneling between adjacent wells can modify the results considerably or even destroy the effect depending on how wide or how high the barriers are. The results are in reasonable agreement with the available experimental data.

The existence of gaps in the energy spectrum of a two-dimensional electron (or hole) system is generally regarded as a prerequisite for the observation of the quantized Hall effect (QHE).[1-3] At very low tempeatures and high magnetic fields the hall conductivity σ_{yx} is equal to ie^2/h, where i is an integer. Quantization of the Hall conductivity has also been observed in superlattices which contain a number of identical, strictly two-dimensional (2D) electronic systems, separated from one another by impenetrable barriers;[4] in this case σ_{yx} is equal to jie^2/h, where j is the number of the 2D systems (wells).

In all these cases, the electronic spectrum is purely two dimensional without any dispersion in the direction normal to the 2D plane. However, recently the QHE has been observed in GaAs-(Al,Ga)As superlattices, with a finite dispersion in the direction normal to the 2D plane, and with barrier widths and heights of about 40 Å and 135 meV, respectively.[5] The deviations from the traditional QHE, observed in 2D GaAs-(Al,Ga)As structure have been discussed in Ref. 5, the main one being that σ_{yx} is equal to $j'ie^2/h$, with $j'<j$. In such superlattices strong tunneling is expected[5] between neighboring wells and the distinction between layers is not very good. It is therefore of interest to study the influnece of tunneling and of finite dispersion (in the superlattice direction) on the QHE. In this Communication we study this influence on the QHE quantitatively. The results obtained are mentioned in the abstract.

To start with, let us consider an electron system described by the Hamiltonian

$$H=H^0+\lambda V-\mathbf{A}\cdot\mathbf{F}(t) \ . \tag{1}$$

In Eq. (1) H^0 is the largest part of H which can be diagonalized (analytically), λV is the interaction ($\lambda V \ll H^0$), and $-\mathbf{A}\cdot\mathbf{F}(t)$ is the external field Hamiltonian. When an electric field $\mathbf{E}(t)$ is applied, $\mathbf{F}(t)=q\mathbf{E}(t)$, $\mathbf{A}=\sum_i(\mathbf{r}_i-\mathbf{r}_{\mathrm{eq}})=\sum_i\mathbf{a}_i$, q is the charge of the carriers (electrons), and $\mathbf{r}_{\mathrm{eq}},\mathbf{r}_i$ are the positions of the carriers before and after the application of the electric field.

In our formalism,[6-8] H^0 contains the periodic part of the interaction λV, expressed by the effective mass m^*. The electrical conductivity is given as a sum of two terms: $\sigma_{\mu\nu}=\sigma^{\mathrm{I}}_{\mu\nu}+\sigma^{\mathrm{II}}_{\mu\nu}$; $\sigma^{\mathrm{I}}_{\mu\nu}$ depends on the interaction (nonperiodic part) but $\sigma^{\mathrm{II}}_{\mu\nu}$ does not. In the presence of a magnetic field (included in H^0) σ^{I}_{yx} is shown to vanish identically.[6,7] Thus the Hall conductivity is given by $\sigma^{\mathrm{II}}_{yx}$. Its dc version reads [cf. Ref. 7, Eq. (2.11)]

$$\sigma_{yx}=(ie^2\hbar/V_0)\sum_{\substack{\zeta_1,\zeta_2\\ \mathrm{spin}}}[(1-e^{\beta\Delta})/\Delta^2]\langle n_{\zeta_2}\rangle_{\mathrm{eq}}[1-\langle\eta_{\zeta_1}\rangle_{\mathrm{eq}}](\zeta_2|\dot{\alpha}_x|\zeta_1)(\zeta_1|\dot{\alpha}_y|\zeta_2),\ \Delta=\varepsilon_{\zeta_2}-\varepsilon_{\zeta_1} \ . \tag{2}$$

In Eq. (2) $|\zeta)$ and ε_ζ are the eigenstates and eigenvalues of the one-electron Hamiltonian h^0 ($H^0=\sum h^0$), respectively; $\langle n_\zeta\rangle_{\mathrm{eq}}$ is the average occupancy of the state $|\zeta)$, given by the Fermi-Dirac distribution function, and $V_0=L_xL_yL_z$ is the volume of the system. Equation (2) leads to the usual Hall effect at high temperatures,[6] to the oscillatory Hall effect at very low temperatures,[8] and to the integral QHE.[7]

Equation (2) will be used to evaluate the Hall conductivity σ_{yx} in a GaAs-(Al,Ga)As superlattice when a magnetic field B is applied in its direction ($\mathbf{B}=B\hat{\mathbf{z}}$). The superlattice consists of n identical wells (GaAs) of width d, separated by n identical barriers [(Al,Ga)As] of width b and constant height W, for $|z-j(d+b)|\le b/2$, where j is an integer ($1\le j\le n$). In the Landau gauge the one-electron Hamiltonian, states, and eigenvalues read ($\mathbf{p}$ is the momentum operator)

$$h^0=(\mathbf{p}+e\mathbf{A})^2/2m^*+W,\ \mathbf{A}=(0,Bx,0) \ , \tag{3}$$

$$|\zeta)\equiv|N,k_y)\otimes|n,k_z)$$
$$=\phi_N(x+l^2k_y)(e^{ik_yy}/L_y^{1/2})\otimes|n,k_z) \ , \tag{4}$$

$$\varepsilon_\zeta=(N+\tfrac{1}{2})\hbar\omega_0+\varepsilon(k_z),\ N=0,1,2,\ldots, \tag{5}$$

where $\omega_0 = eB/m^*$ is the cyclotron frequency, $l^2 = \hbar/m^*\omega_0$, $\mathbf{A}$ is the vector potential, and where $|n,k_z)$ is the wave function in the z direction.[9] $\phi_N(x)$ are the harmonic-oscillator wave functions, N is the Landau-level index, and k_y, k_z are the wave vectors in the y and z direction, respectively. Using Eq. (4) and setting $l' = l\omega_0/(2)^{1/2}$ we obtain

$$(\zeta|\dot{\alpha}_x|\zeta') = il'[-(N+1)^{1/2}\delta_{N',N+1} + (N)^{1/2}\delta_{N',N-1}](n,k_z|n',k_z')\delta_{k,k'} , \quad (6)$$

$$(\zeta|\dot{\alpha}_y|\zeta') = l'[(N+1)^{1/2}\delta_{N',N+1} + (N)^{1/2}\delta_{N',N-1}](n,k_z|n',k_z')\delta_{k,k'} , \quad (7)$$

where $\delta_{k,k'} = \delta_{k_y,k_y'}\delta_{k_z,k_z'}$. Using Eqs. (5)–(7) and proceeding as in Ref. 7, Sec. II, we find that for N fully occupied Landau levels σ_{yx} is given by

$$\sigma_{yx} \approx (2e^2/hL_z) \sum_{N,n,k_z} \{|(n,k_z|n,k_z)|^2 + [|(n,k_z|n+1,k_z)|^2 + |(n,k_z|n-1,k_z)|^2]/2\} \times (N+1) f_{N,k_z}(1 - f_{N+1,k_z})(1 - e^{-\beta\hbar\omega_0}) , \quad (8)$$

where $f_{N,k_z} = \langle n_{N,k_z}\rangle_{\text{eq}}$. This result has been obtained by writing

$$\sum_{n'} \phi(n') = \phi(n) + \sum_{n' \neq n} \phi(n') \approx \phi(n) + \phi(n-1) + \phi(n+1) ,$$

i.e., by assuming that the parameters d, b, and W are such that only the wave functions centered on adjacent wells have a significant overlap. The stronger this overlap the larger the tunneling probability is. Now the quantities $I(k_z) = |(n,k_z|n \pm 1,k_z)|^2$ are independent of the well index n (see below). Using the normalization of $|n,k_z)$ and observing that

$$f_{N,k_z}(1 - f_{N+1,k_z})\exp(-\beta\hbar\omega_0) = (1 - f_{N,k_z}) f_{N+1,k_z} ,$$

we can write[10] Eq. (8) as

$$\sigma_{yx} \approx 2n(e^2/hL_z) \sum_{N,k_z} [1 + I(k_z)] f_{N,k_z} . \quad (9)$$

The integral $I(k_z)$ has been evaluated approximately in Ref. 11 [see Eq. A(11) for $q_z = 0$] and the result, when only the lowest miniband (assumed to be very narrow) is occupied, reads

$$I(k_z) \approx I(k_1) = 4\left(\frac{(a)^{1/2}Wb + 4(W-\varepsilon_1)^{1/2}}{(a)^{1/2}Wd + 2(W-\varepsilon_1)^{1/2}}\right)^2 \times (\varepsilon_1/W)^2 \exp\{-2b[a(W-\varepsilon_1)]^{1/2}\}, \quad (10)$$

where ε_1 is the energy of the miniband $\hbar^2 k_1^2/2m^*$ and $a = 2m^*/\hbar^2$.

To proceed further we need to know the k_z disperison of the energy spectrum. We evaluate it numerically using the Kronig-Penney model which entails that we neglect a small difference in m^* between GaAs and (Al,Ga)As and the effect of band bending due to charge transfer from (Al,Ga)As to GaAs layers.[12] The effect of the nonparabolicity of the conduction band is approximately incorporated into the dispersion relation in the manner of Ref. 13. We consider separately the cases of zero and finite dispersion $\Delta\varepsilon_z \equiv \Delta\varepsilon(k_z)$. We assume throughout that only the lowest miniband is occupied. In Fig. 1 we plot the unperturbed density of states for $\Delta\varepsilon_z = 0$:

$$N(\varepsilon) = (A_0/\pi l^2) \sum_N \delta(\varepsilon - (N + \tfrac{1}{2})\hbar\omega_0 - \varepsilon_0) ,$$

(dashed lines), and for $\Delta\varepsilon_z \neq 0$ in the tight-binding model[14]

$$\varepsilon(k_z) = \varepsilon_0 - (\Delta/2)\cos[k_z(d+b)] ,$$

$$N(\varepsilon) = (4m^* A_0/\hbar^2) \sum_N \{(\Delta/2\hbar\omega_0)^2 - [N + \tfrac{1}{2} + (\varepsilon_0 - \varepsilon)/\hbar\omega_0]^2\}^{-1/2} ,$$

(hatched regions). In the second case, ε_0 is the middle of the miniband whose width is Δ and

$$|(N + \tfrac{1}{2})\hbar\omega_0 + \varepsilon_0 - \varepsilon| \leq \Delta/2 < \hbar\omega_0/2 .$$

A_0 is the area: $A_0 = L_x L_y$ [the exact form of $N(\varepsilon)$ depends on the parameters b and W which modify $\varepsilon(k_z)$].

(i) $\Delta\varepsilon_z = 0$. As shown in Fig. 2, this is the case of wide and not too low barriers ($b \geq 120$ Å, $W \approx 85$ meV) or of high but not too thin barriers ($W \geq 200$ meV, $b \geq 40$ Å). $I(k_z)$ and f_{N,k_z} are independent of k_z. At zero temperature Eq. (9) becomes [$\sum_{k_z} \to L_z \int_{-k_0}^{k_0} dk_z/2\pi$, $k_0 = \pi/(d+b)$],

$$\sigma_{yx} = 2n(N+1)[1 + I(k_1)]e^2/h . \quad (11)$$

In the presence of interactions (λV) we assume, in analogy with the two-dimensional case,[15,16] that the unperturbed energy levels (dashed lines in Fig. 1) are broadened into bands whose center consists of extended states while localized states exist at the top and bottom of these bands. Equation (11) then makes the quantization of σ_{yx} ap-

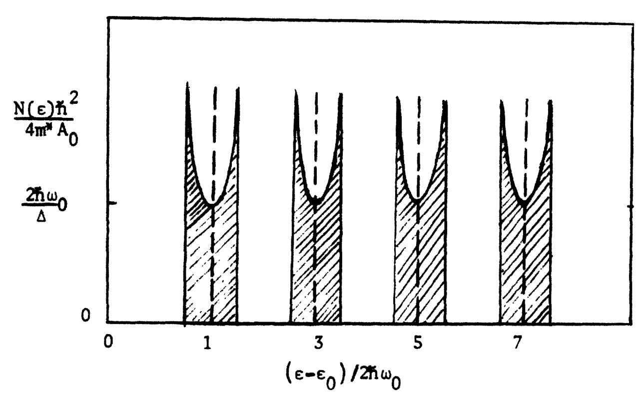

FIG. 1. Plot of the unperturbed density of states $N(\varepsilon)$ vs $(\varepsilon - \varepsilon_0)/2\hbar\omega_0$. The width Δ of the miniband is zero (dashed lines) or finite (hatched regions, $\Delta < \hbar\omega_0$).

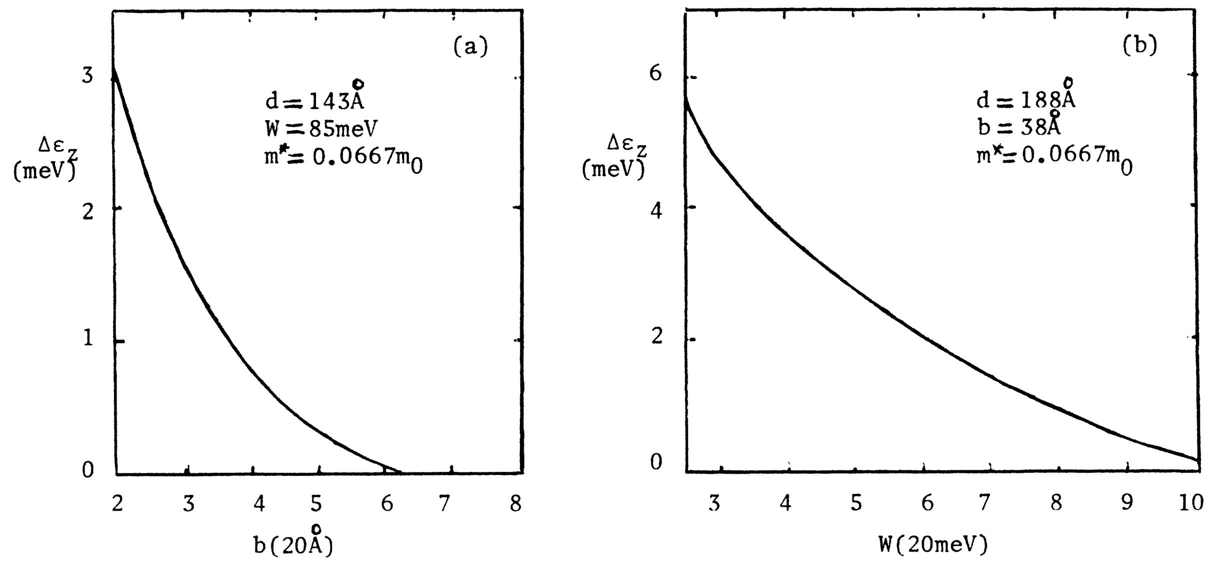

FIG. 2. Lowest miniband width $\Delta\varepsilon_z$ as a function of (a) the barrier's width b and (b) the barrier's height W.

parent whenever the Fermi level lies in the region of these localized states as the magnetic field varies. Since $I(k_1)$ is independent of the magnetic field its value will shift the quantization but it will not affect it further.

At finite temperatures and strong magnetic fields we assume $\exp(-C)\ll 1$, $C=\beta(\varepsilon-(N+\frac{1}{2})\hbar\omega_0-\varepsilon_1)$. Equation (11) must then be multiplied by $1-\exp(-C)$, as is easily seen by expanding f_{N,k_z}. As in the two-dimensional case:[7] (a) the deviation $\Delta\sigma_{yx}(T)$ from the zero temperature value, $\sigma_{yx}(0)$, shows an activated behavior[5] with activation energy C/β and (b) the plateaus of Eq. (11) shrink with increasing temperature due to the "washing out" of the Fermi function, cf. Eq. (9).

The data of Ref. 4 have been obtained at $T=0.2$ K. We assume that $b=d=143$ Å and $W=85$ meV (b and W are not given in Ref. 4). We obtain $\varepsilon_1\approx 14$ meV, $I(k_1)\approx 6\times 10^{-5}$ and $e^{-C}\approx 10^{-4}-10^{-6}$ for $\varepsilon_N+\varepsilon_1<\varepsilon_F<\varepsilon_{N+1}+\varepsilon_1$. Assuming $\sigma_{xx}=0$ ($\rho_{xx}\sim 10^{-10}\Omega/\square$), we find from Eq. (11) that $\rho_{yx}\approx 1/\sigma_{yx}=h/ine^2$, within one part in 10^4 which is the reported accuracy [$i=2(N+1)$, spin included, $n=172$].[4]

(ii) $\Delta\varepsilon_z\neq 0$. As shown in Fig. 1 (hatched regions) the Landau levels develop into bands which are assumed to be broadened in the presence of interactions. The energy spectrum again exhibits gaps when the miniband width $\Delta\varepsilon_z$ is smaller than $\hbar\omega_0$ and the reasoning of (i) can be repeated.

In Fig. 3 we show the dispersion relation in the z direction for the first two bands; the parameters shown are taken from Ref. 5. $\Delta\varepsilon_z$ is about 2.5 meV for the lowest miniband. The bands are broadened slightly when the nonparabolicity of the conduction band is taken into account. For example, using Eqs. (7) and (9) of Ref. 13 with $E_T=(N+\frac{1}{2})\hbar\omega_0+\varepsilon_z$, at $B=8.6$ T, and $E_{g_2}=5E_{g_1}$, we find that the bandwidths increase by about 15% and 10%, respectively. The effect is weaker for smaller B or $E_{g_2}>5E_{g_1}$.

The data of Ref. 5 have been obtained at $T=0.15$ K. Using the reported activation energy, $C/\beta=0.26$ meV, we obtain $e^{-C}\sim 10^{-8}$ and 10^{-7} for $\varepsilon(k_z)$ evaluated at the bottom and top of the miniband, respectively. For $N=0$ we have $\hbar\omega_0\approx 15$ meV. Since the bandwidth $\Delta\varepsilon_z\approx 2.5$ meV is small we again use Eq. (10) for $I(k_z)$ and obtain $I(k_z)\approx 1.5\times 10^{-4}$.

As Fig. 3 shows the dispersion relation is almost linear and can be reasonably well described by $\varepsilon(k_z)\approx\alpha k_z$, $\alpha=(2.5\text{ meV})(d+b)/\pi$. Equation (9) then, for $N=0$, gives (within one part in 10^4) $\sigma_{yx}\approx 60e^2/h$ whereas the experimental result for ρ_{yx}, assuming $\sigma_{xx}=0$, leads to $\sigma_{yx}=48e^2/h$ to five parts in 10^5. The result $\sigma_{yx}=60e^2/h$ remains unaffected (to one part in 10^4) when the nonpara-

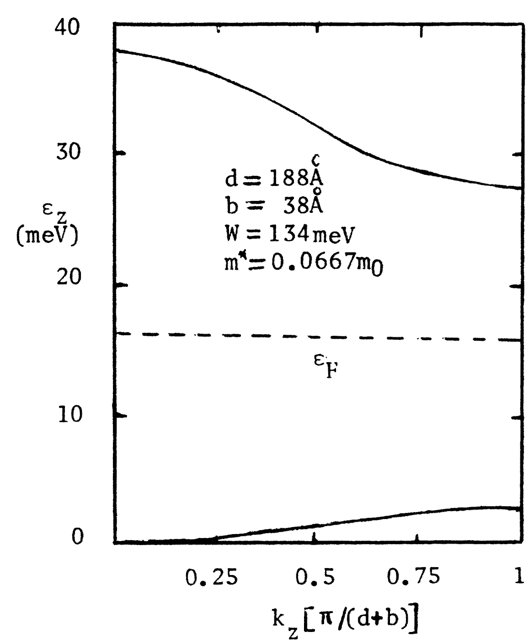

FIG. 3. Dispersion relation in the z direction; ε_z is measured from the bottom of the first miniband.

bolicity of the conduction band is taken into account. Inclusion of electron-electron interaction leaves Eq. (2) unchanged[17] but, in general, broadens the bands of the Kronig-Penney model[12] slightly. This could affect slightly the activation energy but not the value $60e^2/h$. The discrepancy between these two values of σ_{yx} could be probably associated with depletion of several top and bottom layers of the superlattice due to pinning of the Fermi level, as discussed in Ref. 5.

In the examples considered here the effect of tunneling, as reflected by Eq. (10) and the Kronig-Penney model, is very weak and does not seriously affect the reported accuracy of the QHE (which, however, is poorer than in the two-dimensional case). However, as Eq. (10) indicates and as one intuitively expects, tunneling can drastically affect the accuracy of the effect or even destroy it when one reduces the width or the height of the barrier. In this case $\Delta\varepsilon_z$ increases (cf. Fig. 2), the gaps of Fig. 1 shrink, and Eq. (10) becomes a progressively poorer approximation.

In conclusion, we have shown that the QHE can occur in three-dimensional[18] systems when their energy spectrum exhibits gaps and the Fermi levels lies in them. For superlattices this happens for values of the barrier width and height such that the bandwidth is much smaller than $\hbar\omega_0$; in this case the effect of tunneling between adjacent wells affects the accuracy to about one part in 10^4. The results are in reasonable agreement with the experimental data.

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[18]Experimentally, the three-dimensional character of the sample of Ref. 5 is proven by measuring a finite current in the direction of the superlattice.