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Statistical model for inhomogeneities in a two-dimensional electron gas implying a background density of states between Landau levels

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We discuss a statistical model for a spatially inhomogeneous two-dimensional electron gas in a quantizing magnetic field which simulates the effect of Poisson's equation and some essential properties of self-consistent screening. The model yields an effective background density of states between Landau levels and may explain a number of recent experimental observations.

Simple theories of a two-dimensional electron gas (2 DEG) in a quantizing magnetic field predict for the density of states (DOS) broadened Landau levels (LL's) of an elliptical or a Gaussian shape, depending on the approximation in which the interaction of the electrons with randomly distributed scatterers is taken into account.[1,2] For a sufficiently strong magnetic field B, the LL's are energetically well separated and the DOS is expected to be zero or exponentially small in the gap between the LL's. A number of recent experiments[3-12] produce, however, strong evidence for an unexpectedly large DOS in the Landau gaps for the 2 DEG in both GaAs-(GaAl)As heterostructures and silicon metal-oxide-semiconductor structures. For the evaluation of the thermally activated transport in the quantum Hall regime[3-7] the following model DOS has been used:

$$D(E;x)=(1-x)\left[\frac{2}{2\pi l^2}\sum_{n=0}^{\infty}G(E;\varepsilon_n,\Gamma)\right]+xD_0\,, \quad (1)$$

where only a fraction $1-x$ of the total states is described by a Gaussian-shaped Landau DOS. Here $\varepsilon_n=\hbar\omega(n+\frac{1}{2})$ is the spin-degenerate Landau-level energy, $1=(\hbar c/eB)^{1/2}$ the magnetic length, Γ the level broadening, and

$$G(z;\bar{z},\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma}\right)^2\right]\,, \quad (2)$$

a normalized Gaussian distribution. A constant background is represented by a fraction x of the zero-field DOS $D_0=m/\pi\hbar^2$. Measurements of equilibrium properties such as magnetization,[8] specific heat,[9] or capacitance[10] seem to support this picture, although the physical origin of the background DOS remains unclear.

On the other hand, the importance of inhomogeneities has been emphasized,[11,12] especially for an understanding of the measured capacitance,[6,11] and a statistical model with a Gaussian distribution of the electron density has been proposed[11,12] in order to describe their effect. In the present Communication we discuss several aspects of this statistical inhomogeneity model and demonstrate that the statistical averaging procedure can create an effective background DOS between the Landau levels. Thus, this model might provide a physical explanation for the apparent background DOS observed in many experiments. A systematic model study of several quasiequilibrium properties of the quasi-two-dimensional electron system in GaAs-(AlGa)As heterostructures is left for a future publication. Here we consider a strictly two-dimensional model with given electron density n_s, i.e., given mean value $\bar{n}_s$ in the statistical model.

The physical situation one has in mind is a heterostructure which contains subregions with different n_s values of the 2 DEG in the GaAs, for instance, owing to large-scale fluctuations of the donor concentration in the (AlGa)As barrier. These "subregions" are assumed so large that they can be treated as homogeneous systems with well-defined, constant thermodynamic variables. The statistical model considers the electron density n_s as a random variable and replaces the physical observables of the inhomogeneous system by the average values of the corresponding observables over an ensemble of homogeneous systems with the same DOS functions $D(E)$ but with different densities. For the homogeneous systems one has the usual functional relationship

$$n_s=N(\mu)\equiv\int_{-\infty}^{\infty}dE\,D(E)f(E-\mu) \quad (3)$$

between density and electrochemical potential μ, with $f(E)=[\exp(E/kT)+1]^{-1}$ the Fermi function. Thus, together with n_s, μ is also a random variable and for a given probability distribution $G(n_s)$ of n_s one can calculate the corresponding distribution,

$$P(\mu)=G(N(\mu))\frac{dN}{d\mu} \quad (4)$$

of μ and vice versa.

In an inhomogeneous physical sample, the electrochemical potential has a constant value throughout the sample. Fluctuations of μ must then be interpreted as fluctuations of the electrical subband energy along the interface owing to local fluctuations of the depth of the electrostatic potential well confining the electrons to the interface. In a real system, spatial fluctuations of the electrostatic potential and those of the electron density must be consistent with Poisson's equation. This physical aspect is not included in the statistical model, which considers an ensemble of independent homogeneous "subregions" rather than spatial fluctuations within a single sample. For $B=0$, the two-dimensional DOS is constant, $D(E)=D_0\theta(E)$, Eq. (3) reduces (for $\mu\gg kT$) to the linear relation $n_s=D_0\mu$, and both n_s and μ have the same probability distribution. In a strong magnetic field, however, Eq. (3) is strongly nonlinear, and according to Eq. (4) only one of the random variables, either n_s or μ, can be Gaussian distributed (cf. Fig. 1). Consequently, we consider two different versions of the Gaussian statistical model for $B \neq 0$.

We define the "n_s-Gaussian" model by the Gaussian distribution $G(n_s;\bar{n}_s,\delta n_s)$ of the density with B-independent fluctuation δn_s. As seen from Fig. 1, this model leads to a pinning of the most probable values of μ to the LL's, since the thermodynamical density of states (TDOS) $dN/d\mu$ is practically zero in a finite range of μ values between the LL's. As a consequence, the mean value $\langle dN/d\mu \rangle$ of the TDOS is finite (not exponentially small) even if the mean value $\bar{\mu}$ ($\equiv \langle \mu \rangle$) of the chemical potential is well between the LL's. Thus, $\langle dN/d\mu \rangle$ plotted as a function of $\bar{\mu}$, which are the quantities the statistical model attributes to an inhomogeneous system, will show a nonzero background between the LL's. For a homogeneous system, on the other hand, one would plot $dN/d\mu$ vs μ and see no background.

In a real inhomogeneous sample we do not expect such a pinning of μ, which would imply large potential variations of the order of $\hbar\omega$ between adjacent subregions of the sample pinned to different LL's, whereas within each subregion the variation of the potential must be much less than $\Gamma \ll \hbar\omega$ to keep the density fluctuation small. Such a behavior of the electrostatic potential would result in a large curvature at the boundaries of the subregions and be in conflict with Poisson's equation.

As an alternative, we consider the "μ-Gaussian" model with a Gaussian distribution $G(\mu;\bar{\mu},\delta\mu)$ of μ and assume again that δn_s, not $\delta\mu$, is independent of B. If we would assume a small $\delta\mu$ independent of B and $\bar{\mu}$ between two LL's, we would obtain $\delta n_s = 0$ (cf. Fig. 1), that means constant n_s in an inhomogeneous sample, and no local screening of the fluctuating donor charges. This again conflicts with Poisson's equation which then requires large $\delta\mu$ in contradiction to the assumption. Poisson's equation requires large but smooth variations of the electrostatic potential, nonzero δn_s and, for $\bar{\mu}$ in a Landau gap, a considerable probability for n_s being pinned to an even integer value of the filling factor $\nu = 2\pi l^2 n_s$. These requirements are simulated by the μ-Gaussian model which for a Gaussian DOS $D(E;0)$ [cf. Eq. (1)] yields

$$\langle n_s \rangle \equiv \langle N(\mu) \rangle = \int_{-\infty}^{\infty} dE\, D_{\text{eff}}(E) f(E-\bar{\mu}) , \tag{5}$$

where $D_{\text{eff}}(E)$ is given by $D(E;0)$ with Γ being replaced by

$$\Gamma_{\text{eff}} = [\Gamma^2 + (\delta\mu)^2]^{1/2} . \tag{6}$$

One also finds $\langle dN/d\mu \rangle = \partial \bar{n}_s / \partial \bar{\mu}$ for the average value of the TDOS.

Qualitatively, the physical content of the μ-Gaussian model is easily seen within the simple approximation

$$\delta n_s \approx \left\langle \frac{dN}{d\mu} \right\rangle \delta\mu , \tag{7}$$

which leads to an effective DOS with a broadening parameter

$$\Gamma_{\text{eff}} = [\Gamma^2 + (\delta n_s / \langle dN/d\mu \rangle)^2]^{1/2} . \tag{8}$$

Thus, the linewidth of the effective DOS depends on the value of the TDOS at the chemical potential $\bar{\mu}$. Figure 2 shows a typical result obtained from the self-consistent solution of Eqs. (5), (6), and (8). The effective DOS shows narrow, well-separated peaks if $\bar{\mu}$ is in the center of a level but broad overlapping peaks for $\bar{\mu}$ in a Landau gap. The resulting TDOS as function of $\bar{\mu}$ exhibits an apparent

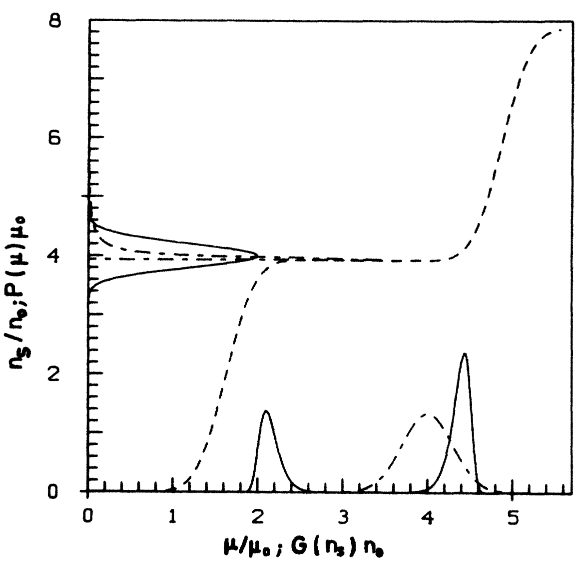

FIG. 1. The electron density n_s as a function of chemical potential μ in the region of the lowest two Landau levels is shown as dashed line. The solid (dash-dotted) lines indicate a Gaussian distribution of $n_s(\mu)$ and the corresponding distribution of $\mu(n_s)$. The parameters used are $m = 0.067 m_0$, $B = 5$ T, $T = 1.64$ K, $\Gamma = 0.3 \times \sqrt{5}$ meV, $\delta n_s / \bar{n}_s = 0.05$, and $\bar{\mu} = 10.7$ meV. The normalization constants are $n_0 = \frac{1}{4}\bar{n}_s$ and $\mu_0 = \frac{1}{4}\bar{\mu}$.

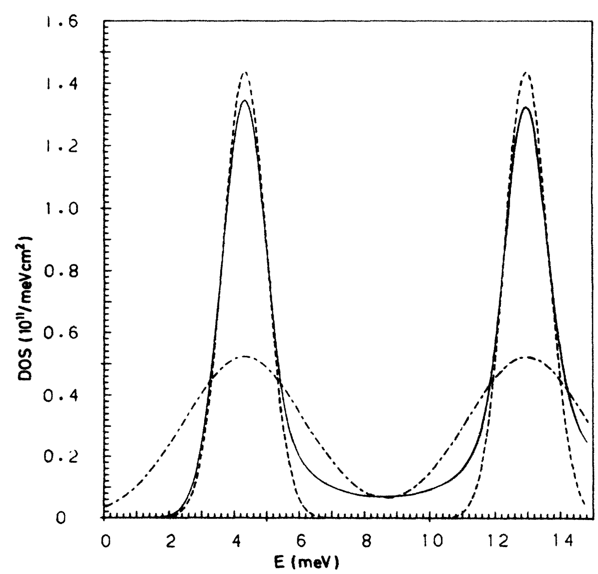

FIG. 2. Average TDOS $\langle dN/d\mu \rangle$ vs $\bar{\mu}$ (solid line) and D_{eff} vs E for $\bar{\mu}$ in the center of the lowest Landau level (dashed line), and of the lowest Landau gap (dash-dotted line), calculated using Eq. (7); parameters as in Fig. 1.

background.

The simple approximation of Eq. (7) nicely demonstrates that the μ-Gaussian model for inhomogeneities leads to a self-consistency relation for the average TDOS and the linewidth of the effective DOS. This aspect is similar to recent self-consistent calculations of the level broadening due to scattering of the electrons by charged impurities.[13-15]. The screening of the impurity potentials, which determines the level broadening and therefore the DOS, depends itself on the DOS at the Fermi level. The calculated level broadening is an oscillating function of the Fermi energy[13,14] and becomes large if the Fermi energy is well between two LL's. Values of the level broadening which correspond to an overlap of adjacent LL's have even been reported.[14,15] This indicates a finite DOS in the Landau gap. The calculations are, however, based on an oversimplified approximation which yields an elliptical line shape for the broadened LL's and, in the case of overlap, unrealistic sharp structures but no flat regions of the DOS between adjacent levels.[2]

For an accurate numerical evaluation of the μ-Gaussian model, the simple approximation, Eq. (7), is not appropriate. For instance, it does not take into account that $\delta\mu$ has an upper bound, say $\delta\mu \lesssim \hbar\omega$, and thus overestimates the value of the apparent background DOS. In Fig. 3 we compare the calculated TDOS for this simple approximation with that for the μ-Gaussian model and that for the n_s-Gaussian model.

Figure 3 demonstrates that both versions of the statistical model for inhomogeneities, which take the density fluctuation δn_s independent of the magnetic field and assume either n_s or μ to be Gaussian distributed, lead to an apparent background in the average TDOS as function of the chemical potential $\bar{\mu}$. The magnitude of the calculated background DOS and also its dependence on the input linewidth Γ [cf. Eq. (1)], is different for the two distributions, as shown in Fig. 4. This can be understood from Fig. 1. For $\bar{\mu}$ in a Landau gap, the μ-Gaussian model emphasizes the tail regions of the LL's, where the DOS is small and an increasing function of Γ. The n_s-Gaussian model, on the other hand, collects contributions from the interior of the Landau peaks, where the DOS is large and increases with decreasing Γ.

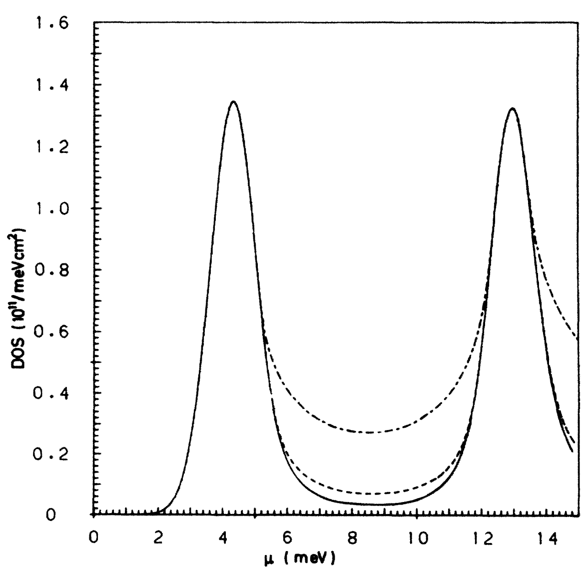

FIG. 3. Average TDOS as a function of average chemical potential for the n_s-Gaussian model (dash dotted), for the μ-Gaussian model (solid curve), and for the simple approximation Eq. (7) (dashed curve), respectively. Parameters as in Fig. 1.

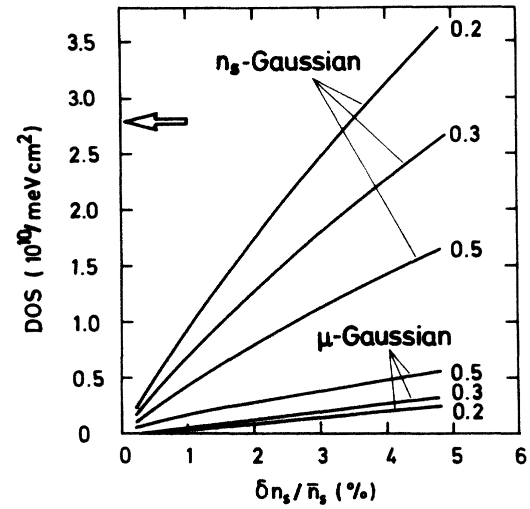

FIG. 4. Effective background TDOS at filling factor $\nu = 2$ as a function of the percentage fluctuation of n_s for different damping $\Gamma = \gamma\sqrt{5}$ meV with $\gamma = 0.2$, 0.3, and 0.5 as indicated in the figure. The parameters m, B, and T are the same as in Fig. 1.

We have checked that typical values of the background DOS determined from experiments can be reproduced by both models with reasonable assumptions for δn_s. A systematic comparison with the different experimental observations[3-11] must, however, be left for a detailed future investigation. It is clear that both versions of the statistical model are simplified limiting cases which cannot fully account for the effects of spatial inhomogeneity in a real sample, where the distribution and the fluctuation of n_s and μ (i.e., the electrostatic potential) are related by Poisson's equation and depend on the magnetic field.

As we discussed, the n_s-Gaussian model which has been applied previously[6,11,12] is in conflict with the requirements of Poisson's equation. Moreover, activation energies much larger than the level width Γ, as are observed in experiments,[4-6] are hard to understand within this model, since pinning of the Fermi level to the LL's means that in the "subregions" thermally activated processes take place within a LL. These problems do not occur with the μ-Gaussian model which yields an oscillating level broadening similar to recent self-consistent screening approaches,[13-15] but in contrast to the approximations used in the latter, also a reasonable, flat background DOS between adjacent LL's. Therefore, there is some hope that this relatively simple μ-Gaussian statistical model, which to the best of our knowledge has not been considered before in the literature, may describe some aspects of real inhomogeneous samples reasonably well.

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